DETERMINABLE NOMINALISM*

1. PROPERTIES AND THEIR ROLES

In our ordinary unreflective talk we refer to properties by name and we quantify over them. This ontological commitment does not seem to be gratuitous. David Lewis has distinguished between what he calls a sparse and an abundant sense of 'property' corresponding approximately to two important and perhaps indispensable theoretical roles that properties are called on to play.[1] Properties in the abundant sense (abundant properties) are required as the semantic values of predicates. There must be an abundant property for every possible predicate in every possible language, however gruesomely gerrymandered that predicate may be, and however impossible that language may be for us to master. Lewis:

> There is one of them for any condition we could write down, even if we could write at infinite length and even if we could name all those things that must remain nameless because they fall outside our acquaintance. In fact the [abundant] properties are as abundant as the sets themselves, because for any set whatever, there is the property of belonging to that set.[2]

Properties in the sparse sense of 'property' (sparse properties) are required as that which makes for genuine, objective, "qualitative" similarity among their instances.[3] Sparse properties are said to "carve up reality at the joints" and it is their inter-relations that natural science aims to codify.[4]

---

* Earlier versions of this paper were read at the University of Massachusetts at Amherst, Dartmouth College and Tufts University. I would like to thank the audiences for their helpful suggestions on those occasions. I would also like to thank Ed Gettier, Michael Molloy, Helen Morris Cartwright, Lynne Rudder Baker, Walter Sinnott-Armstrong, a referee for this journal and especially Neil Feit. Above all I would like to thank Phillip Bricker without whom this paper could not have been written. He carefully read numerous early drafts and there is hardly a page in it that does not bear the imprint of his suggestions and trenchant criticisms. To reduce clutter, I have suppressed further footnotes acknowledging this. I hasten to add, however, that all errors are my own.

Lewis points out that if our ontology already includes all the properties in the abundant sense, there will be one for each property in the sparse sense. So

> we may as well say that the sparse properties are just some -- a very small minority -- of the abundant properties. We need no other entities, just an inegalitarian distinction among the ones we've already got.[5]

Members of the minority he calls "natural" properties. Whether or not some property is natural is supposed to be a non-conventional, mind- and language-independent fact about reality.[6] Pending an adequate analysis, Lewis says he is willing to take this distinction as primitive. He adds, however, that

> Probably it would be best to say that the distinction between natural properties and others admits of degree. Some few properties are <u>perfectly</u> natural. Others…are at least somewhat natural in a derivative way...The colours, as we now know, are inferior in naturalness to such perfectly natural properties as mass or charge; grue and bleen are inferior to the colours; yet even grue does not plumb the real depths of gruesomeness.[7]

The idea is that the properties whose inter-relations science aims to codify are all perfectly natural, and the properties expressed by most of our predicates all have some degree of naturalness, though in many cases this will be less than a perfect degree. One way to think of the naturalness of a property is as a measure of the degree of genuine similarity it imposes on its instances.

In this paper I develop and examine a theory of properties I will call "Determinable Nominalism" (DN). I will follow Lewis's strategy: first, I identify entities suited to playing the abundant property role, and then I appeal to a primitive notion of naturalness to pick out entities suited to playing the sparse property role from among them.

2. DETERMINABLES AND DETERMINATES

In 1921 W. E. Johnson introduced the now familiar distinction between determinable and determinate properties.[8] Properties can be grouped into families, for instance, the masses, colors, charges, and shapes. Roughly, the properties corresponding to these families – having a mass, being colored, etc. -- are determinables, and their members -- being 1kg, being red, being positively-charged, being round, and so on -- are their determinates.

Traditionally, metaphysical theories of properties have concentrated on identifying entities to play the role of individual determinate properties. Candidates have included classes of their instances, Forms, classes of exactly resembling tropes, and universals. Typically however such theories consider determinates largely in isolation from each other and the determinables to which they belong. And it is usually only after determinates have been dealt with that attention is turned to determinables, if at all. The distinctive feature of DN is that it takes entire determinables-together-with-their-determinates rather than individual determinates as its units of analysis.

One reason for this change of focus has to do with the logical relations among properties. Some properties exclude others: if, for instance, a particular is red all over, then it cannot also be blue all over (at the same time); if something is exactly 1kg, it cannot also be exactly 10kg; and so on. And some properties entail others: if a particular is red, it must be colored; if it is 3kg, it must have a mass; and if it has a mass, it must be either 1kg, or 2kg, or 257.3kg, or ...; if it has a charge, it must be either positively-charged or negatively-charged; and so on. Finally, some properties stand in relations of comparative similarity to one another: redness, for instance, is more similar to orangeness than it is to blueness, and being 1kg is more similar to being 2kg than it is to being 100kg.[9]

Now, the general principles governing these relations seem to be bound up with the division of properties into determinables and determinates. In fact, it seems that typically properties exclude one another and stand in relations of comparative similarity iff they are determinates drawn from the same determinable; and it seems that a particular falls under a determinable iff it instantiates some determinate or other of that determinable.[10] Since an adequate theory of properties should accommodate any logical relations among them, this suggests that the focus of such a theory should be on determinables-together-with-their-determinates rather than on individual determinates in isolation. Traditional theories fail to accommodate these relations.

A second reason has to do with naturalness. Naturalness comes in degrees and applies to abundant properties; it measures the degree of similarity that such a property imposes on its instances. As Lewis points out, properties seem to differ greatly in how natural they are. Some few properties, however, are perfectly natural -- the sparse properties -- and it is these that are supposed to "carve up reality at the joints" and whose inter-relations are encoded in the laws of nature.

But notice that it is entire determinables such as mass, temperature and charge rather than individual determinates such as being 3kg or being positive that typically figure in the laws of nature. Notice also that determinates drawn from the same determinable are, intuitively, all equally natural; they seem to impose exactly the same degree of similarity on their instances. Although redness is less natural than being 1kg, for instance, it seems to be exactly as natural as greenness. Notice finally that sparse determinables seem to be better candidates than sparse determinates for being that which carves up reality at the joints. To divide particulars into 1kg particulars and non-1kg particulars, for instance, is at best to carve at a single joint; it divides reality into one group of things similar to each other and a miscellaneous group consisting of everything else. A determinable like mass, however, divides up everything (or at least every physical thing) into various groups. It is like a way of dividing the entire cake into slices rather than simply cutting a single slice. These observations suggest that Lewis's inegalitarian distinction among the abundant properties that selects out the sparse minority may be better drawn at the level of determinables than determinates; they suggest that naturalness be thought of as applying in the first instance to determinables and only derivatively to their determinates. An adequate theory of properties should also get the facts about their naturalness right. And the fact that they too seem to be connected with the division of properties into determinables and determinates also suggests focusing on determinables-together-with-their-determinates instead of on determinates in isolation.

