Issues on the Operability of Multivariable Non-Square Systems

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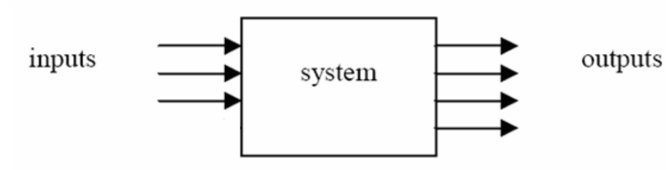
Presentation Outline

- Problem Definition
- Objectives & Proposed Approach
- Motivating Example
- Process Operability
  - Set-Point Operability
  - Interval Operability
- Conclusions

Problem Definition and Motivation



- Non-square Systems:
  - More Outputs (CVs) than Inputs (MVs)
  - Set-point Control NOT Possible for ALL Outputs
    - Fewer Degrees of Freedom
  - ***Interval Control Needed***
- Model Predictive Control (MPC):
  - Very Tight Constraints Make Control Infeasible
- Need Methodology for Design of Non-square Controllers

Research Project Objectives

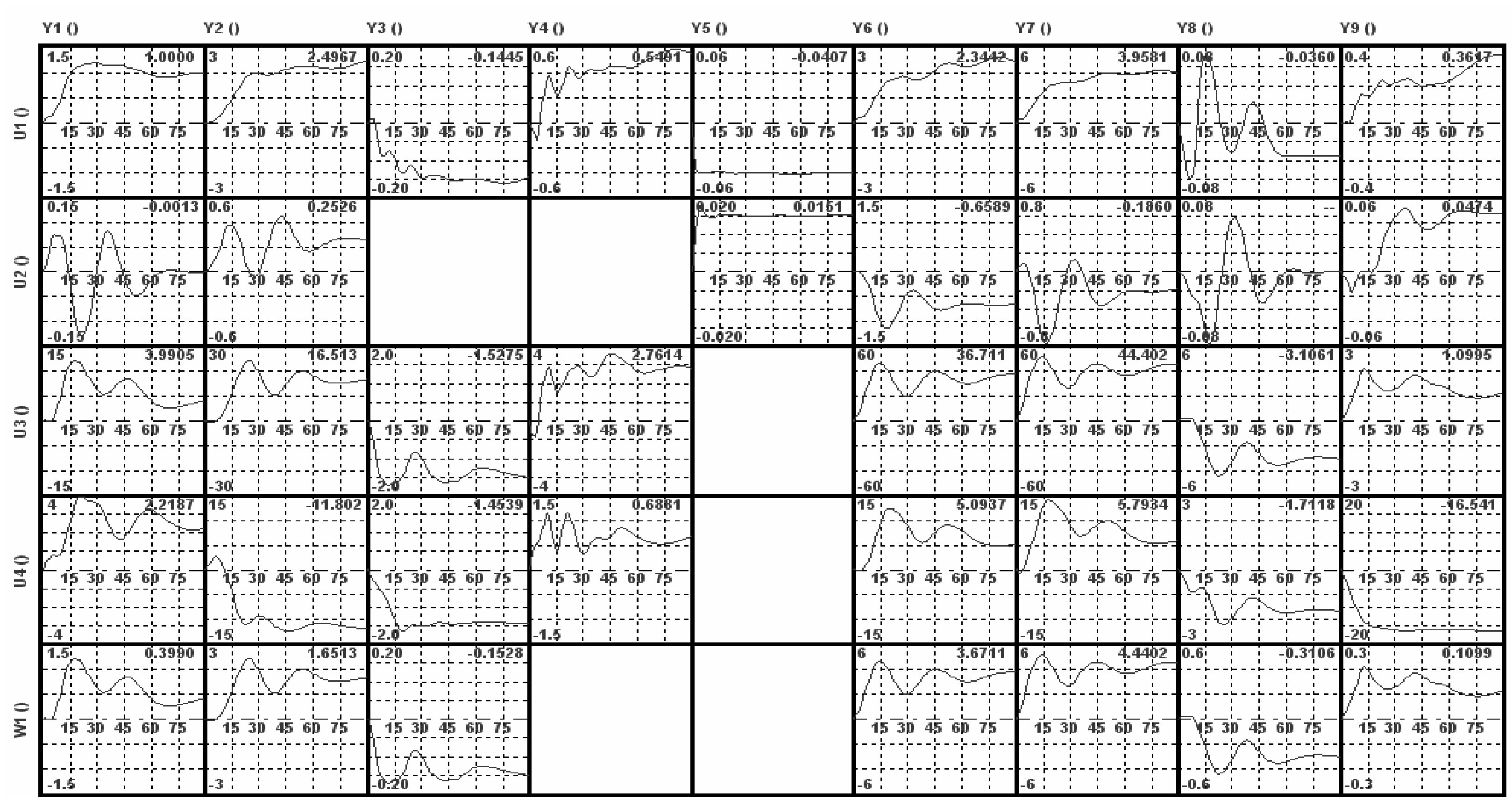
- Demonstration of Need to Properly Design the Output Bounds on Non-square MPC Controllers
    - Use Dynamic Matrix Control (DMC)
- Development of a Process Operability Methodology
    - For Multivariable Non-square Systems
- Apply to Design of Controllers

