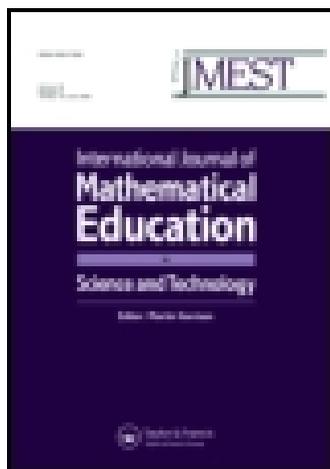


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## **A Chinese young adult non-scientist's epistemologies and her understandings of the concept of speed**

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Past research has investigated students' epistemologies while they were taking courses that required an integrated understanding of mathematical and scientific concepts. However, past studies have not investigated students who are not currently enrolled in such classes. Additionally, past studies have primarily focused on individuals who are native English speakers from Western cultures. In this paper, we aim to investigate whether Hammer and his colleagues' claims concerning learners' epistemologies could be extended to individuals who lack advanced mathematics and science training, have had different cultural and learning experiences, and have grown up speaking and learning in another language. To this end, we interviewed a participant with these characteristics about her understandings of the concept of speed. Our findings show that previous theoretical frameworks can be used to explain the epistemologies of the individual examined in this study. The case suggests that these theories may be relevant regardless of the learner's mathematics and science background, language, educational experience, and cultural background. In the future, more cases should be examined with learners from different academic backgrounds and cultures to further support this finding.

**Keywords:** epistemology; young adult; novice

### **1. Introduction**

In education studies, researchers have investigated the epistemologies held by students who are taking courses that require an integrated understanding of mathematical and scientific concepts. These researchers have reached a consensus that students' epistemologies regarding the nature of knowledge and learning affect how students approach disciplinary problems and understand mathematics and science concepts. For example, Hammer and his colleagues have found that many students, when studying physics in college, frame the learning activity as memorizing a collection of equations and using them to solve the physics problems given in the textbook and exams.[1,2] In addition, for students, solving a typical textbook problem usually involves finding a mathematical solution, such as a number or an expression. These research findings, however, have yet to be explored with individuals who are not currently enrolled in mathematics and science classes. Additionally, these studies have primarily focused on individuals who are native English speakers from Western cultures. The participant in our study reported in this paper was originally from China and was not enrolled in mathematics or science classes at the time of the study.

In this study, we interviewed a young woman named Klara to explore whether the research claims in relation to students' epistemologies could be extended to consider

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learners with different characteristics. Klara's characteristics are unlike those examined in previous studies. She had not taken a single mathematics course in the past five years, nor a science course in the past eight years. Klara took algebra in high school and one year of introductory-level calculus in her first year of college. She took algebra-based physics in 10th grade, which addressed the concept of velocity.<sup>1</sup> After that, however, she had no further schooling in science or mathematics. Her education until the end of her undergraduate career was in her native country China. Given these characteristics, we wanted to examine whether the framework about learners' epistemologies described in prior studies by Hammer and his colleagues (e.g., [1,2]) could help explain the perspectives of individuals such as Klara. We examined two research questions:

- (1) What epistemologies does Klara exhibit when interviewed about the concept of speed?
- (2) How do Klara's epistemologies affect her understandings of speed?

In general, our aim was to investigate whether Hammer and his colleagues' claims concerning learners' epistemologies could be extended to individuals who lack advanced science and mathematics training, have had different learning experiences, and grew up speaking and learning in another language and culture.

Paying primary attention to Klara's epistemologies, we also examined Klara's conceptual understandings of the concept of speed, because we assume that these are interrelated. Rather than asking Klara about her epistemologies explicitly, we chose to investigate her epistemologies through her talking about a particular concept and related phenomena. We think this is a more authentic, grounded, and context-specific way of probing epistemologies than adopting self-report assessments that ask explicit questions about epistemology. Using this approach, we paid attention to Klara's understandings of the concept of speed and the ways she sought to mathematically represent it; we examined her epistemologies and explored how stable they remained during the interview; and we also analysed the relation between her conceptual understandings and epistemologies.

## 2. Theoretical framework

### 2.1. *Epistemological beliefs and resources*

The work we review in this subsection, while mainly drawing from science education, is closely related to mathematics learning. For example, the physics courses mentioned in these studies are algebra-based and calculus-based, and students taking these courses use their mathematics understandings. In a physics course, key concepts in mathematics, such as rate of change and derivatives, are widely applied to represent physics concepts, such as velocity. Therefore, we adopt the perspective that epistemological frameworks that have been used to explain students' learning of physics are relevant to both mathematics and science learning.

Scholars in this field have developed two perspectives regarding learners' epistemologies: a beliefs perspective and a resources perspective. Hammer et al. [2] contrast these two approaches, emphasizing that epistemological beliefs are unitary and stable, whereas epistemological resources are finer grain-sized, sensitive to context, and subject to shifts due to various triggers in the situation, such as the instructor's prompts.

In his early work on epistemologies of college students taking introductory algebra-based and calculus-based physics courses, Hammer [1] claimed that college students held

epistemological beliefs that can be characterized along three dimensions linked to the structure, content, and learning of physics. Using Hammer's terminology for different types of epistemologies along the structure of physics, if a student sees it as a system with coordinated components, his or her belief is categorized as *Coherence*. If a student sees the structure of physics as loosely related fragments, his or her belief is categorized as *Pieces*. In the second dimension related to the content of physics, if a student sees physics as a collection of terms and equations, and these verbal and mathematical symbolisms are used for answering questions and solving problems, this belief is categorized as *Formulas*. If a student believes that physics equations must have underlying conceptual explanations, then this belief is categorized as *Concepts*. Along the third dimension related to the learning of physics, a student's belief can be categorized as *By Authority* if he or she believes that learning physics is receiving information from the teacher or the textbook, or as *Independent* if he or she believes that learning physics involves constructing one's own understandings.<sup>2</sup> When students hold epistemological beliefs, these beliefs are, according to Hammer [1], unitary and consistent across contexts. These beliefs lead students to approach physics problems in particular ways: to make sense of phenomena and try to explain them in everyday language, or to pull out formulas from a textbook, plug in numbers, and get the answer.

In his later work, Hammer, together with his colleagues, proposed an alternative way of interpreting students' epistemologies from a resources perspective.[2,3] From this perspective, students are seen as activating sets of epistemological resources. These resources include the

ones for understanding the source of knowledge (Knowledge as transmitted stuff, Knowledge as fabricated stuff, Knowledge as free creation, and others); forms of knowledge (Story, Rule, Fact, Game, and others); knowledge-related activities (Accumulation, Formation, Checking, and others); and stances toward knowledge (Acceptance, Understanding, Puzzlement, and others). [2,p.8]

Compared to epistemological beliefs, epistemological resources are more numerous, less formalized, and much smaller in grain size. They are loosely connected, sensitive to context, and subject to change easily. Here we elaborate on this resources perspective by giving a hypothetical example. In a traditional lecture class focused on the concept of speed, students may activate epistemological resources including that the concept of speed is a group of formal definitions and equations, and learning this concept is accepting those definitions and equations delivered by an instructor and then practicing problem solving. Activating these resources, students may choose to approach problems by looking in textbooks or their notebooks for the definitions and equations they consider relevant to the problem and using them to solve the problem. On the other hand, in a hands-on activity in which students are exploring the speeds of moving objects, students activate other epistemological resources, such as the measurement of speed is an issue, and making sense of the phenomena at hand is important to learning. These resources may lead students to approach problems by finding ways to measure speed and interpreting the phenomena in their own words according to their genuine understanding.

The connection between the beliefs perspective and the resources perspective is, according to Hammer et al., that if a resource, or a set of resources, appears to be consistently used across different contexts, and shifts or variations are not identified, then that set of resources can be, at least functionally, equal to a particular stable belief or a particular combination of beliefs.[2] In the present study, our goal was to examine Klara's epistemologies through her responses during the interview: did she hold stable epistemological beliefs (and if so, what were they?), and/or did she activate epistemological resources?

## 2.2. Stability of ideas and the mechanisms of stabilization

In learning sciences, one goal of studying and modelling learners' epistemologies is to find out the roles they play in conceptual learning, and thus to inform pedagogy. There are many kinds of roles that epistemology can play in learning. The present study mainly focuses on examining a claim that Hammer and his colleagues made: epistemologies can help to stabilize a set of learning resources.[2] We briefly explain this claim later.

Hammer et al. [2] view learners as possessing a rich repertoire of learning resources and responding to questions by activating a locally coherent set of these resources. When Hammer et al. discuss learning resources, these resources include both conceptual and epistemological ones, and they are usually intertwined. Epistemological resources, a subset of learning resources, have been reviewed in the previous section. Conceptual resources are those relevant to the cognitive aspects of understanding. Instances of conceptual resources are intuition, common sense, and semi-formalized rules, such as p-prims.[4] Contrasting conceptual resources with unitary (mis)conceptions, and epistemological resources with unitary epistemological beliefs, Hammer et al. emphasize the instability of resources or combinations of resources. According to Hammer and his colleagues, in certain situations, what resources are activated, to what extent they are structured, and how they are structured is temporary and sensitive to context. When students respond to questions, they activate resources, and often shift among different resources in a very short period of time. Triggers for these shifts are many, such as interviewers' or instructors' prompts, a student's own reflection, or other aspects of the situation.[2]

However, a resource combination could be stabilized over time. Hammer and his colleagues [2] discuss three mechanisms for stabilizing a set of resources: *contextual*, *deliberate*, and *structural*. According to the authors, a *contextual* mechanism is passive. In this mechanism, few epistemological aspects are involved. A set of resources becomes stable simply because the questions these resources intend to answer are always put in the same context. The *deliberate* mechanism involves epistemologies. In this mechanism, the subject deliberately monitors her argument and checks the resources she has activated, trying to make more globally coherent explanations. Over time, a globally coherent structure emerges and stabilizes. When the structure is crystallized, the learner becomes less and less mindful about checking coherence. The *structural* mechanism uses the same structured set of resources repeatedly, thus ensuring stability. When a set of resources becomes stable, this set could be viewed as a unitary conception/belief.[2]

For the present study, we will focus mainly on the *deliberate* mechanism. By presenting the participant phenomena that relate to the concept of speed and asking her questions about it, we wanted to identify how Klara understood the concept as well as how she approached questions. Furthermore, we aimed to find out how stable her understandings were, and how her epistemologies affected their stability.

## 3. Methodology

The goal of this study was to investigate Klara's epistemologies when she explored the concept of speed. We decided to focus on speed for a number of reasons. First, speed is a widely used word in people's everyday life. Therefore, one may have a wealth of understandings of the concept of speed from daily life, and these understandings may affect his or her epistemologies, and vice versa.

Second, in school, velocity is a concept that permeates both the science and mathematics curricula, with its connections to rate of change. Learning about velocity in science and

math classes may also add a facet, or layer, to people's understandings of this concept and its related epistemologies.

Third, the concepts of speed, velocity, rate of change, and related issues such as interval, instant, and infinitesimal have fascinated mathematicians, scientists, and philosophers throughout history. For example, the famous paradoxes of Zeno in ancient Greece addressed the concept of limits and its relation to a moving object. Newton provided us with a definition of momentum, a concept closely related to the concept of velocity, in its calculus form for the first time, giving a mathematical description for velocity at an instant. Finally, in modern physics, Einstein redefined the concept of velocity from a relativistic perspective. Reading and hearing people talking about speed in relation to its historical evolution may play a role in one's conception and epistemologies of speed.

Fourth, and perhaps for the above reasons, researchers in the field of the learning sciences have long been interested in and explored learners' understandings of speed. For example, Piaget thoroughly investigated young children's ideas of the concept of speed.[5] After Piaget, studies have investigated students' ideas of speed, from elementary grades [6–9] through college.[10,11]

Past studies about students' understandings of speed have focused on their conceptual understandings without relating them to epistemology. The present study extends beyond past studies by using the concept of speed as a vehicle to examine epistemology, providing a new and unique contribution.

### **3.1. Participant**

The participant in this study, