Children's Invented Notations as Insights into Mathematical Thinking: A Review of "Mathematical Development in Young Children: Exploring Notations"
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# *Reviews*

## Children's Invented Notations as Insights Into Mathematical Thinking: A Review of *Mathematical Development in Young Children: Exploring Notations*

*Mathematical Development in Young Children: Exploring Notations.* (2004). Barbara M. Brizuela. New York: Teachers' College Press, 124 pp. ISBN 0-8077-44522 $24.95 (pb).

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Mathematical notation has recently been viewed as an integral part of the teaching and learning of mathematics (NCTM, 2000). The notation systems discussed in this book are the external representations made by pen and paper and which have external existence, commonly called "external representations" (e.g., Goldin, 1998, Marti & Pozo, 2000). These representations correspond to the widely adopted conventions, as distinct from internal representations or mental models. External representations include writing, numerical notations, drawings, and maps (Lee & Karmiloff-Smith, 1996). The notations, or external representations, explored in this book involve writing numbers, the role of commas and periods in numbers, fractions, data tables, and the interrelationship between different types of notations including graphs and number lines. The book is arranged chronologically in terms of the increasing age of the children and the increasing complexity of the mathematical content dealt with in the notations. The main hypotheses presented are as follows: (a) examinations of children's representations help us understand children's learning; (b) representations are invented and constructed by the child, and these processes help him or her understand the convention of notations systems; and (c) the interplay between various external representations not only helps children reach deeper conceptual understanding but also reach a deeper understanding of the representation itself.

In the research reported in this book, the author takes the stance of viewing notations as conceptual objects, things about which children think and develop ideas, and upon which they reflect. Thus, children's learning of mathematical notations is not simply seen as the development of perceptive-motor skills but is viewed as socially constituted objects with certain characteristics and a logic that underpins them. This is a refreshing approach to examining children's understanding and use of external representations. In this framework, children are perceived to be trying to understand the relationship between notations and the way in which they work together. It is also posited that children reconstruct social notation systems. These constructions make sense to them within the context of their own mathematics and are elements of the conventional ideas that they later develop. The stance taken in this book reflects Piaget's (1969, 1973) perspective of learning as being active and complex. Thus, learning is posited as occurring not only with children simply receiving or copying information but as involving effort on their part to construct their understandings and interpretations.

The important dimension that appears to be underrepresented in the discussions and examples presented is the interplay between the children's invented written notation and their description of their notation. It is suggested that the notation gives insights into children's understanding. Although this is an interesting and important perspective, the dialogue between the researcher and the child as they construct the notation systems is what gives real insights into how they are thinking about number. The notation systems alone, as the author acknowledges, are at times difficult to interpret. In fact, if one examined the notations in isolation, one could conclude that these children knew little about numbers or how they operated. As children describe their own invented notation systems to the interviewer, the discourse provides a window into the children's thinking.

The ideas in the book are presented using a series of case studies with young children. The data are predominantly gathered through clinical interviews and classroom events. Chapters 2, 3, and 4 focus on how three young children (aged 5 and 6) cope with ways of writing numbers, writing numbers beyond 12, and the role and function of commas and periods in numbers. Chapters 5, 6, and 7 reflect on some of the work that emerged from the Early Algebra project, directed by Analucia Schliemann and David Carraher. Some of these ideas have been presented in earlier journal articles and conference proceedings. The children in these chapters are all either year 2 or year 3 students. The external representations in these chapters exemplify how one child develops notations to contend with fractions, children's representations of data tables, and the different ways that children used representations, particularly graphs and data tables, when exploring a problem involving functions.

The recurring argument throughout this book is that the child's ability with written representations is not simply a process of learning how to make the shapes and how to perfect perceptive-motor skills but is about developing ideas about the representation itself. I In some cases, these ideas are quite sophisticated. For example, in the case of George in chapter 2, although his writing of numbers was imperfect, it did appear to indicate that the position of numbers played some role in his determining whether numbers were larger than or smaller than others. Brizuela used this example and others to hypothesise that although place value is difficult for many children, we know little about the genesis of place value and the capabilities of young children to distinguish the importance of place in written numbers. Brizuela conjectures that primitive notation systems begin to assist us in accessing this information, but it could also be suggested that this only occurs in conjunction with the child's description of these notation systems and the external influences that have come into play in their development.

In chapters 3 and 4, the discussion proffers the notion that children's invented notations are of utmost importance to children's learning. Thus, numbers are not simply primitive copies of notation systems that children experience in classroom settings. In fact, it is suggested that the interplay between the child's invented notation and conventional notation is central to supporting children's learning and helps in making sense of written number systems. In a constructivist framework, knowledge of the notational aspects of mathematics is created through the interaction between what one brings to the situation and what is presented to the learner. Paula, a 5-year-old, exemplifies this interaction by her conversations about sense-making with regard to two-digit numbers. She draws on many outside life experiences to support her understandings, including letters and names. In her invented world, 30 is the capital of 3. Drawing on theories of both Piaget and Ferreiro (1996), it is suggested that children need to invent in order to understand and assimilate information. For example, Paula's capital letters are products on the way to understanding two-digit numbers. Not only do numbers in different positions have different meanings in her world, they also have different notations. Brizuela suggests that this construction of knowledge parallels that of the history of the number system itself in that inventions become conventions and inventing and creating are of utmost importance to knowledge construction. This suggestion is also supported by the case of Thomas, a 6-year-old child. His evolving use of periods and commas in number representation exemplifies the relationship between evolving conceptual understandings about written numbers and the notations used to support these understandings.

The last three chapters extend the arguments about how children come to construct their understanding of different aspects of written number systems to include notations such as fractions, data tables, and Cartesian coordinates graphs—the notations commonly used in the context of solving algebraic problems. Here, notation systems are seen not only as tools to represent their understanding and thinking but also to further this understanding and thinking. It is claimed that the interplay between various representations in fact helps deepen one's understanding of each. The children featured in these three chapters are in year 3, and the work reported was conducted with the author's colleagues, Analucia Schlieman and David Carraher (e.g., Schliemann, Carraher, & Brizuela, 2001). The aim was to document issues of learning and teaching in an "algebratized" (Davydov, 1991) arithmetical setting, how reasoning evolves, and what role symbolic notations play in the course of this evolution of thinking.

In chapter 5 it is argued that the data presented support the theory that notation systems can help compress the meaning of problems and that the data collected show evidence of both socioconstructivist and constructivist theories of learning. The problems presented in chapter 3 were designed to probe children's understanding of fractions and how notations supported the development of this understanding. They were also designed to ascertain whether the notation system that children chose represented actions or were notation systems that quickly went to the heart of the problem, for example, whether they represented specific arithmetic changes within the problem as actions or they chose an external representation that supported more general thinking such as data tables. All problems had two key components, a fraction name (e.g., third, fourth, two thirds) and an amount ($3, $6, 10 pounds). The children also had had recent prior experiences representing fractions using pie graphs. It could be suggested that in each instance the fraction mentioned triggered the use of pie graphs as a representative system. In each problem, the fraction word mapped directly onto the representation that helped reach a solution in the best way possible. The task was thus reduced to identifying which number represented which fraction part. There is certainly evidence indicating that the fraction model helped thinking but no evidence of how this thinking evolved. Although they did choose general representations for their problems, it could be suggested that the structure of the problem itself suggested this more general representation. Chapters 6 and 7 shed more light on this important issue. Also the argument that the data show thinking somewhere between socioconstructivist and constructivist theories of learning is tenuous. It is suggested that the first was evidenced by the choice of the representation, a representation discussed in class, and the second evidenced by the way in which Sarah thought about this diagram as she solved the problem. Although the model had been presented in class, the mathematical community, Sarah constructed it in a way that exhibited deep understanding of how the model helped her solve problems and her internal construction of its features, thus exemplifying both socioconstructivist and constructivist learning.

In chapters 6 and 7, the focus turns to data tables and how children learn about what they consider relevant in their construction and perceptions of the additive functions as reflected in their self-designed tables. Formal tables were presented to the whole class by the teacher of the Early Algebra project. Examples are given to illustrate what features of these tables children used for their own representations of data tables and graphs and how these representations interact and contribute to the understanding of the other. One predominant feature that all children tended to include in their data tables was explicit information in the form of some kind of referent or owner of the data. In some cases, they tended to prescribe an owner to each entry. It is suggested that the tables they produced revealed their workings of the logic of the problem at hand, and thus gave some insight into children's conceptions and understandings of additive relations. But what cannot be drawn from the data are the interactions between their own constructions of data tables and their development of their understanding of additive relations. The examples clearly demonstrate that young children can effectively use data tables and that these representations help to clarify and articulate their thinking. This is an important finding.

Chapter 7 focuses on the interactions between different representations and the understanding that occurs in representing problems in multiple ways. It begins to address the important issues of (a) what is gained when children establish relationships between different external representations of problems, (b) the impact that these relationships have on each other, and (c) how children relate their representations with those proposed by others. The children's work illustrates how they coordinate multiple representation systems to reason simultaneously about linear functions. The examples also illuminate how, when appropriating others' representations, children use them to corroborate their own understandings and reformulate them for their own requirements. Thus, notation systems both compress meanings and enable children not only to communicate their own understanding but also support children in developing this understanding. They permit understandings to be shared, and sharing assists children in reformulating and refining their own understandings.

The examples delineated in this book illustrate (a) construction of notation systems supporting the construction of conceptual aspects of mathematics, (b) rich and important interactions between children's invented notation systems and conventional notation systems, and (c) sharing of notation in the same register and different notation from different registers assisting in reconstructing and deepening children's conceptual understanding. As Piaget (1969, 1973) suggested, children learn by actively engaging in the process, and through this activity, they are continually assimilating and accommodating new perspectives. Brizuela suggests that Piaget's theories are relevant to the study of mathematical notations because of the role he assigns to the learner. Learners are active participants who try to understand the world that surrounds them. They learn through their actions and construct categories of thought. They do not appropriate knowledge by copying reality or information transmitted to them. Thus, in learning mathematics, notation systems are neither the added extras nor the perceptive-motor exercise that relies on copying. Brizuela claims that they are central to the learning itself, a claim that needs further investigation. The book provides many rich examples that illuminate differing uses of notation systems. Its strength lies not in the notation themselves but in the excerpts of how children interpret their own invented notation systems, as it is here that illustrations of differing thinking occur.

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