Proposed Approach

- Motivating Simulations of a Non-square Process:
  - Steam Methane Reformer (SMR):
    - 4 manipulated, 1 disturbance and 9 controlled variables
  - DMCplus^{TM} controller (AspenTech)
- Process Operability Concept
  - Analyses for square systems*
  - Extended to Non-square Linear Systems:
    - Can be Used in Choosing Possible Ranges for Outputs
  - Application Examples Using Simple Non-square Systems

(*) Vinson, D. R.; Georgakis, C. *Journal of Process Control* 2000, *10*, 185-194.

Motivating Example: Steam Methane Reformer (SMR): Step Response Model*



Dynamic Matrix for the SMR problem (DMCplus™)

(*) from Dave Vinson, PhD thesis at Lehigh University

SMR: Nominal Feasible Constraints

MV/CV	Low Limit	High Limit
u_1	-19	19
u_2	-40	40.0
u_3	-0.9	0.9
u_4	-0.85	0.85
y_1	-1.35	1.35
y_2	-67.2	67.2
y_3	-0.7	0.7
y_4	-21.5	21.5
y_5	-0.1	0.1
y_6	-6.75	6.75
y_7	-80.65	80.65
y_8	-2.15	2.15
y_9	-15.6	15.6

Feasible Set of Input and Output Constraints for the SMR problem

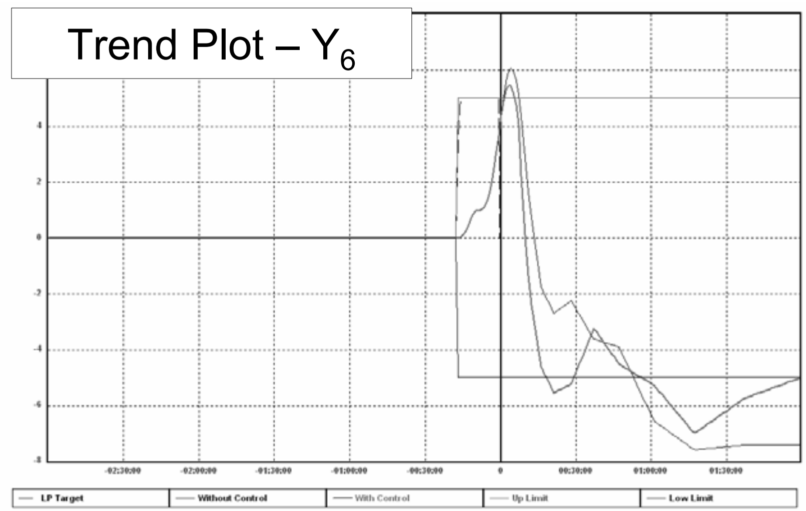
DMCplus™ Simulations – Case 1

- ► Making the Y_6 constraints tighter:

CV	Original Low Limit	Original High Limit	New Low Limit	New High Limit
y_6	-6.75	6.75	**-5.00**	**5.00**
y_7	-80.65	80.65	-80.65	80.65