A final reason has to do with the analysis of modality. A property's place in the network of exclusions, entailments and similarities and the degree of similarity it imposes on its instances constitute an important part of what is essential and distinctive about it, what makes it the very property it is. But the traditional focus on individual determinates at the expense of determinables leaves little room to explain these features. Indeed, most philosophers write as though it were just a brute metaphysical fact that each property possesses some kind of unique sui generis nature or "quiddity" that is responsible for them. And this is problematic. In particular, as we will see below, the existence of quiddities seems to undermine the possibility of reductively analyzing modality in terms of "recombinations" of actual materials. If, however, the distinctive features of properties can be explained instead in terms of a general division of properties into determinables and determinates, where the former are the primary bearers of naturalness, then quiddities may be rendered theoretically superfluous thereby enhancing the prospects for an actualist analysis of modality. It seems to me that such an analysis should be possible. So this constitutes a third reason for DN's change of focus.

3. SOME RESTRICTIONS ON DN

There is not space in this paper to develop a full theory of properties and the scope of DN is restricted in three ways.

First, it is intended to cover only intrinsic properties -- roughly, those “qualitative” properties that a particular has per se, in and of itself, regardless of the state or existence of any other distinct thing e.g. the shapes, colors, masses, and lengths.[11] This restriction is supposed to exclude properties that are “extrinsic”, “implicitly relational” or “vectorial” such as being five miles from Boston, being an uncle, being the bookies’ favorite and traveling eastwards at 30 knots. According to Lewis, all the sparse properties are intrinsic.[12]

Second, it is intended to cover only those sparse properties that are basic in the sense that there are “just enough of them to characterize things without redundancy”.[13] This restriction is supposed to exclude “disjunctive” properties like the property of being either round or red, “conjunctive” properties like the property of being both round and red, “structural” properties like the property of consisting of a proton and an electron a certain distance apart, and so on.

Finally, it is intended to cover only properties capable of instantiation by simple concrete particulars (whether or not these properties could also be instantiated by complex or abstract particulars). To be six feet tall it is necessary to be complex, but this is not so for charges and masses, or at least not obviously. So heights fall outside the scope of the theory but charges and masses are probably included. Perhaps being a property, or being spatially unextended are properties that only abstract entities could instantiate, so these too are excluded.[14]

However, having stated these restrictions, for the sake of vividness I will sometimes employ as examples properties that are not officially covered.[15] And for clarity of exposition, I will suppress reference to the restrictions; I will use ‘property’ to mean ‘intrinsic, basic property capable of instantiation by simple concrete particulars’.

4. DN STATED

Now let us find entities suited to play the theoretical roles of the properties. And to accommodate the logical relations among properties and the facts of naturalness while avoiding quiddities, let us focus on entire determinables-together-with-their-determinates. Set theory seems to me to be the best place to look for candidates.[16] For an ontology of classes should be granted Moorean status,[17] the identity-conditions and mathematics of classes are well understood, and set-theoretic structure seems a promising candidate for doing at least some of the work of quiddities.

First then the abundant determinables:

F is a <u>determinable$_a$</u>[18] iff F is a pair <S, f> where:

(i) S is a metric space; and

(ii) f is a function from concrete particulars into the points of this metric space.

The idea is to think of an abundant determinable as a “classification” of particulars. Its first member, a metric space, is a collection of points together with a “distance” relation on these points; it defines the possible classificatory locations and how they are related.[19] Each point corresponds to a determinate of that determinable, and the distance relation provides a measure of the similarities among them. Its second member, a function, maps each concrete particular in its domain to a unique point in the metric space; it defines which particulars go where on the classification. It thereby determines the extension of each point according to that determinable, the collection of particulars mapped onto that point.[20] Intuitively, determinables typically classify very large collections of particulars. But this account does not require a determinable's function to map every particular to some point in its metric space. Nor does it require that it map some particular to every one of the points.[21]

Notice that for every determinable <S, f>, the collection, $P_{<S, f>}$, of non-empty extensions of the points in its metric space constitutes a partition of its domain; that is, the members of $P_{<S, f>}$ are collectively exhaustive of the domain of f and mutually exclusive. Equivalently: for every determinable <S, f> there is a corresponding equivalence relation $R_{<S, f>}$ viz. the relation that a bears to b iff f(a) = f(b). Let us call $P_{<S, f>}$, the characteristic partition of <S, f>, and $R_{<S, f>}$ the characteristic relation of <S, f>.

Next, the abundant determinates. A natural way to capture the idea that the identity of any determinate is bound up with its determinable is to identify it with a determinable-extension pair, to think of it, in effect, as a "centered" classification of particulars:

F is a determinate$_a$ iff F is a pair <D, E_i> where

(i) D is a determinable$_a$; and

(ii) E_i is the extension of point i in the metric space encoded by D.

Some more terminology: let us call a determinate's first member its determinable, and its second member its extension. And let us say that each determinate belongs to its determinable. According to DN, determinables and determinates are quite different. So it may prove helpful to distinguish the relation that a particular bears to a determinable from the relation it bears to a determinate. Let us say that a particular a falls under F iff F is a determinable <S, f> and a is in the domain of f; and let us say that a particular a instantiates F iff F is a determinate <<S, f>, E_i> and f(a) = i. But let us continue to use the more colloquial ‘is’ and ‘has’ indiscriminately to refer to either relation: a is F or a has F(-ness) iff either a falls under F or a instantiates F.

Each determinate is associated with a specific point in the metric space encoded by its determinable, viz. the one indicated by the subscript on its second member.[22] But a metric space also encodes a distance relation among its points. Let us say that a determinate <<S, f>, E_i> is more similar to determinate <<S, f>, E_j> than it is to determinate <<S, f>, E_k> iff $d(i, j) < d(i, k)$, where d is the distance relation encoded by S.[23] Thus the relations of similarity among properties are interpreted as expressing the distances among the points with which they are associated. Notice that these similarity relations are defined only for determinates belonging to a common determinable.

Third, let us characterize the abundant properties (simpliciter):

> F is a property$_a$ iff F is a pair <w, x> where w is a world, x is either a determinable$_a$ or a determinate$_a$, and x exists at w (where a set exists at a world iff its members exist there).

For any property <w, x>, let us call w its world, x its determinable or determinate, and the collection of particulars having x, its extension at w. And let us say that a property belongs to its world.[24]

Notice that this entails that properties are world-bound sets. However, we will see below how to adapt Lewis’s “method of counterparts” to interpret modal claims in a way that allows the determinable nominalist to say that properties exist at more than one world and, unlike sets, have their extensions accidentally. We will also see (in section eight) how the determinable nominalist can construct the worlds themselves from materials available at the actual world without invoking any modal notions.

The next step is to characterize the sparse properties. This is where naturalness enters the picture. The idea is to think first of each determinable$_a$ and determinate$_a$ as having a certain degree of naturalness at every world at which it exists, and then to identify the sparse properties with those abundant properties <w, x> such that x is perfectly natural at w:

> <w, x> is a property$_s$ iff if x is a determinable, then x is perfectly natural at w, and if x is a determinate, then its determinable is perfectly natural at w.

Naturalness has not been analyzed; it is a conceptual primitive of DN. DN's plausibility does, however, require several assumptions about naturalness: that it comes in degrees and has a highest degree ("perfect" naturalness);[25] that it applies at any world to the abundant determinables of that world; and that (sets qualifying as) determinables may vary in their degrees of naturalness from world to world.[26] I will not attempt to dispute or defend these assumptions here.[27] But a couple of comments are in order concerning the assumption that naturalness applies to determinables. First, this does not preclude us from thinking of determinates as also being bearers of naturalness. For since each determinate belongs to a unique determinable, we can think of a determinate as simply inheriting a degree of naturalness from its determinable. Let us say then that the degree of naturalness of determinate <<S, f>, E$_i$> at world w = the degree of naturalness of <S, f> at w. In this way, determinates can also be thought of as bearers of naturalness, albeit derivatively. Second, notice that when a determinable <S, f> is perfectly natural at some world, its characteristic relation $R_{<S, f>}$ corresponds to a relation of genuine similarity among particulars there. If <S, f> is the mass determinable, for instance, then $R_{<S, f>}$ is the relation that a bears to b iff a and b have the same mass – the mass-similarity relation. And this suggests identifying the similarity relations among particulars with DN’s characteristic relations. Let us say then that a is genuinely similar to b (in some respect) iff a $R_{\langle S, f\rangle}$b where <S, f> is perfectly natural at w.

The final step consists in characterizing a “counterpart” relation among properties.[28]

Any set has various features at any world at which it exists. Some are what might be called set-theoretical features: having a certain membership, satisfying DN’s definition of a ‘determinable’ or a ‘determinate’, encoding a certain sort of metric space, and so on. Intuitively, any set that has such a feature has it essentially. But if the determinable nominalist is right about what properties are, then certain sets – those that satisfy its definition of a ‘determinable’ or a ‘determinate’-- also have what might be called property-theoretical features: a certain degree of naturalness, a certain sort of nomological or causal role, and so on. Any set that has such a feature has it accidentally.[29]

Now, by virtue of their set- and property-theoretic features, sets existing at various worlds will be linked by numerous similarities. And it is these that collectively determine the counterpart relation among the properties themselves:

> Property <w*, x*> is a counterpart of property <w, x> iff x* resembles x enough, and at least as closely as any other of w*’s sets y* resembles x, where the respects of similarity may include any of their set-theoretic features, and any property-theoretic features that x* and y* have at w*, and x has at w.

(It is more convenient to talk simply of features of properties than of features of sets-at-worlds. So let us say that a property <w, x> has a certain feature iff x has that feature at w.)

The point of introducing the counterpart relation is to permit the interpretation of de re claims about properties: a property exists at a world w iff it has a counterpart that belongs to w;[30] it might have some feature iff it has a counterpart that has the feature; it has some feature accidentally iff it has a counterpart that lacks it; and it has some feature essentially iff all of its counterparts have it. The idea is that when <w*, x*> is a counterpart of <w, x> it is to be thought of as the property that <w, x> would be, and its features as the features <w, x> would have, if things were how they are at w* rather than how they are at w.[31]

This interpretation allows the determinable nominalist to say that a property typically exists at multiple worlds and has some of its features accidentally, even though DN identifies properties with world-bound entities. For since counterparthood is merely a similarity relation, nothing precludes a property from having counterparts that belong to different worlds or that lack some of its features. It also allows her to distinguish the accidental features of properties from the accidental features of sets, even though DN identifies properties with sets. For one property may qualify as a counterpart of another even if it differs in some feature (e.g. its extension) that is essential to any set that has it; and it may qualify only if it has some feature (e.g. a certain degree of naturalness) that is accidental to any set that has it.

This interpretation also allows the determinable nominalist to explain the "softness" in our intuitions about which of a property's features to classify as essential and which as accidental. For counterparthood is a relation of overall similarity, the resultant of a number of diverse respects of similarity. And typically such relations are not governed by a fixed recipe specifying once and for all how the various respects of similarity are to be combined into a single measure. Rather, which are to count, what their relative weights are to be, how much similarity is enough, and so on, usually varies with the context; and even then there is often indeterminacy and vagueness. So we should expect the standards of similarity governing the counterpart relation – and hence the classifications -- also to be context-dependent, vague and indeterminate. The fact that some features of properties seem clearly to be essential (e.g. their degrees of naturalness), while others seem clearly to be accidental (e.g. their extensions) can be attributed to the fact that property-talk typically presupposes standards of similarity that require a similar degree of naturalness but give no weight to similarity of extension. The fact that our intuitions are sometimes inconclusive concerning how certain other features of properties are to be classified can be attributed to the vagueness or indeterminacy in the standards of similarity. And the fact that there are even convincing intuitions supporting rival classifications of certain features can be attributed to the context-dependence of these standards. Perhaps, for instance, most contexts presuppose standards that require similarity of nomological or causal role for counterparthood. And if we concentrate on such contexts, these roles will seem to be essential. But some contexts – e.g. discussions of familiar properties at worlds with radically different laws or no laws at all – clearly do not. And if we concentrate on these contexts, the roles will seem to be merely accidental.

As we will see in section seven, DN entails the identity of properties at any world where they lack certain distinguishing features. And this may at first seem to be incompatible with certain modal intuitions: intuitively, distinct properties might have shared all such distinguishing features. Moreover, "exchanging" all these features generates what are, intuitively, new possibilities for those properties. It is worth noting, therefore, that when interpreted counterpart-theoretically, these intuitions are perfectly compatible with DN. This is because counterparthood is a relation of comparative similarity. And so, like such relations generally, it can be expected to lack many of the formal features of identity e.g. symmetry and transitivity. In particular, we can expect distinct properties belonging to one world sometimes to have a common counterpart belonging to another. For sometimes <w*,

x*> will resemble both <w, x> and <w, y> enough, and more than any other property belonging to w* resembles either, even when x and y are distinct. And we can expect a single property belonging to one world sometimes to have distinct counterparts at another. For sometimes <w*, x*> and <w*, y*> will both resemble <w, x> enough, equally – though perhaps in different ways – and more than any other property belonging to w* despite belonging to the same world. And we can even expect a property <w, x> sometimes to have a distinct counterpart <w, y> at its _own_ world. For sometimes the context will determine standards of similarity that give no weight to the ways in which x and y differ at w. We can also expect combinations of all of the above.

One consequence of this is that if our intuitions about the identity and distinctness of properties at one world are interpreted counterpart-theoretically, they do not in general entail anything about the “identity” and “distinctness” of these properties (i.e. their counterparts) at any other world. So DN is not to be faulted for distinguishing properties at some world on the grounds that it entails their “identity” (i.e. the identity of their counterparts) at another, or vice versa. In particular, because properties can have a common counterpart, the determinable nominalist can say that distinct properties _might_ have shared all their distinguishing features, even though DN entails their identity at any worlds where they do share all these features.

A second consequence is that distinct possibilities for properties will not in general correspond to distinct worlds. For when a _single_ world contains multiple counterparts of a property, it thereby represents _multiple_ possibilities for that property. A single world containing two counterparts of some one actual property, represents _both_ the possibility that it have the first’s features (perhaps a certain extension, a certain causal or nomological role) _and_ the possibility that it have the second’s features instead (perhaps a quite different extension or role). In the same way, distinct possibilities for _pairs_ of properties (or triples, or…) may also be represented by a single world. Perhaps one possibility for coexistent properties F and F* is that the former have feature Φ and the latter, feature Φ*; perhaps another is that the latter have Φ and the former have Φ* instead. Now, if a world contains two properties one of which has Φ and the other Φ* and each of which qualifies as a counterpart of both F and F*, then it represents both possibilities: the former because it contains a counterpart of F having Φ coexisting with a counterpart of F* having Φ*; and the latter because it also contains a counterpart of F having Φ* coexisting with a counterpart of F* having Φ. So DN is not to be faulted for failing to distinguish a world for each possibility for a property, or pair of properties (or a triple, or…). In particular, because each member of a pair of properties can qualify as a counterpart of the other, the determinable nominalist can say that "exchanging" certain features among otherwise exactly similar properties generates new possibilities for those properties, even though DN entails that such properties differ only by differing in these features. Perhaps, for instance, there is a world at which F has one instance and F* has two instances but at which there is nothing else for the determinable nominalist to appeal to distinguish them; maybe it is a world without laws or causal roles etc. Now, since properties typically have their instances accidentally, it seems that we should also recognize a second possibility according to which things are exactly as they are at this world except that F has two instances and F* has just one instance. And this may seem to present a problem for the determinable nominalist; for clearly she cannot recognize a second world corresponding to this second possibility. However, in a context like this where similarity of extension is being given no weight, each of F and F* qualifies as a counterpart of the other. So she can take the one world to represent both possibilities: the first because it contains a counterpart of F – F itself – that has one instance, and a counterpart of F* -- F* itself – that has two instances; and the second because it contains a counterpart of F -- F* -- that has two instances, and a counterpart of F* -- F – that has just one instance.[32]

5. SOME CONSEQUENCES OF DN

The following are all logical consequences of DN: no particular instantiates more than one determinate belonging to a given determinable; if a particular instantiates some determinate, it also falls under its determinable; and if a particular falls under a determinable, it instantiates some determinate or other of that determinable. But DN entails no constraints on the instantiation of determinates belonging to distinct determinables, or on which determinate of a given determinable a particular falling under it must instantiate. This can all be confirmed by a brief inspection of the definitions.[33] These consequences reflect pre-theoretic intuitions about the general principles governing the relations of exclusion and entailment among properties, principles we employ when we argue that a red particular cannot be blue but must be colored, or that a colored particular must be red or blue or…, or that a red particular may instantiate any determinate mass, charge or length. And by interpreting similarity relations among properties as expressing the distance relation among the points in a determinable's metric space, DN also entails that similarity relations hold only among determinates of the same determinable. So it also reflects our intuitions about these relations, for instance, that it is out of place to ask whether being 1kg is more or less similar to being red than it is to being round.

According to DN, a determinate inherits its degree of naturalness at any world from its determinable. So DN entails that determinates of the same determinable are all equally natural. But it entails no constraints on the relative naturalness of distinct determinables and their determinates. It thereby reflects our pre-theoretic intuitions concerning principles about the relative naturalness of properties, principles that we employ when we argue that redness is exactly as natural as greenness and that being 1kg is exactly as natural as being 10kg, but that the relative naturalness of the colors and masses is not to be decided a priori.

DN identifies relations of genuine similarity among particulars with the characteristic relations of perfectly natural determinables. So it entails that particulars are genuinely similar (in some respect) at a world iff they instantiate the same determinate of some determinable that is perfectly natural there. And it thereby provides a clear sense in which the characteristic partitions of such determinables "carve up reality at the joints"; for they divide up their domains in such a way that particulars are classified together iff they are genuinely similar.

In sum, DN has consequences that mirror the pre-theoretic intuitions about the logical relations among properties, the facts about naturalness, and the role of sparse properties in providing for similarities among particulars and in carving up reality. Its set-theoretic constructions then are promising candidates for playing the various property-roles.

6. THE SIGNIFICANCE OF DN

But so what? Given the enormous versatility of set theory it should come as no surprise that various intuitions about properties can be modeled set theoretically. And since DN was tailored specifically to capture these intuitions, it can hardly be said to have explained them. Indeed, taking naturalness as primitive and simply assuming that it picks out just those set-theoretic constructions that have features that match these intuitions is tantamount to taking the intuitions themselves as primitive.

However, DN also has several further features. And it is in these rather than in the fact that it captures certain intuitions it is specifically designed to capture that its real philosophical significance lies.

For one thing, it encodes a general metaphysical picture of properties. And this is a picture that differs from traditional pictures in several ways. First, it elevates the distinction between determinables and their determinates from a local curiosity to a central feature of the nature of properties generally. Second, it treats determinables and determinates as fundamentally different – one as a pair of a metric space and a function, the other as a pair of a determinable and an extension. Third, it reverses the traditional order of ontological priority between determinables and determinates. Determinables, if dealt with at all, have usually been thought of mere Boolean compounds of determinates. But according to this picture a determinate has an entire determinable as a constituent and a determinable typically has more structure – in the form of its metric space's distance relation – than can be recovered by Boolean operations on determinates. Moreover, it treats the characteristic features of a determinate – its relations to other properties, and the degree of similarity it confers on its instances – as deriving wholly from features of its constituent determinable. Fourth, it reverses tradition by taking the primary bearer of naturalness – that which should have been taken to correspond to a universal, we might say – to be the determinable rather than the determinate. In effect, this is to take the fundamental property-unit to be that which comprehensively classifies various heterogeneous particulars drawn from a single world rather than that which unites a homogeneous collection of particulars drawn from various worlds. It is to take the fundamental job of a property to be the provision of a natural carving up of the whole of one world (or at least a large, perhaps typically category-sized, portion of it), rather than a partial carving up of many.

For another thing, DN is highly economical. Its primitive ontology includes only concrete particulars and classes; it eschews additional categories of Universals, Forms, or tropes. Its primitive ideology contains only some fundamental notion(s) of set theory and the notion of naturalness (that comes in degrees); in particular, it invokes no primitive modal notions.[34] And it identifies each property with a set-theoretic construction from the particulars of its own world; in particular, unlike the form of class nominalism discussed by Lewis, it eschews mere possibilia in accounting for properties that actually exist.[35] Moreover, even if it does not really explain the target intuitions, at least it accommodates them on the basis of a mere handful of definitions (plus the axioms of set theory). It is not obvious how theories that focus on determinates in isolation can even manage this much, at least not without adopting additional primitives (e.g. 'determinate', 'determinable', 'similarity', some modal notion) or a huge – perhaps infinite – number of additional axioms (e.g. 'if something is red, it is not blue', 'if something is red, it is colored', 'redness is exactly as natural as blueness').

