AC 2008-1824: INTEGRATING ALGEBRA AND ENGINEERING IN THE MIDDLE SCHOOL CLASSROOM

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Integrating Algebra and Engineering in the Middle School Classroom

Abstract

The Building Math curricula project was originally titled "Integrating Algebra and Engineering in the Classroom." It resulted in the development of three middle school instructional units that uniquely integrates inquiry-based mathematics investigations and engineering design challenges. The engineering design challenges provide meaningful and engaging contexts to learn and use mathematics, and to develop students' teamwork, communication, and manual skills. The mathematics investigations yield useful results to help students make informed design decisions. In particular, special focus was given to the development and use of algebraic reasoning. This paper seeks to answer the questions: "What algebraic learning is evidenced in the student work and classroom videos collected in the pilot studies?" and "How was the engineering design is informed by the mathematics research phase of design challenge?" The major finding claims that when engaged in Building Math design challenges, middle school student at different grade levels use algebraic reasoning when analyzing changing rates of an exponential function, interpret slope in a meaningful context, and use a mathematical model to make reasonable predictions. They then use this understanding to inform their engineering designs to meet the criteria and constraints of the challenge.

Algebra and Engineering

There is widespread consensus that algebra is important as a "gatekeeper" to higher levels of math and careers in science, technology, math, and engineering fields (Moses, 1993[1]; Pelavin & Kane, 1988[2]). Also, prominent organizations such as the National Academy of Engineers and the International Technology Education Association have been calling attention to the need to increase technological literacy for all people, even those who may not enter or are not in quantitative professions. "To take full advantage of the benefits and to recognize, address, or even avoid the pitfalls of technology, Americans must become better stewards of technological change" (Pearson, 2004[3]).

The Building Math project sought to address the demonstrated needs described above by developing activities that integrate algebra and engineering. This was not an easy endeavor, as existing activities tended to emphasize one subject over the other, or require a team of teachers (i.e., technology, science, and math) to coordinate over a fairly lengthy period of time. After several iterations of implementing activities in pilot classrooms, activities that successfully integrated algebra and engineering had these qualities:

(1) the activities are embedded in some greater context that makes the design work have a purpose, and
(2) the activities make mathematics a necessary means to designing an effective product or process.

For example, in the Amazon Mission unit (consisting of three week-long design challenges that can be done throughout the year), students read a one page introduction that invites students to imagine that they are planning to visit an indigenous people group in the Amazon rainforest.

Students learn that many Yanomami people suffer from malaria and that their first design challenge is to design a prototype of a medicine carrier that can keep the medicine insulated within a temperature range. Students analyze a graph showing the change of temperature over time that models the performance of an existing poorly functioning medicine carrier. They then conduct hands-on controlled experiments to learn about the insulation performance of different materials, collect data and create graphical representations to make predictions and inform their designs. This is the first of three design challenges in the Amazon unit that utilize the same storyline.

The storyline describes needs or problems for which students need to engineer solutions, and the math investigations support and inform the designs. This is in contrast to many traditional engineering design challenges where students are simply told to design an object like a bridge or tower without any real-world context that expresses a need for a bridge or a tower. This is also in contrast to open-ended design challenges where students are simply given materials to tinker with, usually by trial and error, until they find something that "works." Building Math design challenges are research-based – the math investigations develop a base of knowledge that students draw upon to make design decisions.

Other guiding principles for designing the units included:

(1) activities should be relatively short (maximum 1 week),
(2) math topics should be appropriate for and important ones in middle school,
(3) students work in groups,
(4) algebraic relationship and reasoning are highlighted,
(5) students build a product or design a process that can be tested,
(6) there are quantitative methods of measuring performance,
(7) students use the 8-step engineering design process during challenges (see Figure 1), and
(8) a variety of solutions are possible.

These principles are in line with five strategies for contextual learning used by nationally recognized science and mathematics teachers as identified by The Center for Occupational Research and Development (CORD)[4,5] and Crawford[6]

- Relating – learning in the context of one's life experiences or existing knowledge
- Experiencing – learning by doing, through exploration and discovery
- Applying – learning by putting the concepts to use
- Cooperating – learning in context of sharing, responding and communicating with other learners
- Transferring – using knowledge in a new context

The three Building math units are described in Table 1.

Table 1: Overview of Building Math Units

Unit Name and Storyline	Grade Level	Engineering Design Challenges	Math Content
Everest Trek: Students imagine that they are climbing Mt. Everest.	6	• A coat • A bridge to cross a crevasse	• Graphing • Measurement • Representation

		• A zip-line transporter to descend the mountain	• Data collection and analysis • Relating two variables • Proportional reasoning
Stranded! Students imagine that they are stranded on a deserted island in the South Pacific.	7	• A shelter • A water collector • A canoe loading plan	• Scale models • Proportions • Measurement • Symbolic representation • Graphing • Nets • 3-D shapes • Volume • Surface area
Amazon Mission: Students imagine that they are on a community service trip to visit the Yanomami people in the Amazon region of South America.	8	• A medicine carrier • A water filter • An intervention plan to slow the spread of a virus	• Graphing • Measurement • Data collection and analysis • Representation • Relating two variables • Compound probability • Exponential patterns • Converting units • Compound inequalities

Middle School Algebra

Traditionally, algebra is taught as a separate course in the late middle school to early high school grades. However, educators and researchers like Cathy L. Seeley, a former president of the National Council for Teachers of Mathematics (NCTM[7]), would describe the development of algebraic thinking as "a process, not an event." Kaput observed in an address given at a national symposium on algebra in 1997 that traditional school algebra focuses on "algebra as syntactically-guided manipulation of formalisms" at the expense of other forms of algebraic reasoning, such as generalizing and formalizing patterns, modeling, and "the study of structures and systems abstracted from computations and relations" (Kaput, 1997[8]). In other words, a traditional introductory algebra course tends to focus on manipulation of symbols – e.g., collecting like terms, factoring, simplifying, and solve equations.

The *Principles and Standards for School Mathematics* (NCTM, 2000[9] stated, "All students should: (1) understand patterns, relations, and functions; (2) represent and analyze mathematical situations and structures using algebraic symbols; (3) use mathematical models to represent and understand quantitative relationships; and (4) analyze change in various contexts." Algebra is a content strand that cuts across all the grades, from Pre-K-12. Elementary school students can explore patterns in geometry and arithmetic, and even begin using symbolic representation (Schliemann, Carraher, Brizuela, 2007). At the middle school level, students continue their symbolic work with equations and expressions. There is particular focus on proportionality – expressed as ratios, proportions, percents, rates, constant change in a number series or table of values, a straight line on a graph, relationship between two variables, slope, etc.

In Building Math units, students engage in algebraic reasoning by modeling physical phenomena, analyzing change in both linear and non-linear relationships, extrapolate and interpolate data based on trends, describe the shapes of graphs within meaningful contexts, represent data in tables and graphs, and generalize patterns. For example, in the Everest Trek unit, students conduct experiments using foam bridges and pennies to see how width relates to bridge strength (quantified as the amount of “sag” produced by a fixed live load), and how thickness relates to bridge strength. The former shows a linear relationship while the latter shows a quadratic relationship.

Purpose of Study

Given that the intent of developing Building Math is to engage students in algebraic reasoning within the context of engineering design challenges, it is appropriate to direct our research study to answering these questions: What kind of algebraic reasoning and learning are evidenced in the student work and classroom discussions? And specifically, what are students’ understandings of change in exponential functions? What are their misconceptions? What strategies did 6th grade students use to extrapolate data values based on the exponential pattern? How did the story context help 8th grade students understand the meaning of slope?” The next series of questions are related to the engineering design of the prototype: How was the engineering design informed by the mathematics research phase? What specifically did the students use from the research phase to inform their design choices?” What other criteria and considerations did the students integrate into their design? How creative were the student prototypes?

Tasks

Research Phase

The first research phases of two units were chosen for comparison. They both involve analyzing an exponential function by students from opposite ends of the middle grades – 6th and 8th grades. In the 6th grade Everest Trek unit, students examine a table of values and it corresponding graph that shows how temperature changes over time of a mannequin warmed to body temperature and wearing a T-shirt that is then placed in a room with an air temperature of - 15ºF (see figure 1).

Table 2.1: Temperature Under Cotton T-Shirt Over Time

TIME (SECONDS)	0	5	10	15	20	25	30	35	40	45	50	55	60
TEMPERATURE (°F)	80.0	65.3	52.9	42.4	33.6	26.1	19.7	14.3	9.8	6.0	2.7	0.0	-2.3

Graph 2.1: Temperature Under Cotton T-Shirt Over Time (in -15°F Lab Room)

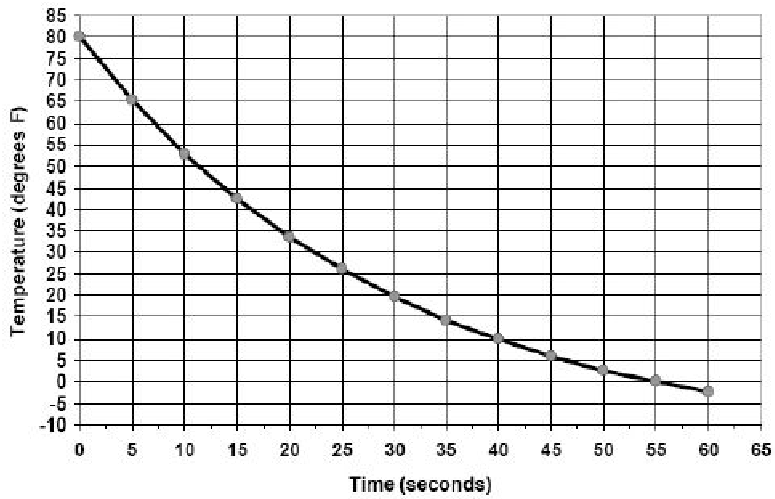

Figure 1. Temperature vs. time for mannequin in cold room

The student workbooks guide students through a series of questions to help them analyze change in the graph: how the temperature changed between certain times and to describe the overall change. We will focus our analysis on question 2.4, where students are asked to predict what the temperature will be at 65 seconds (see figure 2).