- ► ***Problem Becomes infeasible***
  - DMCplus™ Controller:

Trend Plot – Y_6

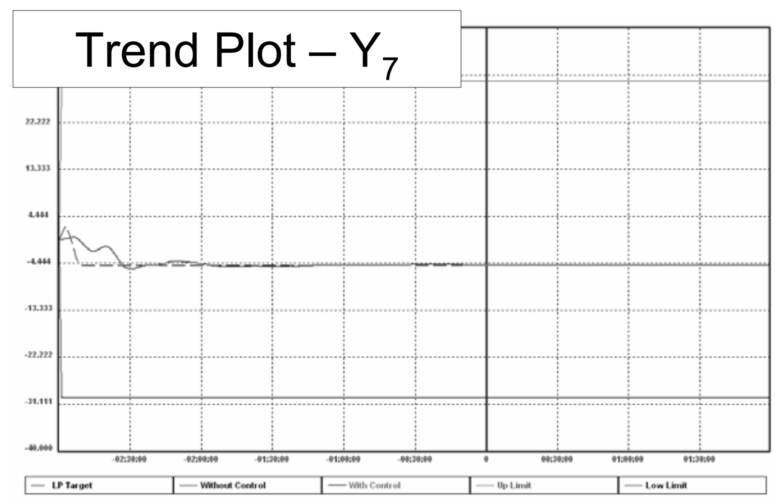


DMCplus™ Simulations – Case 2

- Making the Y_7 constraints tighter:

CV	Original Low Limit	Original High Limit	New Low Limit	New High Limit
y_6	-6.75	6.75	-6.75	6.75
y_7	-80.65	80.65	**-30.00**	**30.00**

- Problem Feasible!
- Need Methodology to Design Output Bounds



Process Operability Definition

- ►Vinson and Georgakis (2000)*:
  - Process is Operable if ...
    - ►Available Inputs are Capable of Satisfying Desired Steady-state & Dynamic Performance Requirements in Presence of Expected Disturbances
- ►Set-Point Operability
  - We Want to Reach Every Point in Desired Output Space (DOS)

(*) Vinson, D. R.; Georgakis, C. *Journal of Process Control* 2000, *10*, 185-194.

Steady-State Operating Spaces

- Available Input Space (AIS)
  - Ranges of the inputs that are available to manipulate
- Desired Output Space (DOS)
  - Ranges of the outputs that are desired to be achieved
- Expected Disturbance Space (EDS)
  - Ranges of the disturbances that are expected to affect the process
- Achievable Output Space (AOS)
  - Ranges of outputs that can be achieved with the available inputs in AIS

Operability: Shower Example

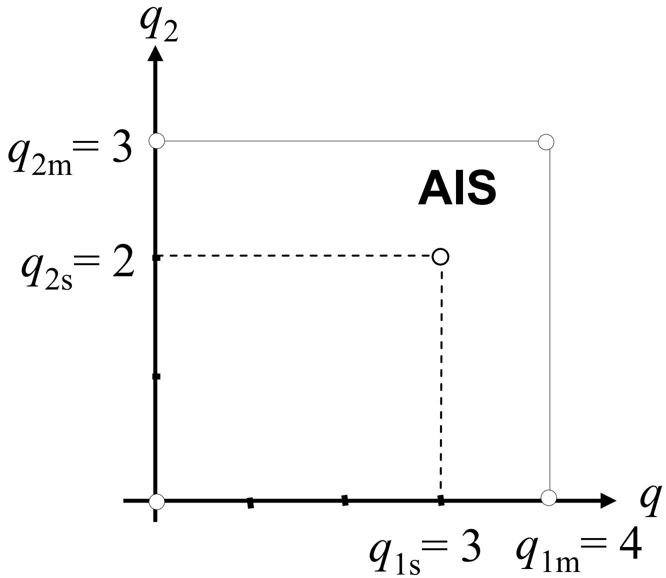
Two Streams of Hot and Cold Water:

$T_{1f} = 60^0 F; \quad 0 \le q_1 \le 4$ gal/min

$T_{2f} = 120^0 F; \quad 0 \le q_2 \le 3$ gal/min

Steady State Values:

$q_{10} = 3$ gal/min; $\quad q_{20} = 2$ gal/min

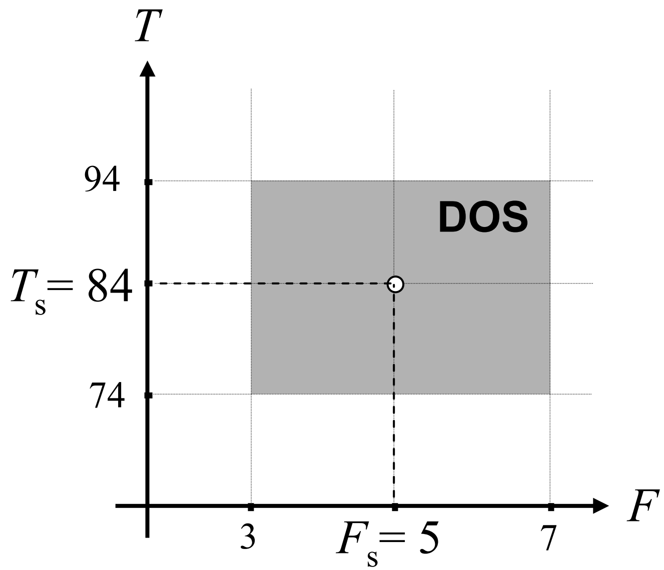


Nonlinear Steady State Model

Total Flow: $F = q_1 + q_2$

Output Temperature:

$$T = \frac{q_1 T_{1f} + q_2 T_{2f}}{q_1 + q_2}$$

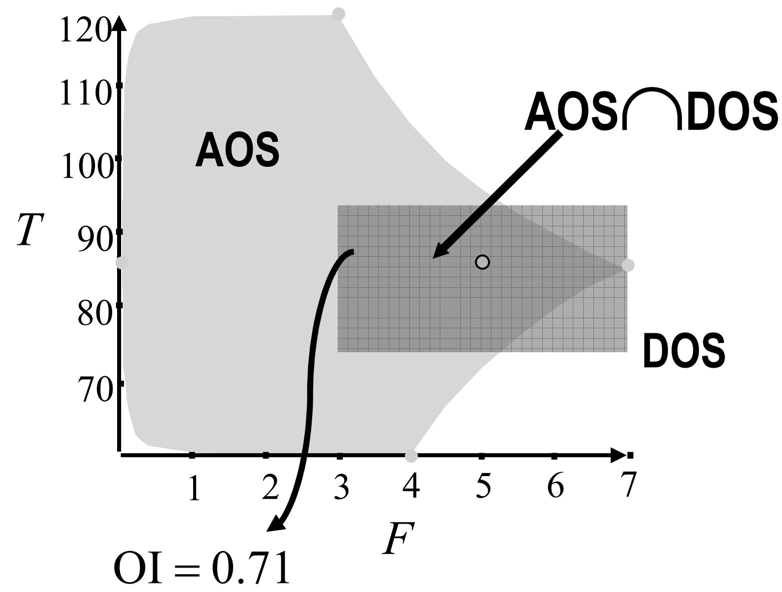


Set-Point Operability Index

- Operability issues:
  - Expectations (DOS) were unrealistic
  - Can clearly see what expectations will be realistic

- Operability Index (OI)
  - Fraction of the DOS that can be achieved

$$OI = \frac{\mu(AOS \cap DOS)}{\mu(DOS)}$$



Interval Operability

- Fix Some Outputs at Set-points,
- Allow Others to Vary Within their Intervals
  - To be Operable in Intervals:
    - *Need One Feasible Operating Point*
- Achievable Output Interval Space (AOIS)
  - Smallest Set of Output Constraints
    - That Can be Achieved with
      - Available Input Space (AIS) and
      - Expected Disturbance Space (EDS)