A third noteworthy feature of DN is its "anti-quidditism". According to DN, a property is merely a set-theoretic construction having various features -- a certain degree of naturalness, a certain nomological or causal role, etc. Its distinctive place in the network of logical relations among properties is a matter of its internal set-theoretic structure. And its other features are either not unique to it (e.g. its degree of naturalness), or are not a matter of its nature but are at least partly extrinsic (e.g. its nomological or causal role). These various set- and property-theoretic features determine the counterpart relation among properties and thereby their essences. Thus quiddities – unique, sui generis property-natures – are rendered theoretically superfluous. In effect, DN has assigned their work instead to additional set-theoretic structure and the counterpart relation.

In sum, DN is no mere exercise in set-theoretic modeling; it has real philosophical significance. This lies in the alternative way to think about properties it generates, in its accommodation of the target intuitions on a finite basis within a minimal and actualist metaphysics, and in its bypassing the need to posit quiddities. DN also has considerable philosophical utility; it suggests, for instance, a way to formulate an actualist recombinatorial analysis of modality that avoids two particularly recalcitrant problems. But before looking at this, let us see how the determinable nominalist responds to a traditional problem for any theory that identifies properties with sets.

7. THE PROBLEM OF EXTENSIONALITY

Consider a cruder theory of properties, CN. Unlike DN, CN identifies determinates and determinables alike simply with unstructured classes of particulars. Otherwise it is just like DN: it identifies abundant properties with world-set pairs; it identifies sparse properties with those world-set pairs <w, x> such that x is perfectly natural at w; and it endorses a counterpart-theoretic interpretation of de re claims about properties.

Now, since classes differ only by differing in extension, CN entails the identity of any properties belonging to the same world that are coextensive; in particular, it entails that there is no world containing distinct but coextensive determinate properties. But this is wrong; clearly the property of having a mass of 3kg and the property of having a temperature of 3K, for instance, are distinct even at worlds where they are coextensive.

But what it is not so obvious is what is wrong with it. Indeed, identifying determinates whenever they are coextensive at some world would seem to enhance theoretical simplicity. And given the counterpart- theoretic interpretation of our modal intuitions, such an identification is perfectly compatible with many such intuitions that may seem at first to be incompatible with it. Since distinct properties at one world can have a common counterpart at another, it is compatible with the intuition that distinct properties might have been coextensive, for instance. And since each member of a pair of properties can qualify as a counterpart of the other, it is also compatible with the intuition that "swapping" the extensions of otherwise similar properties generates a new possibility for those properties.

Here is a naïve diagnosis: what is wrong with the identification of coextensive determinates is that it ignores the fact that they may belong to distinct determinables. Even if, e.g., the property of being 3kg and the property of being 3K were coextensive, they would still be distinct because one is a mass and the other a temperature. And the reason that matters is that the fundamental natural laws often involve entire determinables rather than individual determinates. So, for one thing, identifying coextensive determinates will not in general enhance theoretical economy after all; it will tend to complicate the laws. For another, since the laws of a world fix which causal powers are associated with which of its determinates, determinates belonging to different determinables will typically be associated with quite different causal powers even at worlds where they are coextensive. Because the entire mass and temperature determinables actually play quite different nomological roles, the various mass and temperature determinates confer on their instances different causal powers. So those effects of a particular due its being 3kg would have to be distinguished from those due to its being 3K, even if it should turn out that these determinates are actually coextensive.

DN, of course, takes each determinate to have an entire determinable as a constituent. It thereby avoids CN's general identification of coextensive determinates. And, if the naïve diagnosis is correct, it distinguishes them on the right basis -- the determinables to which they belong.[36] However, if determinables encode the same metric space and classify all the particulars of some world in exactly the same way (for short: if they are isomorphic), then even DN entails that they, and hence their corresponding determinates, are identical there. If, for instance, all the corresponding mass and temperature determinates were coextensive at some world, then, according to DN, they would be identical there.[37]

But this consequence is not obviously objectionable. It is hard to think of any counterexamples. And, as we saw with CN, even modal intuitions that may seem to be incompatible with it will typically turn out not to be so when interpreted counterpart-theoretically. Moreover, the above justification for distinguishing coextensive determinates does not extend to isomorphic determinables and their determinates. This is because the laws of a world characterize any properties they mention "only up to isomorphism": any properties that classify all particulars in exactly the same way at some world would be equally eligible candidates for playing any given nomological role there; nothing about the behavior of that world's particulars could decide among them. In particular, any determinables that are isomorphic at a world would be equally eligible candidates for any nomological role defined by its fundamental natural laws. But then, since the laws determine which causal powers are associated with which determinates, corresponding determinates belonging to isomorphic determinables would automatically be equally eligible candidates for any causal roles there too. In short, the natural laws and the nomological and causal roles they define do not provide grounds for distinguishing among isomorphic determinables or their corresponding determinates as they do in the case of non-isomorphic determinables and their determinates. So it seems that identifying isomorphic determinables and their determinates really would enhance theoretical economy. It would also be compatible with any possible observational evidence. For the intrinsic properties of any particular – and recall that DN is a theory only of intrinsic properties -- are not directly observable;

observation is a matter of how a particular affects our instruments and our senses and that is a matter that is extrinsic to it, a matter of which causal powers are associated with the various determinates it instantiates.

In sum, there is no obvious reason why the determinable nominalist should be worried by any problem of extensionality. For the identification of isomorphic determinables and their corresponding determinates seems to be compatible with our modal intuitions, if these intuitions are interpreted counterpart-theoretically; it may enhance theoretical economy; and it is compatible with any possible observational evidence. This does not of course show that this identification is correct, but it does show that the onus is on the opponent of DN to say what's wrong with it.

8. THE ANALYSIS OF MODALITY

Modal idioms are often interpreted in terms of quantification over worlds: roughly, it might have been that so-and-so iff there is a world that represents that so-and-so. If the key terms in this schema – 'world' and 'represents' – can be defined without referring to any non-actual entities or invoking any modal notions, then it can be regarded as an actualist analysis of modality. In this section, I will outline how DN can guide us in the formulation of such definitions, and thereby demonstrate one aspect of its philosophical utility.

However, DN is very limited in scope; it is a theory only of basic intrinsic properties capable of instantiation by simple concrete particulars. As such it is silent on a number of issues that a fully general analysis of modality must decide. It has no implications, for instance, concerning which alternative spatiotemporal frameworks or collections of particulars there might have been; or how the properties of complex particulars are related to the properties of and relations among their parts; or how the extrinsic and non-basic properties are related to the basic intrinsic properties. To avoid entanglement in these issues, let us concentrate only on those aspects of modality to which DN is relevant. Let us ignore possibilities that involve non-actual spatiotemporal frameworks or non-actual stocks of particulars; and let us ignore anything other than "base facts" – those facts about which simple concrete particulars have which basic intrinsic properties.