2.4 Using the graph, what do you think the temperature under the cotton clothing will be at 65 seconds? ____________________ °F

How did you come up with this prediction?

____

____

____

Figure 2. Copy of question about predicting temperature from graph

In the 8th grade Amazon Mission unit, students are also given a table of values and a corresponding graph that shows an exponential function showing change of temperature over time, but this one concerns warming rather than cooling (see figure 3).

Table 2.1: Temperature of Medicine over Time in Current Container (in 98°F Lab Room)

TIME (MIN)	0	10	20	30	40	50	60 (1 hr)	70	80	90	100	110	120 (2 hr)
TEMP (°F)	59.0	68.4	75.5	80.9	85.0	88.1	90.5	92.3	93.7	94.7	95.5	96.1	96.6

Graph 2.1: Temperature of Medicine over Time in Current Container (in 98°F Lab Room)

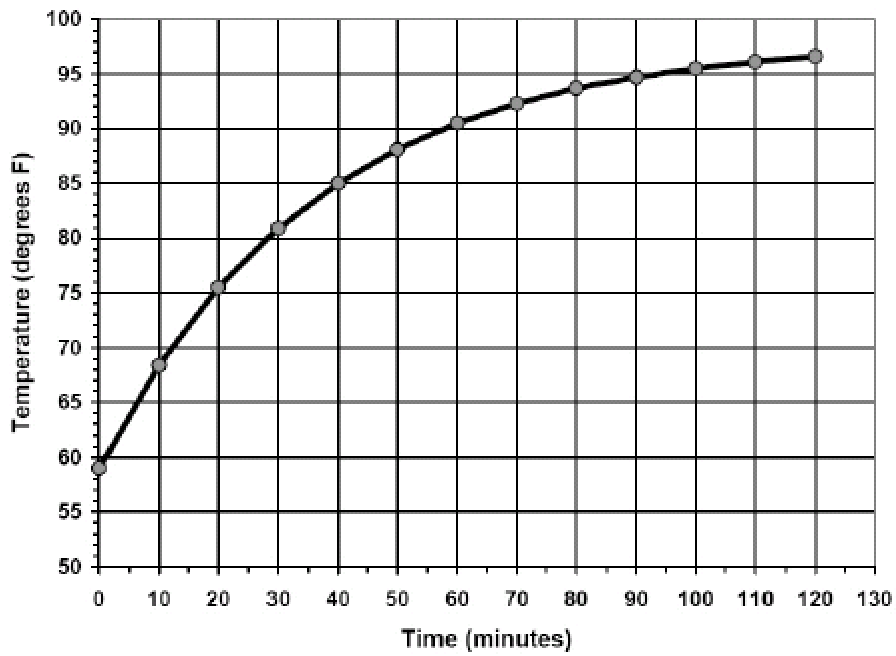

Figure 3. Copy of table and graph for Amazon unit.

The malaria medicine starts off at a chilled temperature of 59ºF and the medicine carrier is then placed in an environment of 98ºF to simulate the tropical Amazon climate.

As in the Everest Trek unit, the 8th grade students answer a series of questions in the Amazon Mission unit that helps them analyze change in the graph, including drawing straight lines between two sets of points on the graph and finding the slope of each line, and describing the shape of the graph. We will focus our analysis on these questions (see figure 4), and examine students' procedures for finding slope, their interpretation of the meaning of slope, and compare their answers to 2.1 and 2.4. These two questions appear to be similar, but we wonder if the intermediary questions about slope help students to describe the change in the graph in more precise and nuanced ways when answering 2.4.

2.1 Based on the graph, describe how the temperature is changing as time goes on.

2.2 Using a ruler, draw a straight line that goes through the data points at 0 and 10 minutes.
What is the slope of this line? ____________
What does this slope tell you about what is happening to the medicine's temperature between 0 and 10 minutes?

2.3 Using a ruler, draw a straight line that goes through the data points at 110 and 120 minutes.
What is the slope of this line? ________
What does this slope tell you about what's happening to the medicine's temperature between 110 and 120 minutes?

2.4 Based on the graph and your answers to 2.2 and 2.3, describe how the rate of change in the temperature changes over time.

Figure 4. Copy of questions from Amazon unit

Idea Generation: Brainstorming and Choosing

In the engineering design process of Building Math, there are eight steps: identify the problem, research, brainstorm, choose, build, test, communicate, and redesign. In this section we consider the students brainstorming work and what they choose as their potential solution. These two tasks can be considered as their personal idea generation phase of the design challenge. For the Everest unit, the student's brainstorm page can be seen in Figure 5.

3. BRAINSTORM POSSIBLE SOLUTIONS: GEARING UP!

You've done some great research and are ready to create some possible coat designs. As you brainstorm possible solutions, keep these design requirements in mind.