Interval Operability Example - I

► Non-square linear model: 1 input & 2 outputs

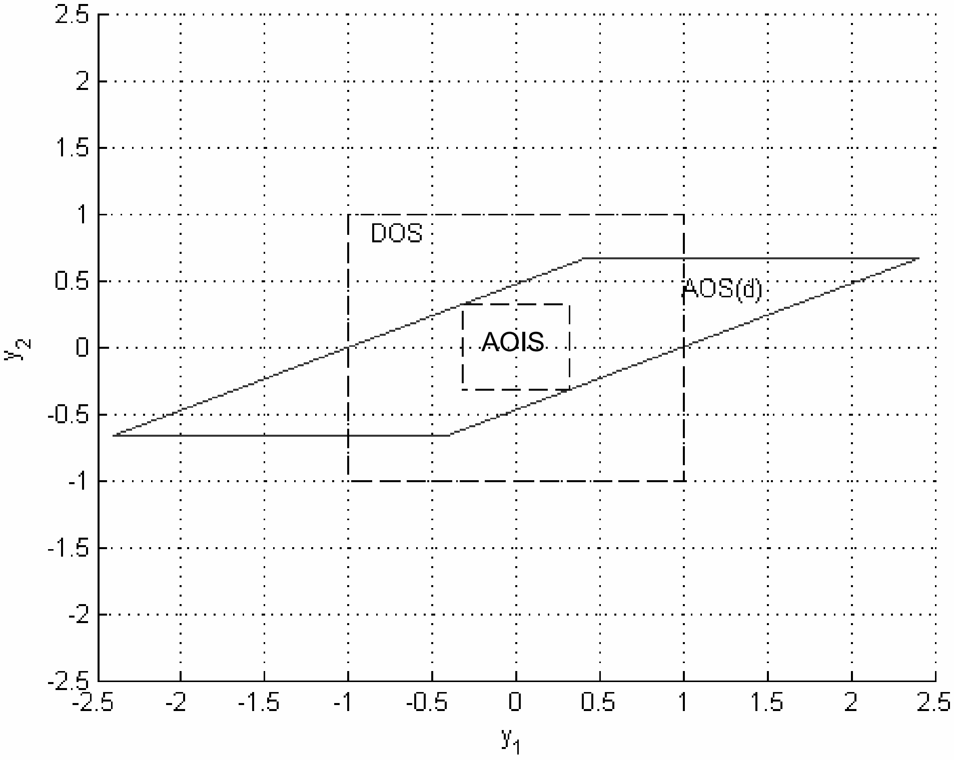
$$y = G u_1 + G_d w_1$$

$$\Rightarrow \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} u_1 + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} w_1$$

$$AIS = \{u_1 \mid -1 \le u_1 \le 1\}$$
$$EDS = \{w_1 \mid -1 \le w_1 \le 1\}$$
$$DOS = \{y \mid \|y\|_\infty \le 1\}$$

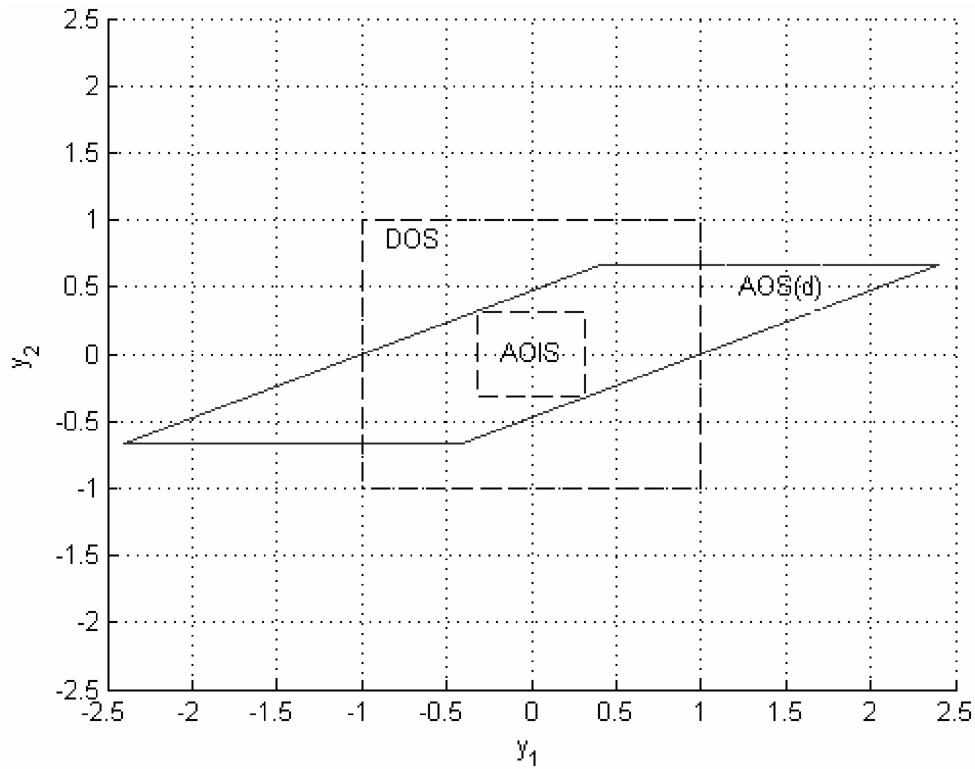
Steady-state gain matrices:

