An Operability-Based Methodology for the Feasible Output Ranges in the Control of Non-Square Systems

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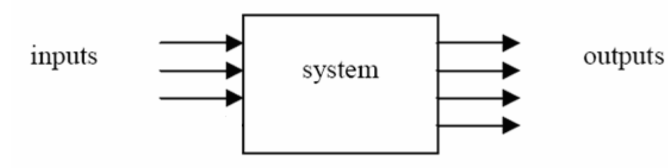
Presentation Outline

- Problem Definition and Motivation
- Motivating Example
- Objectives & Proposed Approach
- Process Operability
  - Interval Operability
    - Iterative Methodology
    - Industrial Example
- Conclusions

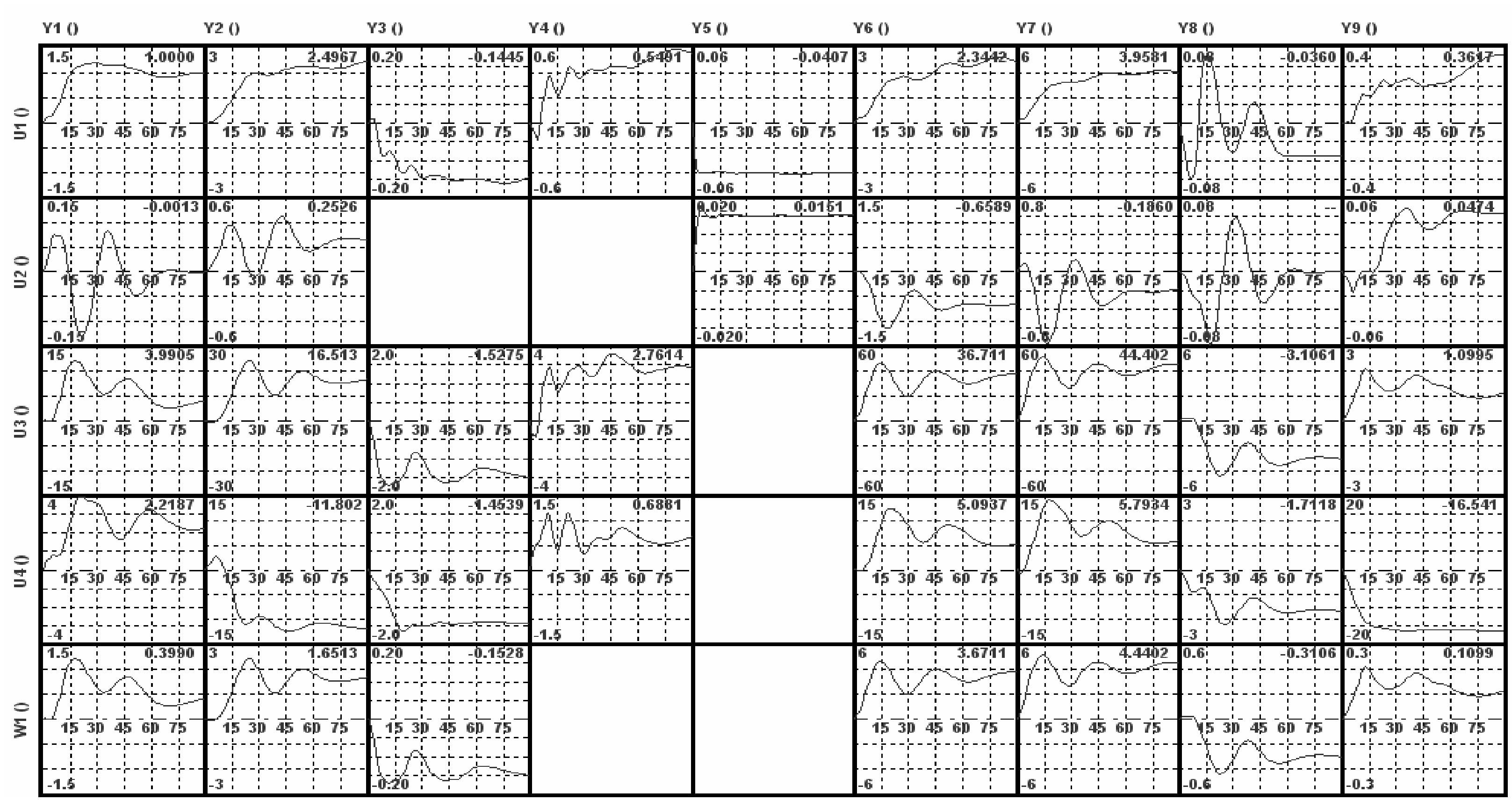
Problem Definition and Motivation



- Non-square Systems:
  - More Outputs (CVs) than Inputs (MVs)
  - Set-point Control is NOT Possible for ALL Outputs
    - Fewer Degrees of Freedom
  - ***Interval Control is Needed***
- Model Predictive Control (MPC):
  - Very Tight Constraints Make Control Infeasible
    - Soften Output Constraints*
- Need Methodology for Design of Non-square Controllers

(*) Rawlings, J. B. (2000). *IEEE Control Systems Magazine*, *20*(*3*), 38-52.

Motivating Example: Steam Methane Reformer (SMR): Step Response Model*



Dynamic Matrix for the SMR problem (DMCplus™ - AspenTech)

(*) from David R. Vinson, PhD Thesis at Lehigh University

SMR: Nominal Feasible Constraints

MV/CV	Low Limit	High Limit
u_1	-19	19
u_2	-40	40.0
u_3	-0.9	0.9
u_4	-0.85	0.85
y_1	-1.35	1.35
y_2	-67.2	67.2
y_3	-0.7	0.7
y_4	-21.5	21.5
y_5	-0.1	0.1
y_6	-6.75	6.75
y_7	-80.65	80.65
y_8	-2.15	2.15
y_9	-15.6	15.6

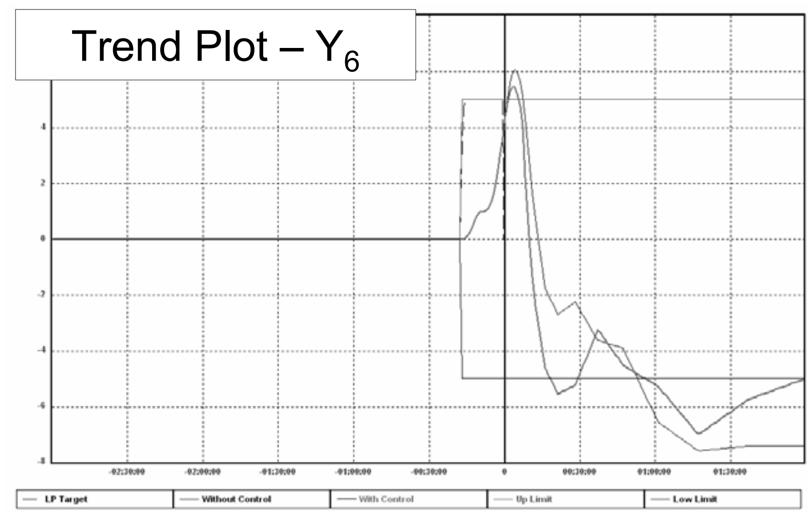
Feasible Set of Input and Output Constraints for the SMR problem

DMCplus^{TM} Simulations – Case 1

- ► Making the Y_6 constraints tighter:

CV	Original Low Limit	Original High Limit	New Low Limit	New High Limit
y_6	-6.75	6.75	**-5.00**	**5.00**
y_7	-80.65	80.65	-80.65	80.65

- ► ***Problem Becomes infeasible***
  - DMCplus^{TM} Controller:

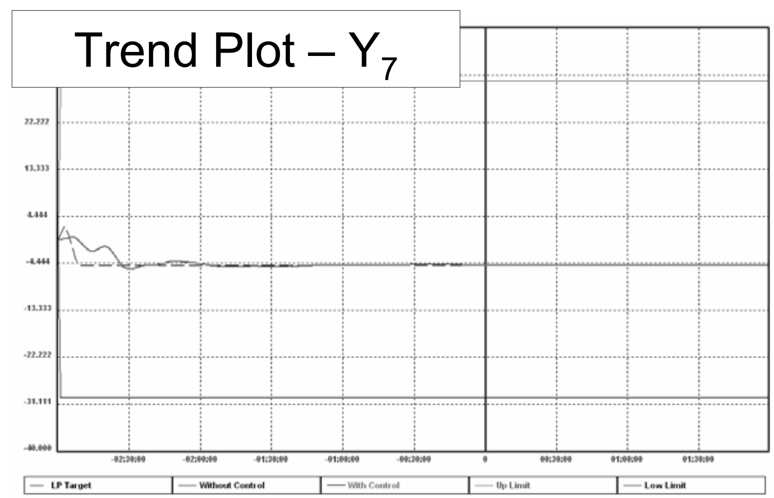


DMCplus™ Simulations – Case 2

- Making the Y_7 constraints tighter:

CV	Original Low Limit	Original High Limit	New Low Limit	New High Limit
y_6	-6.75	6.75	-6.75	6.75
y_7	-80.65	80.65	**-30.00**	**30.00**

- Problem Feasible!
- Need Methodology to Design Output Bounds



Objectives and Proposed Approach

- Development of a Process Operability Methodology
  - For Multivariable Non-square Systems

- Process Operability Concept
  - Analysis for Square Systems*
  - Extended to Non-square Linear Systems:
    - Can be Used in Choosing Possible Ranges for Outputs
  - Application Examples Using Non-square Systems
    - $n \geq m + 1$ (n: # outputs, m: # inputs, 1 disturbance)

(*) Vinson, D. R.; Georgakis, C. (2000). *Journal of Process Control*, *10*, 185-194.

Process Operability Definition

- Vinson and Georgakis (2000)*:
  - Process is Operable if ...
    - Available Inputs are Capable of Satisfying Desired Steady-state & Dynamic Performance Requirements in Presence of Expected Disturbances
- Set-Point Operability
  - We Want to Reach Every Point in Desired Output Set (DOS)

(*) Vinson, D. R.; Georgakis, C. (2000). *Journal of Process Control*, *10*, 185-194.

Steady-State Operating Sets*

- Available Input Set (AIS)
  - Ranges of the Inputs that are Available to Manipulate
- Desired Output Set (DOS)
  - Ranges of the Outputs that are Desired to be Achieved
- Expected Disturbance Set (EDS)
  - Ranges of the Disturbances that are Expected to Affect the Process
- Achievable Output Set (AOS)
  - Ranges of Outputs that can be Achieved with the Available Inputs in AIS

(*) Sets Previously Defined as Spaces by the Authors

Interval Operability

- ►Fix Some Outputs at Set-points

- ►Allow Others to Vary Within their Intervals
  - To be Operable in Intervals:
    - ►*Need One Feasible Operating Point*

- ►Achievable Output Interval Set (AOIS)*
  - Tightest Set of Output Constraints
    - ►That Can be Achieved with
      - Available Input Set (AIS) and
      - Expected Disturbance Set (EDS)

(*) Lima, F.; Georgakis, C. (2006). *ADCHEM Proceedings*, 989-994.

Interval Operability Example – I*

- Non-square linear model: 1 input & 2 outputs

$$\mathbf{y} = \mathbf{G}\,\mathbf{u_1} + \mathbf{G_d}\mathbf{w_1}$$

$$\Rightarrow \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} u_1 + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} w_1$$

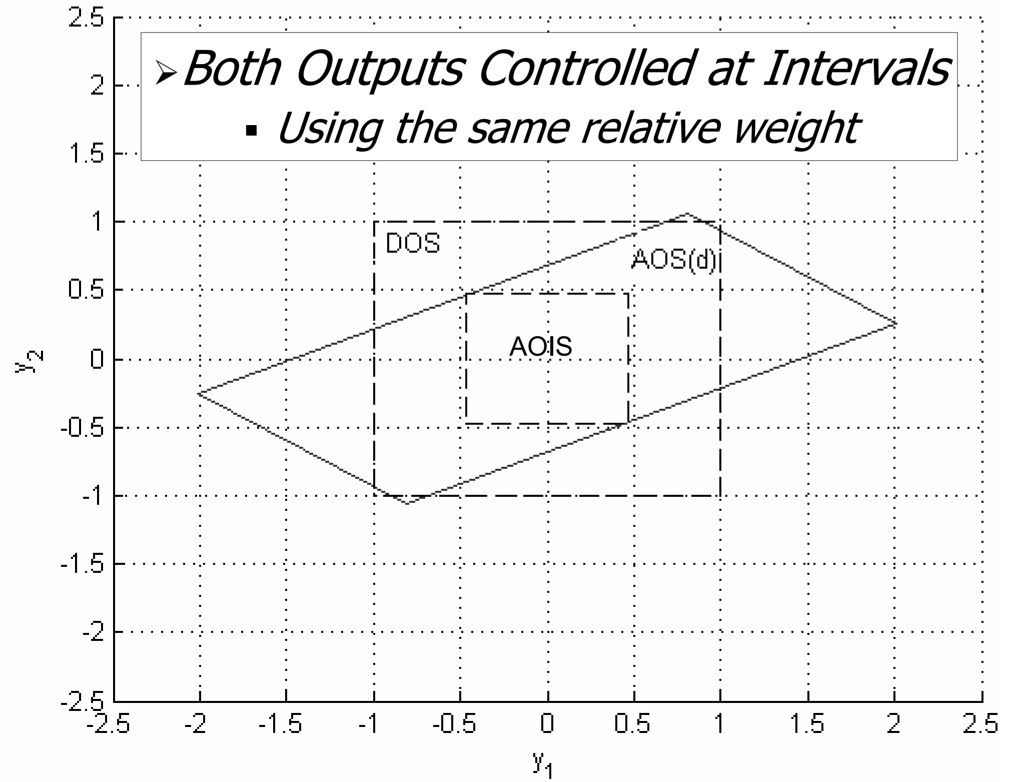
$$AIS = \{u_1 \mid -1 \le u_1 \le 1\}$$

$$EDS = \{w_1 \mid -1 \le w_1 \le 1\}$$

$$DOS = \{\mathbf{y} \in \Re^2 \mid \|\mathbf{y}\|_\infty \le 1\}$$

Steady-state Gain Matrices:

$$G = \begin{bmatrix} 1.41 \\ 0.66 \end{bmatrix}; \quad G_d = \begin{bmatrix} -0.6 \\ 0.4 \end{bmatrix};$$

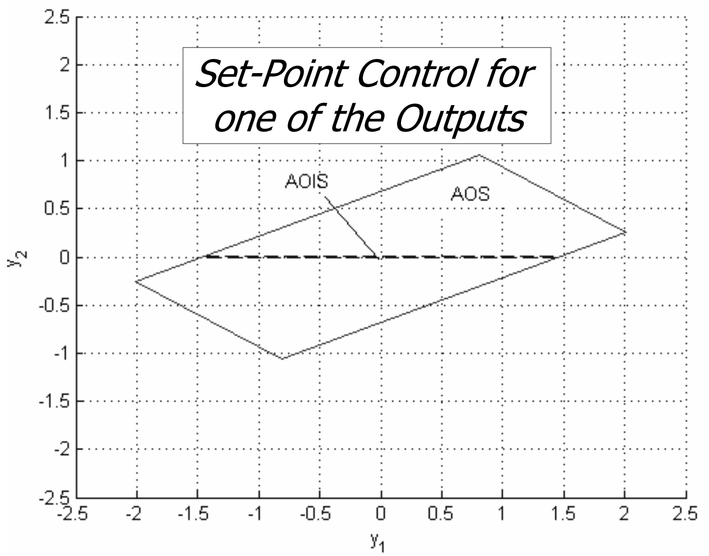


(*) Lima, F.; Georgakis, C. (2006). *ADCHEM Proceedings*, 989-994.

Changing Output Weights and Steady-State

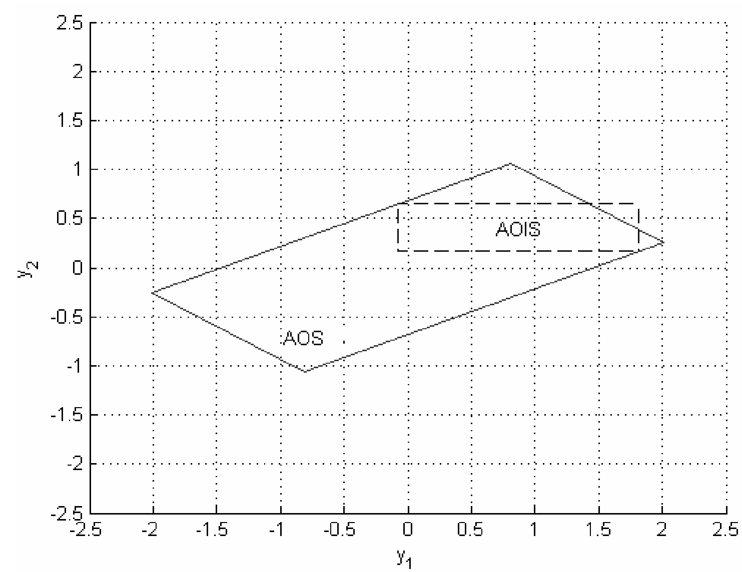
- Changing the Relative Output Weights
  - Influences the AOIS Aspect Ratio:

$$r_{12} = \frac{\Delta y_1}{\Delta y_2} = \frac{w_2}{w_1}$$



AOIS calculated using r_{12} = 1000:1

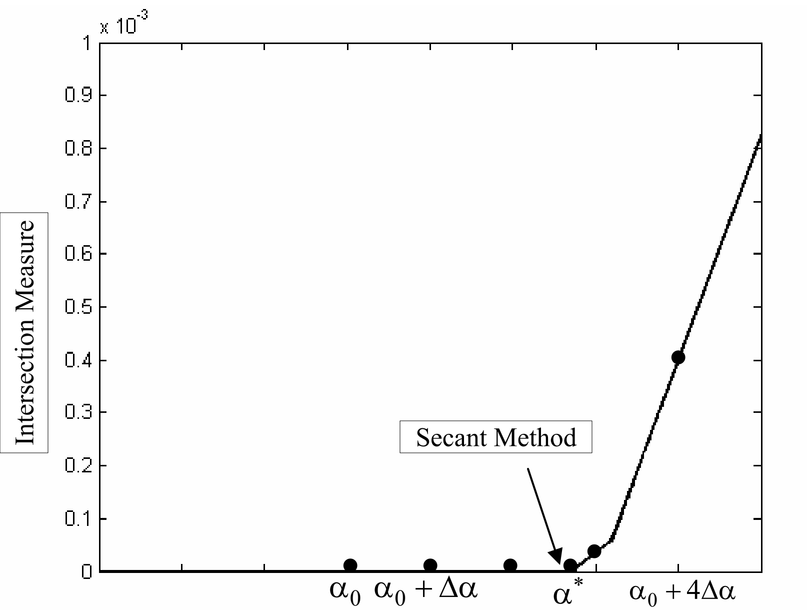
- Moving the Steady-state from the Origin
  - Asymmetric Problem

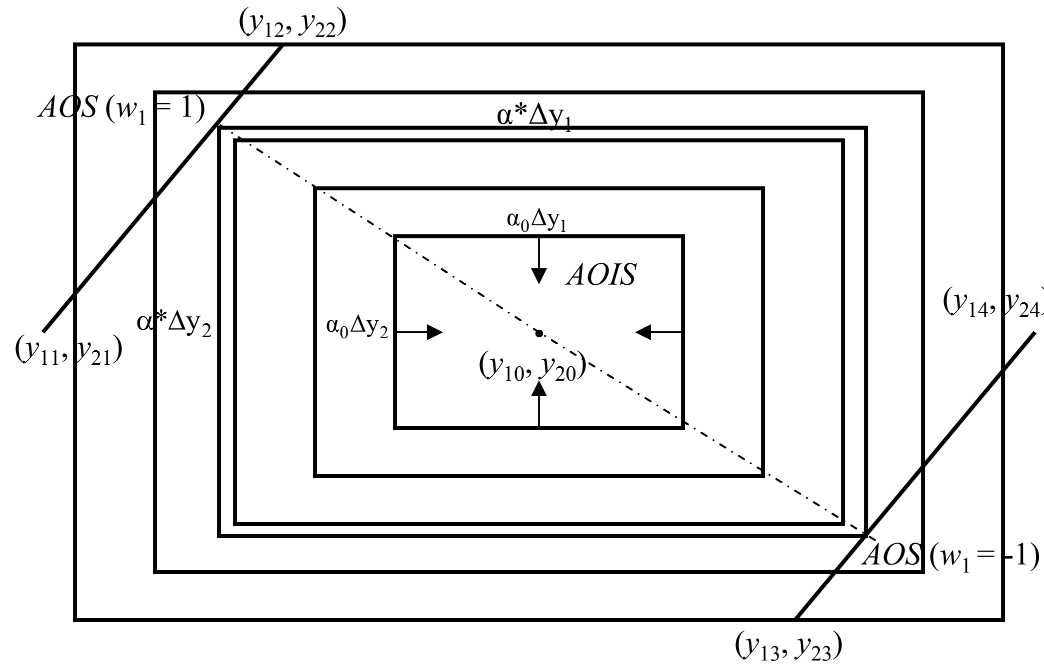


AOIS for y_0 = (0.5, 0.5) *and* r_{12} = 4:1

High-Order Systems: Iterative Methodology

- **Enlarging AOIS**
  - Touches Both Extreme Disturbance Lines of the AOS($w_1 = \pm 1$)
- **Minimize Intersection Measure:** $\mu(AOS(w_1) \cap AOIS)$





Polygon Aspect Ratio: $r_{12} = \dfrac{\Delta y_1}{\Delta y_2} = \dfrac{w_2}{w_1}$

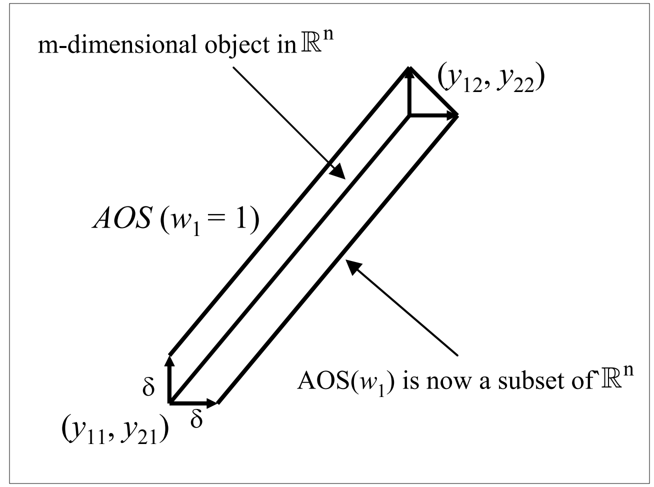
$$AOIS(\alpha) = \{\mathbf{y} \mid \mathbf{b_1} \le \mathbf{y} - \mathbf{y}_0 \le \mathbf{b_2}\}$$

$$\mathbf{b_1} = \left[\frac{-\alpha}{w_1} \;\; \frac{-\alpha}{w_2} \cdots \frac{-\alpha}{w_n}\right]^T; \quad \mathbf{b_2} = \left[\frac{\alpha}{w_1} \;\; \frac{\alpha}{w_2} \cdots \frac{\alpha}{w_n}\right]^T;$$

$$\mathbf{y}_0 = [y_{01} \; y_{02} \cdots y_{0n}]^T; \quad \mathbf{y} = [y_1 \; y_2 \cdots y_n]^T;$$

Iterative Methodology

- Calculation of Intersections using Geometric Bounding Toolbox (GBT)*:
  - Sets AOS(w_1 = ±1) have to be **Full Dimensional**
  - Convex Hull of all Points is Calculated
    - Polytopes Obtained are used in the Intersections



- Algorithm might be Computationally Expensive and Unstable
  - As Problem Dimensionality Increases
    - Computation of Convex Hulls and Intersections

(*) Veres *et al.* (1996). Geometric Bounding Toolbox (GBT) for MATLAB

Results: Industrial Example

- Steam Methane Reformer (APCI*):
  - 4 CVs, 3 MVs, 1 DV

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 14.07 & -0.04 & 2.66 \\ -3.92 & 0 & -1.96 \\ -7.74 & 6.03 & 0 \\ 6.60 & -3.90 & 4.89 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} + \begin{pmatrix} 2.96 \\ -2.18 \\ 0 \\ 5.44 \end{pmatrix} w_1$$

$$AIS = \left\{ \mathbf{u} \in \Re^3 \mid \|\mathbf{u}\|_\infty \le 1 \right\}$$

$$EDS = \left\{ w_1 \mid -1 \le w_1 \le 1 \right\}$$

$$\mathbf{y_0} = (0.1,\ 0,\ -0.15,\ 0)^T$$

$$\mathbf{w} = (1,\ 1,\ 1,\ 1)^T$$

Iterative Methodology

CV	Original Low Limit	Original High Limit	Designed Low Limit	Designed High Limit
y_1	-1	1	-0.25	0.33
y_2	-1	1	-0.23	0.35
y_3	-1	1	-0.38	0.20
y_4	-1	1	-0.23	0.35

Computational Time: 23.71 s

(*) Air Products and Chemicals, Inc.

Conclusions

- Concept of Operability Extended
  - To High-order Non-square Systems
- Iterative Methodology Developed
  - Successfully Applied to Industrial Example
    - Steam Methane Reformer (APCI)
- Industrial Impact is Two-Fold
  - Advances the Ability of Operability Assessment
    - For Continuous Processes
  - Design of Output Constraints for MPC controllers

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Thank you for your attention

Questions

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