ENGINEERING CRITERIA	
GOOD INSULATOR →	When surrounded by ice, the material must keep the temperature above 65°F for 30 seconds.
THIN →	The total thickness of the materials must not exceed 2 cm.
LOW COST →	Your coat design should be as low in cost as possible.

You may combine the four available materials in whatever manner you choose. The cost of one layer of each material to make one coat is listed in Table 1.1 below. Table 1.2 provides the cost of optional items that you may want to include in your coat design.

Table 1.1: Basic Materials for Coat

MATERIAL FOR COAT	COST PER LAYER
denim	$5.25
fleece	$8.50
nylon	$4.75
wool	$10.50

Table 1.2: Optional Items for Coat

OPTIONAL ITEM	COST PER ITEM
zipper	$0.99
plastic button	$0.10
stainless steel button	$0.15
denim pocket	$0.53
fleece pocket	$0.85
nylon pocket	$0.48
wool pocket	$1.05
denim hood	$1.59
fleece hood	$2.55
nylon hood	$1.43
wool hood	$3.15

Figure 5. Copy of brainstorm phase from Everest

For the Amazon unit, the student brainstorm page can be seen in Figure 6.

3. BRAINSTORM POSSIBLE SOLUTIONS: MALARIA MELTDOWN!

You've done some great research and are ready to think about some possible medicine-carrier designs. As you brainstorm possible solutions, keep these design criteria and constraints in mind.

ENGINEERING CRITERIA	
GOOD INSULATOR →	In an environment that is 98°F, the inside of your model carrier must stay between 59°F and 86°F for 2 hours.***
LOW COST →	Your model should be as low in cost as possible.
RUGGED AND PROTECTIVE →	You will place an egg inside of the carrier, and you will drop the carrier from a height of 1 meter. Both the egg and the carrier must remair intact.

*** Due to time constraints, you will let 1 minute represent 2 hours. Your model will perform to the same level in 1 minute as a carrier that is 11 times thicker would in 2 hours. Therefore, your actual carrier design will be 11 times thicker than your model design.

ENGINEERING CONSTRAINTS

You may combine the five available materials in whatever manner you choose. The cost of 1 square meter (m^2) of each material is listed in Table 1.4 below.

Table 1.4: Cost per Square Meter (m^2) of Materials for Medicine Carrier

MATERIAL	COST FOR 1 SQUARE METER (m^2)
corrugated cardboard	$0.99
foam board	$2.25
bubble wrap	$2.85
aluminum foil	$0.50

Figure 6. Copy of brainstorm phase from Amazon

Participants

The two 6th grade pilot groups were in two independent, suburban schools. One group of 6th graders consisted of 16 students – exactly 8 girls and 8 boys – and was in a class designated as 6th grade honors mathematics. Students in this group included both American and international students. The other group of 6th graders consisted of five classes of 7 to 20 students. The class sizes were particularly small at this school because it is a school for student with reading or language challenges such as dyslexia.

The two 8th grade pilot groups were in two urban and suburban public schools, respectively. One group of 8th graders consisted of three classes of 18-26 students in an urban charter school. Actually, one of those classes was a mix of 7th and 8th graders in an 8th grade level math class. One class was considered "low level" while the other two consisted of student with mid-to-high ability levels. The other pilot group consisted of two classes of 18-23 mid-to-high ability level students in a suburban public school.

Data

Several sources of data were collected: classroom videos, student workbooks, web feedback surveys from teachers, videotaped interviews with teachers, and identical pre- and post- pencil-and-paper assessments that included both multiple choice and open-response questions. This study will focus on analyzing select problems from the student workbooks and relevant classroom videos.

As we examined students' responses to the select problems, we counted correct and incorrect responses, and categorized students' explanations or strategies of finding their answers. We also assessed the student's use of the research to inform their engineering design. We also watched the videos to see how teachers interacted with the students to discuss these problems, and what math ideas they focused on.

Results

6th grade – Everest

Mr. H's uses the Connected Mathematics curriculum (CMP) with his 6th grade math honors class and thus, his students expect the inquiry-based, investigative approach that also characterizes the Building Math activities. The classroom video shows Mr. H reviewing questions for research phase 1 that were assigned for homework. He began the class by observing students had a variety of answers to question 2.4, and asked students to discuss their answers with their groups and share their strategies. During that time, he walked around and listened to the groups. After a short while, he brought the groups back to a whole class discussion. Since he already had a sense of the different strategies they were using when he circulated the room during group work time, he was able to quickly identify which students used different strategies and asked them to share their ideas. There were four strategies that produced reasonable predictions:

- Strategy 1: extending the curve on the graph and reading the value off the graph's scale
- Strategy 2: find the first order differences in the table's numerical values and estimate that the next difference would be slightly smaller than the one before.
- Strategy 3: find first and second order differences in the table's numerical values and average the last three second-order differences.
- Strategy 4: start like strategy 3 but then go further and find a third level of differences, and noticing that the third level of differences are almost constant (-2), subtract 2 for the next third level difference and then work one's way up the levels, subtracting the appropriate difference each time.

All four strategies yielded reasonable estimates of between -4° and -4.4°.

We did not have classroom video that documented the teacher discussing the same problem in the other 6th grade pilot, but from looking at a sample of the student work, we observed a greater range of answers, including ones that fell outside the reasonable range. When asked how they obtained their answers, students used the first strategy of extending the graph, and others, like subtracting a little less than the previous difference, subtracted the same amount as previous difference, and mentioning that the change is "slowing down." None of the samples showed the more precise estimation of using second and third order differences as H's students did. Despite the less precise estimation of the second group of pilot students, all students were able to notice the decreasing or "slowing" rate of change in temperature.

The students from both classes discovered that fleece and wool are good insulators and knew that those materials should be included in their coat design. However, they were also aware of other considerations that were specifically discussed as well other considerations that they wanted to impose – such as comfort, style, and personal interests. In the idea generation phase, the students worked independently and in small groups (three or four). Depending on the group dynamics the designs evolved around their prior knowledge on what is a good coat, their findings from their research, their understanding of the design criteria and constraints, their self-imposed criteria for style and durability, and their creativity. One sample of the student coat designs is shown in Figure 7.

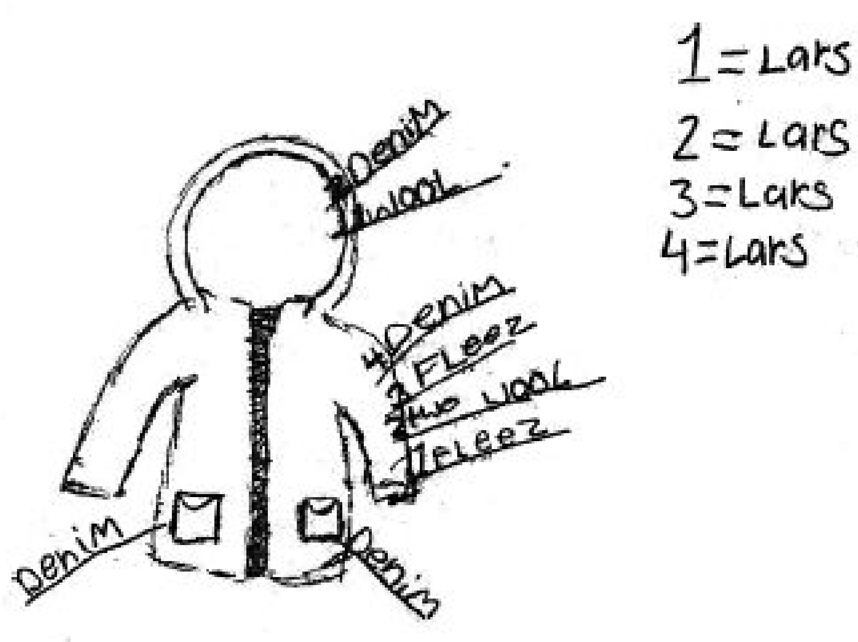

Figure 7. Sample student design for coat.

8th grade - Amazon

Mrs. M, one of our 8th grade pilot teachers, instructed her students to work in groups to analyze the graph shown in figure 3 and to answer questions about the graph, including those shown in figure 4. Then she had one person from each group report back, focusing especially on the values of the slope of lines that connect two pairs of points on the graph. Almost all groups gave the same answer to question 2.2: 9.4º per 10 minutes. Mrs. M neither agreed nor disagreed with the answer but wanted to know what the slope is in per minute, that is, as a unit rate. She then proceeded to do a mini-lesson on how to find the slope when given two points. The workbooks do not specify how students should find the slope, nor does it represent the data points as coordinates. Mrs. M didn't explicitly relate the coordinates to the data points, and perhaps assumed that her students would already understand this connection. In any case, she showed them how to calculate the slope by giving them the formula $(y_2-y_1)/(x_2-x_1)$, labeling the coordinates using those variables, and then instructing her students to calculate the slope by using the formula. She made sure that they attach the correct units to the numerical values. Then she instructed them to simplify by dividing by the denominator, which was 10, to get 0.94º per minute. This was repeated for question 2.3.

Students in her class were all able to explain the meaning of slope, for instance, a slope of 9.4º/10 means that the temperature went up 9.4º during the first 10 minutes. Although some students did describe in 2.1 that the temperature increases more quickly and then the increase slowed over time, others merely said that the temperature increased over time. But almost all students noted in 2.4, after they did the slope calculations, that not only is the temperature increasing over time, the rate of change is changing ("decreasing," "gets smaller," "slows down").

Unlike Mrs. M, who directed her students towards 1 answer for each of the slope calculations, Mr. L gave his 8th grade students a general strategy for finding the slope by extending the line to form the hypotenuse of a right triangle, and estimating the other two sides to represent the "rise" and "run" values. Most of the students' answers did not include units and interpreted the slope in general terms such as, "the temperature between 0 and 10 min is increasing" or "the temp is increasing rapidly." If students do not simplify their fractions, it can appear that there were a variety of different answers. According to the video, students began to answer the questions in research phase 1 in class, and finished it for homework. The following day, Mr. L asked students which questions they want to go over, and started with 2.4 and 2.5, which focused more on the conceptual understanding of slope and how they show a decreasing change in rate. When another student asked about the answer to 2.3, which is to find the value of slope, Mr. L asked the class for the answer and initially, no one answered. When he called on a student, the student gave the wrong answer. Mr. L went through the triangle-estimation strategy of finding slope and estimated that the slope is 1/30, which he said means that it's a rate that says that the temperature would increase 1º over 30 minutes. When he asked for the answer to 2.2, the value of the slope for the line connecting the first two data points, a student answered 2, and Mr. L accepted that estimate without going through the calculation, and just pointed that this value is greater than 1/30, which backs up his point earlier about the relative comparison of steepness and what that shows about relative rates of change. Although students' slope values may be reasonable estimates, the disadvantage of this strategy is that, even with simplifying fractions to the lowest terms, it still appears that students got a variety of different answers, and the different denominators make it hard to compare. Also, these differing results make it difficult to compare the two slope values for 2.2 and 2.3 as rates (comparing 2 to 1/30). Contrast this to Mrs. M's students, who were able to compare the slope 9.4º/10 min and 0.5º/10 min and see the exact magnitude of the change in rate.

When comparing Mrs. M and Mr. L, one can observe a contrast between focusing on procedural fluency and conceptual understanding[10]. The advantage of Mrs. M's approach is that students are able to correctly calculate slope and doing the procedure together in class leads to the same form of the answer. Despite a lack of teacher focus on the meaning of slope, students were nevertheless able to interpret it correctly in the context of change of temperature over time. In Mrs. M's case, procedural fluency provided the correct numerical values so students can see that when the change is great, the slope is almost 1, whereas when the change is not so great, the slope is closer to 0.

Mr. L focused more on the conceptual understanding of relating the visual appearance of steepness to the relative magnitude of change – that is, a steeper line means a greater amount of change in temperature over time, and a less steep line means a less amount of change in temperature over time. His students are also able to correctly observe that the rate of temperature change is decreasing, but their slope values were rough estimates and their interpretation of the meaning of the slope is a lot more vague. That is, they are not able to say with the same amount of precision as Mrs M's students were able to.

This is a case where procedural fluency actually supports a stronger conceptual understanding, which is a conclusion that may be surprising, given that Building Math, and other math curricula that uses the constructivist approach, tends to privilege conceptual understanding over procedural fluency. Without the procedural fluency, students can still grasp the conceptual understanding, but it isn't as robust, and, in this case, students would miss the connection between slope and rates.

The students used their findings from the research phase to inform their "brainstorming and choosing" of designs individually and then in groups. As evidenced in the student workbooks, in Mrs. M's class almost all (35 out of 41) of the students (there were three incomplete books and four books which showed that students insisted on having aluminum in their design) used their research findings as a basis for their medicine carrier. Specfically, they all chose to use the combination of two best performing insulators based on their research phase. Most of them also realized that more layers would insulate better. With this knowledge as well as with the other criteria, such as protection, they would add at least one more layer to their medicine carrier design (e.g., cardboard because of the stiff nature). An example is in figure 8.

It is worth noting that based on the communication phase of the engineering design process considering all the criteria and constraints including price, the students would then remove some layers realizing that they could stay within insulation requirements and decrease costs.

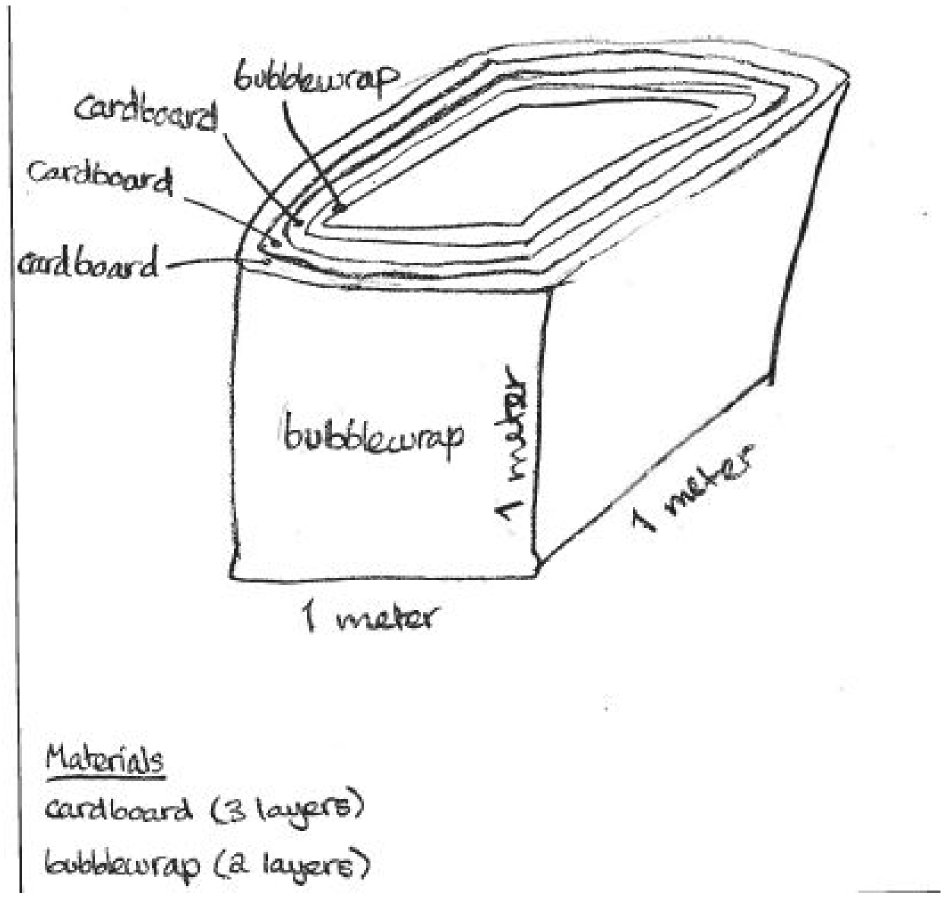

Figure 8. Example of medicine carrier

We know experiments don't always work perfectly. So how do students deal with questionable research results? For one of the groups (2 student workbooks) that chose aluminum foil in their idea, their research phase somehow indicated that it was a good insulator in one of four tests. Thus, they did include aluminum foil in their idea; however, after prototyping and testing, their results indicate that the combination of rubber and aluminum did not work very well which gave them pause. It appears that the students were wrestling with conflicting findings, and eventually their final design did not include the aluminum foil.

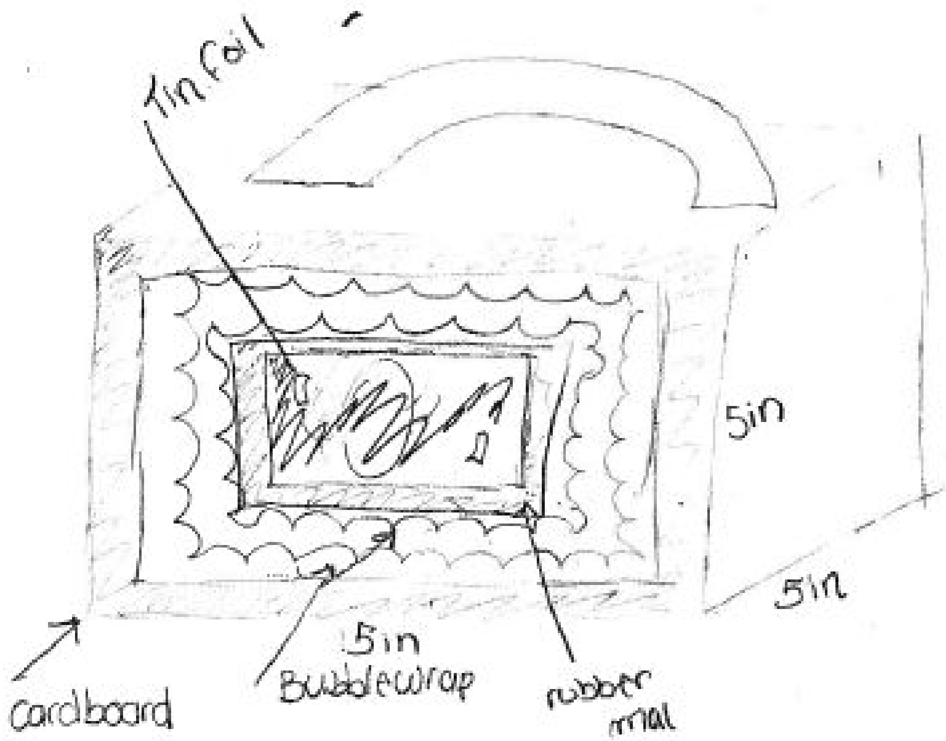

Figure 9. Example of medicine carrier with aluminum foil in design

The other group that used aluminum foil (2 student workbooks) had found that one layer of aluminum foil and one layer of rubber worked well in their research phase. Their idea thus had those elements, but they added bubble wrap and cardboard on top of the foil/rubber. However, they had added these layers to meet the other criteria of protection. So in this case, during the communication phase to the class, the one of the students in the group was insistent on retaining the aluminum/rubber combination in future designs. Although her thought process is logical, we think this is an example of where the teacher can step in and provide more guidance so students do not continue with misconceptions on insulation.

Aside from those two groups, there are a large number of students who persistent with the misconception, even after research evidence shows that aluminum is not a good insulator, that aluminum is a good at insulating. We speculate that the students think that since their food is wrapped in aluminum foil then it must be a good insulator. This would support the notion that student's prior knowledge is difficult to overcome even in scientific investigations.

In Mr. L's class, we found similar trends in informed design related to insulation; so we focused on examining some of the video recordings from this class. We found that the student discussions were highly focused on geometric dimensions - size and shape. The teacher had told the class specifically to consider non-box shapes. One video example showed how the students estimated sizes of the container based on the size of the material that would be placed inside (in the story it was medicine; however, for prototyping it was an egg). So the students knew that an egg was several inches in size and had some discussion about width vs. circumference. In another group, the students used rulers and their prior knowledge to consider geometric sizes –

for example, having about one third of the box for the egg storage and filling the rest with foam. These rules of thumb appear to be based on their prior knowledge of containers and packaging. One creative design is shown in figure 10.

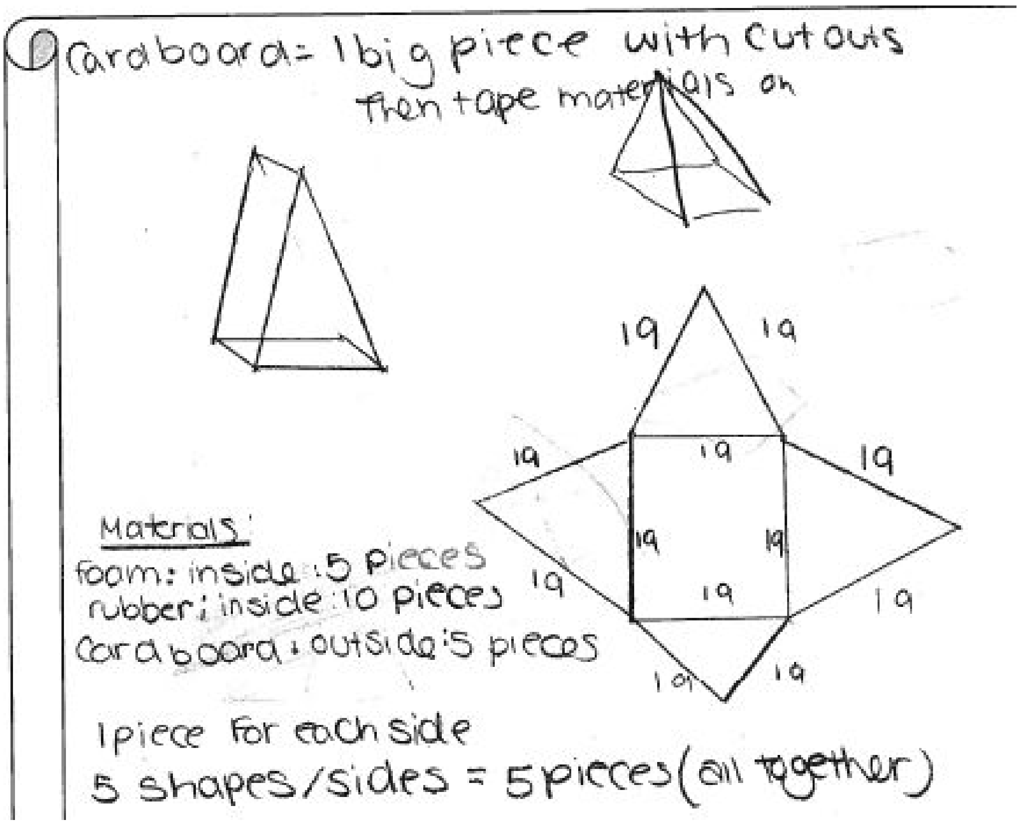

Figure 10. Non-cube medicine container example

Comparing across 6th and 8th grade classes it becomes clear that students are able to understand and compute slopes and changing slopes through more than one approach. These approaches are satisfactory for the engineering design phase in which students use their research findings to inform their prototypes and redesigns. So from an engineering viewpoint, the various algebraic approaches are robust and students are engaged in the learning process and context. From an algebraic teaching standpoint, teachers can use the instructional materials through their preferred approaches (whether more procedural or conceptual). Moreover, teachers can use these instructional materials as applications of prior teaching units on algebra. It is worth showing teachers the various approaches that these pilot teachers used the Building Math units to reflect upon their teaching styles and opportunities to try new approaches.

Conclusion

Integrating algebra and engineering can be done effectively by having math be essential to informed engineering decisions. A contextual approach for the units provides engagement in the activity, especially when students can learn together in small groups. Through the Building Math activities, students can find meeting the engineering design challenges satisfying without being overly competitive. The findings from this analysis indicate that it is possible to make non-linear, exponential functions accessible to students of different grade levels using different approaches. In 6th grade, students analyzed orders of differences using the numerical values from the table and also extending the curve of the graph to recognize the changes in rate. In 8th grade, students related slope to rate and observed that the relative change in slope indicates relative range in rate. These approaches all lead to a better understanding of the changes in temperature with time in order to inform their engineering design prototypes. Their engineering designs also reflected their understanding of the other criteria and constraints that needed to be met.

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