$$G = \begin{bmatrix} 1.41 \\ 0.66 \end{bmatrix}; \quad G_d = \begin{bmatrix} 1 \\ 0 \end{bmatrix};$$



Interval Operability Example - I

- Both Outputs Controlled at Intervals
- Operable: If DOS covers AOIS



Interval Operability Index: $IOI = \dfrac{\mu(DOS \cap AOIS)}{\mu(AOIS)}$

Interval Operability Example - II

► A 3x2 Process:

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} w_1$$

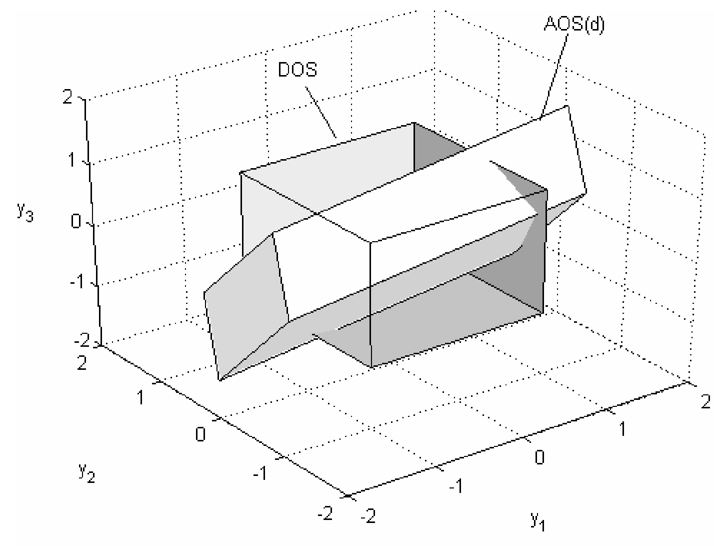
$$AIS = \left\{ u \in \Re^2 \mid \|u\|_\infty \leq 1 \right\}$$

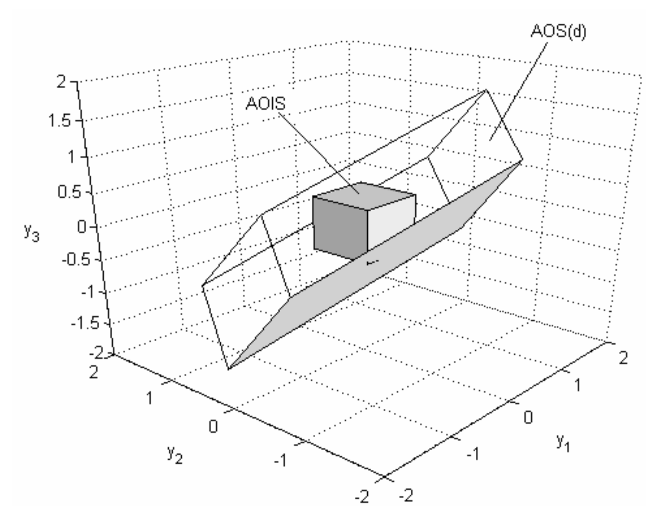
$$EDS = \left\{ w_1 \mid -1 \leq w_1 \leq 1 \right\}$$

$$DOS = \left\{ y \in \Re^3 \mid \|y\|_\infty \leq 1 \right\}$$

Steady-state gain matrices:

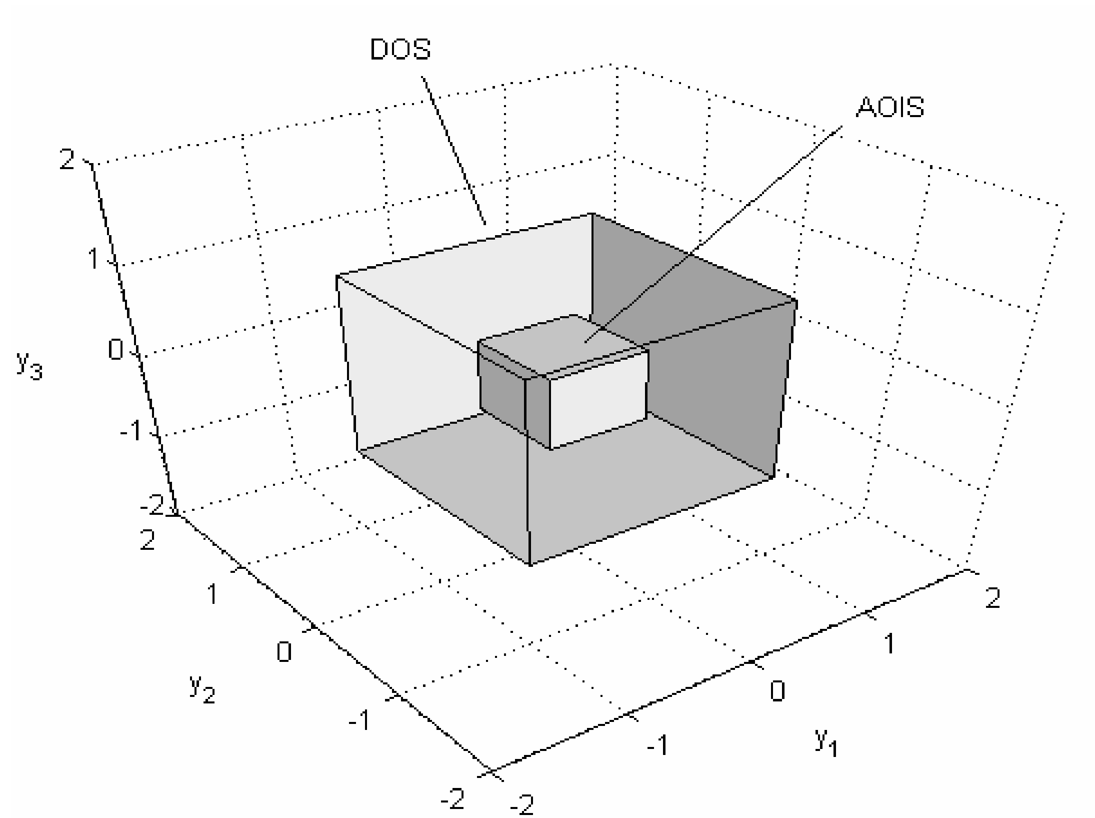
$$G = \begin{bmatrix} 1.41 & 0.27 \\ -0.39 & -0.20 \\ 0.66 & 0.49 \end{bmatrix}; \quad G_d = \begin{bmatrix} 0.2 \\ 0.4 \\ 0.4 \end{bmatrix};$$





Interval Operability Example - II

- ALL 3 outputs Controlled in Intervals
- DOS Need Be Larger than AOIS
- Initial DOS Can Be Reduced



Conclusions

- ► Design of Output Bounds for Non-square MPC (DMC) Controllers is Necessary
- ► Concept of Operability Extended
  - Achievable Output Interval Space defined
- ► We Examined Steady-State Operability
  - Need to Examine Dynamic Operability

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