Now, consider DN's ontological picture of the base facts of these possibilities. If DN is correct, to determine the base facts of a world it suffices to determine which of its concrete particulars have which of its determinables and determinates. DN identifies abundant determinables and determinates with impure sets and the having relation with a set-theoretical relation. But, intuitively, sets and their members are necessarily connected – neither can exist without the other -- and they have their inter-relations essentially. So, if DN is correct, all the possibilities in our scope – those alike in their stocks of simple concrete particulars -- are also alike in their stocks of abundant determinables and determinates, and in which simple concrete particulars have which of these determinables and determinates. DN also entails that to fix which of these determinables and determinates is also sparse at some world it suffices to fix their degrees of naturalness there. But since a determinate inherits its degree of naturalness from its determinable, the possible variations among the possibilities boil down to variations in how naturalness is distributed among its determinables. In short, as far as the base facts are concerned, DN entails that the alternative possibilities in our scope correspond one-one with the "possible distributions of naturalness" among actually existent abundant determinables, the possible ways that nature could be articulated at the joints.

This suggests the following definitions:

w is a world =df. w is a function that assigns to each actually existent abundant determinable D, a degree of naturalness w(D) ("D's degree of naturalness at w").[38]

A world w represents that a simple concrete particular a has a basic intrinsic property F =df. a is in the extension of some property F* that belongs to w and F* is a counterpart of F.

The idea here is simple. The worlds have simply been identified with the possible distributions of naturalness. And, to stay within the bounds of actualism, the distributions themselves have been rendered as functions from naturalness to the collection of abundant determinables that actually exist.[39] Because of our limited focus, 'representation' has been defined in such a way that worlds represent only base facts. And to permit the straightforward application of the schema to our modal claims, it has been defined directly in terms of the counterpart relation.[40]

Now, if DN's ontological picture is correct, set theory alone guarantees that there is a world for every one of the alternative possibilities for the base facts (except, of course, for the possibilities we are ignoring). Indeed, it guarantees that for every compossible collection of base facts there is a world representing just those base facts. Notice also that these definitions refer only to actual entities: simple concrete particulars that actually exist and set-theoretic constructions from them. And, with the exception of the counterpart relation, the only notions they invoke are ones that the determinable nominalist has already shown how to define non-modally in terms of simple concrete particulars, the property of naturalness (that comes in degrees), and various notions drawn from set theory. The counterpart relation raises a minor problem because although the determinable nominalist has shown how to define it in terms of various features of properties, a full account of some of these features, e.g. their nomological and causal roles, lies beyond the scope of DN. However, there does not seem to be any reason why all such features should not ultimately be accounted for actualistically and non-modally. In short, if DN is correct, the definitions seem to be good candidates for converting the quantificational schema into an adequate and genuinely actualist analysis of modality (albeit a very limited one).

This account is "recombinatorialist": it identifies alternative possibilities with recombinations of various elements that actually exist and are actually combined in a certain way, viz. abundant properties and degrees of naturalness. Notice that if DN is right that a sparse property just is an abundant property having a certain degree of naturalness, this is, in effect, to take recombination to operate at the level of sparse property-constituents rather than at the level of the sparse properties themselves.

One consequence of this is a guarantee that no world represents anything inconsistent with the logical relations among properties; none represents of any particular that it instantiates two determinates of the same determinable, for instance. For, if DN is correct, these relations are all consequences of facts about the set-theoretic structure of abundant properties. And these facts will be unaffected by redistributing naturalness.

Another consequence is a guarantee (probably) that some worlds represent facts involving sparse properties that are not actually instantiated, so-called "alien" properties.[41] The actual distribution of naturalness selects out a tiny minority of abundant properties as perfectly natural. These are the sparse properties of the actual world. Intuitively, they exemplify a very limited range of metric spaces, nomological and causal roles. But the reservoir of abundant properties is vast, and alternative distributions will make quite different selections. Since every possible metric space is exemplified by some actually existent abundant property or other, the sparse properties of some worlds will exemplify quite different metric spaces from any sparse property of the actual world. Moreover, it may be that different enough alternative <u>global</u> distributions of naturalness will determine worlds having different laws.[42] If so, alternative distributions will often determine different nomological and causal roles for the abundant properties they select as perfectly natural. But either way it seems that alternative distributions will often select as perfectly natural some properties that are so different from any actual sparse property as to lack counterparts at the actual world. These are alien properties. Hence, it seems likely that many worlds will represent facts involving alien properties.

(In fact, DN already allows for the actual existence of what we might call "weakly alien" properties: actually uninstantiated determinates belonging to determinables some of whose other determinates are actually instantiated, such as Hume's missing shade of blue and the missing physical values e.g. having a mass of exactly 10^{100}kg. For DN does not require that a determinable's function map some particulars to <u>every</u> point its metric space. The weakly alien properties can simply be identified with determinates that correspond to points in their determinables that actually have empty extensions.[43])

Many other proposed analyses of modality are also recombinatorialist.[44] But typically they presuppose accounts of properties that focus on determinates in isolation and attribute their characteristic features to their quiddities. And that makes it hard to see how else to take recombination but as operating at the level of the sparse properties themselves rather than at the level of their constituents. And that in turn makes it is very difficult to see how to secure the consistency of the worlds or to guarantee that some represent facts involving alien properties (without abandoning either actualism or the hope of analyzing modality). Could consistency be secured by formulating for each property a collection of axioms specifying its relations to other properties, and then requiring that each world represent only what is logically consistent with these axioms? It seems unlikely. Just how could we be sure which axioms to include? How could we even state the axioms for properties that do not even exist to be referred to, the alien properties? Wouldn't the list of axioms have to be infinite, rendering the theory incompletable? And even if consistency were no problem, just how could recombining or otherwise utilizing only those materials that actually exist generate alien properties, quiddities and all (or even unique (interpreted) representations for them)?[45]

To summarize. If DN is true, then it is a straightforward matter to formulate non-modal, actualist definitions of 'world' and 'represents' in a way that guarantees that there is a world that represents that so-and-so iff, subject to our limitations, it is possible that so-and-so. In particular, there is no difficulty in guaranteeing that no world represents anything inconsistent with the logical relations among properties and that some worlds represent facts involving alien properties. If DN is not true, securing these two features is a formidable if not insuperable problem. Since it seems to me that such an analysis must be possible, I take this to show that DN has real philosophical utility.

9. TWO PROBLEMS AND A SKETCH OF HOW TO RESPOND

Finally, let us examine two phenomena that seem to show that it will not quite do to take determinables-together-with-their-determinates as the fundamental units of analysis. Due to limited space I will be able only to sketch ways to develop DN to accommodate them.

The first concerns "relative determinates". Armstrong has observed that some properties seem not to be determinates or determinables simpliciter, but determinables relative to some properties and determinates relative to others.[46] Redness for instance seems to be a determinate relative to color, but a determinable relative to scarlet. And scarlet itself seems to be a determinable relative to various completely specific reddish shades. This raises three connected problems for the determinable nominalist. First, DN entails that nothing is both a determinable and a determinate. But redness seems to be a counterexample. Second, DN entails that determinates of the same determinable exclude each other. But although redness and blueness exclude each other and scarlet and aquamarine exclude each other, scarlet and redness do not, nor do blueness and aquamarine. And yet they all seem to belong to the same color determinable. Finally, in such cases there seem to be additional logical relations among properties to those entailed by DN; for something can be scarlet only if it is red, something can be aquamarine only if it is blue, and so on. So DN is incomplete and the plausibility of the DN-inspired analysis of modality of the previous section is undermined.

First, some terminology. Let us say that one determinable D* is a <u>refinement</u> of another D iff any particulars classified alike by D* are classified alike by D, and the metrical structure of D can be embedded in that of D*.[47] Roughly, one determinable is a refinement of another when it provides a more fine-grained classification of particulars, while still preserving the internal structure of the original classification. Let us say that a sequence of determinables is a <u>refinement-sequence</u> when each member (except the first) is a refinement of its predecessor in the sequence. Finally, when one determinable is a refinement of another, let us say that determinates of the latter are <u>subdeterminables</u> relative to the corresponding determinates of the former.

To accommodate Armstrong's observations a determinable nominalist might argue as follows. Strictly speaking, there is not a single color determinable here but a refinement-sequence consisting of three members: the ordinary seven-color determinable, the intermediate determinable (that includes scarlet, aquamarine etc.), and the infinitely-many, completely-specific-hue determinable. It is just a fact about how we speak that typically we give (strictly distinct) determinables the same name e.g. 'color' when, like these three, they constitute a refinement-sequence. Now, since redness belongs only to the first member of this sequence it is only a determinate; strictly, it not a determinable at all. And since redness and scarlet, blueness and aquamarine, and so on belong to distinct determinables, it remains true that determinates of the same determinable always exclude each other. Finally, by taking refinement sequences rather than determinables as the units of analysis and recombination, DN can accommodate the "new" logical relations and the analysis of modality can proceed in a way that respects them. Notice, in particular, that it follows from the definition of 'subdeterminable' that when one determinable (e.g. the intermediate color determinable), qualifies a refinement of another (e.g. the seven-color determinable), any particular that instantiates a determinate of the former (e.g. scarlet) also instantiates the corresponding sub-determinable of the latter (e.g. redness).

The second phenomenon concerns "inter-determinable" relations. Some determinables seem to entail others: if, for instance, a particular is colored, it must also have a shape, an area, etc. And some seem to exclude others: perhaps there are some determinables under which only physical particulars like electrons can fall, and others under which only non-physical particulars like spirits and Cartesian egos can fall. Finally, perhaps there are similarity relations among determinables: maybe, for instance, the former "physical" determinables are more similar to each other than they are to any of the latter "non-physical" determinables. The problem for DN of course is that, unlike the previous "intra-determinable" relations, these relations cannot be captured in terms of the internal set-theoretical structure of determinables.

One way to accommodate such relations might be to think of the particulars of any world as subject to an initial classification by means of a "super-determinable" into fundamental categories. Perhaps the categories include physical substances and mental substances, or perhaps they involve more or other categories. And then the account of a 'determinable' could be revised to require that the particulars falling under any determinable consist of all and only the particulars in the extension of some one determinate of the super-determinable. Roughly, the idea is to think of a determinable as a total classification of the particulars in some one category, rather than as a partial classification of all particulars, as before. The structure of this super-determinable could then be exploited to explain the relations of similarity and exclusion among the determinables. And the totality requirement could be exploited to explain their entailment relations. Thus modified, DN would then entail that every physical particular instantiates a determinate from each of the determinables appropriate to that category – a mass, a shape, an area etc. (Indeed, this may be the way to interpret those vague metaphysical slogans to the effect that every particular must be "fully determinate", that a "bare", or even semi-clothed, particular is impossible.)

In short, Armstrong's observations and the possibility of inter-determinable relations show that DN needs to be extended and complicated. However, they do not provide compelling grounds for abandoning its general thrust, in particular, its anti-quidditism.

REFERENCES

Armstrong, David M. (1978) A Theory of Universals, Volume II: Universals and Scientific Realism (Cambridge, U.K.: Cambridge University Press).

(1983) What Is a Law of Nature? (Cambridge, U.K.: Cambridge University Press).

(1989) A Combinatorial Theory of Possibility (Cambridge, U.K.: Cambridge University Press).

Bricker, Phillip. (1987) "Reducing Possible Worlds to Language", Philosophical Studies 52, pp. 331-355.

Heller, Mark. (1998) "Property Counterparts in Ersatz Worlds", Journal of Philosophy 95, pp. 293-306.

Johnson, W. E. (1921) Logic, Part I (Cambridge, U.K.: Cambridge University Press).

Kaplansky, Irving. (1972) Set Theory and Metric Spaces (New York: Chelsea).

Lewis, David K. (1968) "Counterpart Theory and Quantified Modal Logic", Journal of Philosophy 65, pp. 113-26.

(1983) "New Work for a Theory of Universals", Australasian Journal of Philosophy 61, pp. 343-77.

(1986) On the Plurality of Worlds (Oxford, New York: Basil Blackwell).

(1991) Parts of Classes (Oxford, New York: Basil Blackwell).

[1] See Lewis (1983): 343-77, and Lewis (1986): 50-69.
[2] Lewis (1986): 59-60.
[3] There is also a sense, discussed below, in which properties themselves stand in relations of similarity. I rely on the context to determine whether 'similarity' and 'resemblance' refer to the relation among particulars or the relation among properties.
[4] Roughly, the sparse properties correspond to universals (see Lewis (1983)). For the relation between sparse properties/universals and science see e.g. Armstrong (1978) and (1983).
[5] Lewis (1986): 60.
[6] The sense of 'natural' here is not that of 'not artificial', 'not rare', etc. A property is natural in the intended sense just when it makes for genuine resemblance among its instances.
[7] Lewis (1986): 61.
[8] Johnson (1921).
[9] It might be argued that these relations are better classified as "metaphysical" than "logical" on the grounds that the connections between sentences like 'a is red' and 'a is not blue' or 'a is colored' are "not a matter of the form of the sentences involved" or that the corresponding arguments are not valid "in the strict sense". But 'a is not blue' and 'a is colored' do follow from 'a is red'. And if translation into some formal language fails to reveal this, then so much the worse for this proposed translation. Surely it is better to reject proposed regimentations than to posit extra-logical "metaphysical" necessities.
[10] I will proceed on the assumption that this characterization of the logical relations among properties holds quite generally. However, this may not be the case. In the final section, I return to discuss the exceptions and outline how they might be accommodated.
[11] Completely uncontroversial examples of even intrinsic properties are hard to come by. But since these examples are illustrative only I will simply assume that they all qualify.
[12] Lewis (1986): 62.
[13] Lewis (1986): 60. This restriction may be redundant; Lewis claims that it is already built into our conception of what it is for a property to be sparse.
[14] Again, there is controversy over exactly which properties are excluded by this restriction.

[15] Roughly, the reason for these restrictions is to permit the development of a theory of properties that makes no appeal to any modal notions. Intuitively, the excluded properties all supervene in one way or another on those included, and supervenience is a modal notion.
[16] In calling DN "nominalist" I have in mind the traditional sense of rejecting Universals, rather than Goodman's and Quine's "Harvard" sense of rejecting classes. For the traditional usage, see e.g. Armstrong (1978): 28-43, Lewis (1983): 343-77, and Lewis (1986): 50-69. Because no issues requiring a distinction between sets and classes will be discussed here, I will use 'set' and 'class' as synonyms.
[17] Of course, this is not to say that classes are unproblematic. See Lewis (1992): 57-9 for why an ontology of at least pure classes should be granted Moorean status.
[18] The subscripts 'a' and 's' indicate the abundant and sparse sense of 'property' respectively.
[19] Formally, a metric space is a set M – the collection of points – together with a function D(p, q) that is typically real-valued – the distance relation -- defined for all p, q in M such that: (1) $D(p, p) = 0$; (2) $D(p, q) > 0$ for $p \neq q$; (3) $D(p, q) = D(q, p)$; and (4) $D(p, r) \leq D(p, q) + D(q, r)$. For an introduction to metric spaces see Kaplansky (1972).
[20] Formally: if <S, f> is a determinable and p a point in its metric space S, then the extension of p according to <S, f>, E_p =df. {x: f(x) = p}.
[21] This account permits "unstructured" determinables whose metric spaces encode distance relations according to which any two distinct points are the same distance apart. It also permits determinables whose metric spaces consist of just a single point. There will be at least one of these determinables for each (unstructured) class of particulars. So the abundant determinables are abundant enough. However, it also distinguishes among determinables that differ neither in which particulars fall under them nor in which particulars they classify together, but only in their metric spaces (even when these metric spaces differ only by simple transformation). Below I will introduce a counterpart relation among properties that provides a sense in which such determinables are really "the same".
[22] Intuitively, a property's extension is merely an accidental feature of it. So why identify the second member of a determinate with the extension of a point E_i rather than just the point i itself? One reason is that, as we will see below, the determinable nominalist takes the question of what is and isn't an accidental feature to be a context-dependent matter and one which is sometimes indeterminate. It seems undesirable to rule out a priori the possibility of contexts according to which the extension of a property is essential to it or contexts that leave it indeterminate whether the extension is essential or not.
[23] Do the familiar determinables of the actual world encode the same sort of metric space? It seems not. Some seem to encode "circular" spaces (e.g. the colors), others seem to encode spaces having the structure of the positive reals or the real line (e.g. the so-called extensive and intensive magnitudes -- mass, length, temperature, density, volume and so on), still others seem to encode spaces that have (or could have, if sufficiently generalized) the same sort of structure as the integers (e.g. those involving discrete quantities such as charge). And maybe in some cases the internal structure is more complicated still -- perhaps it is some kind of multi-dimensional space. Since DN aims to be a general theory of properties, it ought to maintain only that there is some internal ordering to each determinable and that this is encoded in the form of a metric space. The nature of the ordering of any given determinable should be a matter for empirical investigation.
[24] Although I also used 'belongs to' for the relation between a determinate and its first member, the context can be relied on resolve any ambiguity.
[25] It might be wondered whether perfect naturalness should be taken to be a world-relative notion (perhaps: a determinable is perfectly natural at a world iff there is no other determinable there with a higher degree of naturalness) or an absolute notion (perhaps: a determinable is perfectly natural iff it has a degree of naturalness above a certain absolute level). For ease of exposition, I will assume that it is an absolute notion (though nothing will be said about the degree of naturalness required to qualify as perfectly natural). However, the theory could easily be rewritten so as to allow it to be a world-relative notion.
[26] As will become clear below, however, this does not mean that a property can vary in its degree of naturalness from world to world (even though properties are sets!).
[27] Lewis's own class nominalism requires similar presuppositions about naturalness. But I do not follow Lewis in thinking of it as having to do with complexity of definitions (see e.g. Lewis (1986): 61). I assume that naturalness is a measure of objective similarity.
[28] So-called because it straightforwardly mimics Lewis's famous counterpart relation for particulars (Lewis 1968). Heller (1998) also introduces a counterpart relation among properties. His paper appeared just as I was finishing this one and I have not yet had time to study it fully. I do not know the extent to which he would agree with what I say.
[29] Or so I will assume. As mentioned above, DN presupposes that the degree of naturalness of a set is an accidental feature of it. And although nomological and causal roles etc. are beyond the scope of this paper, intuitively they should receive the same modal classification as naturalness.

[30] Notice the distinction between belonging to a world and existing at a world.
[31] The definite article here is misleading; as will be seen below, a given property may lack any counterparts at some worlds and it may have multiple counterparts at others.
[32] This account is modeled on Lewis gives a parallel treatment of possibilities for individuals (Lewis (1986): 220ff.).
[33] Consider, for instance, the first claim. By the definition of 'instantiates', a particular instantiates a determinate <<S, f>, E_i> iff it is mapped by f to the point i in the metric space S encoded by determinable <S, f>. Now, to instantiate more than one determinate of that determinable, a particular would have to be mapped to more than one such point. But this cannot happen because, by virtue of the definition of a 'determinate', f must be a function.
[34] Although properties are defined in terms of worlds – they are identified with world-set pairs – as I mentioned above, in section eight I will show how to construct worlds themselves in actualistic, non-modal terms. So, at least if this construction is successful, primitive modality has been avoided.
[35] See especially, Lewis (1983).
[36] Notice moreover that this is a difference that is detectable at the world in question; no access to otherworldly entities is required. Even at worlds where being 3kg and being 3K are coextensive, we could detect the difference between the mass and the temperature determinables by noticing that some 2kg particulars are still 273K, but some 50K, others 2000K, or that some 5K particulars are 10kg, but others 100kg, others 1.9kg.
[37] Assuming it is possible that some determinable could be sufficiently similar to both the mass and temperature determinable to qualify as a counterpart of both. DN of course is not committed to this being the case.
[38] Notice that there is no circularity here. DN defines 'property' in terms of worlds and 'world' in terms of abundant determinables. But the abundant determinables are characterized independently in terms of set-theoretic constructions from actual concrete particulars.
[39] For simplicity, I have ignored the possibility that only certain degrees of naturalness actually exist.
[40] It could have been defined in such a way that worlds represent facts involving only world-bound properties. But given DN's proposed counterpart theoretic interpretation of our modal intuitions the counterpart relation would still have to be applied to test whether the resulting schema is adequate to these intuitions. So it may as well be built directly into the definition of 'representation'.
[41] Only "probably" because of the aforementioned problem about the counterpart relation.
[42] We cannot say for sure because a full analysis of natural laws and the roles they determine lie beyond the scope of DN.
[43] Thus DN distinguishes the instantiation of a property at a world from its existence there, illuminating a distinction that has often either been denied or left obscure.
[44] See, e.g., Armstrong (1989).
[45] For a detailed discussion of the first of these difficulties, see Bricker (1987); for a detailed discussion of the second, see Lewis (1986): section 3.2.
[46] Armstrong (1978): 111.
[47] Formally: determinable <S*, f*> is a refinement of determinable <S, f> iff for any particulars x and y, (1) if $f^*(x) = f^*(y)$, then $f(x) = f(y)$; and (2) there is a function g from the points in S* to the points in S such that for any points p, q and r in metric space S*, if $d^*(p, q) < d^*(p, r)$, then $d(g(p), g(q)) \leq d(g(p), g(r))$.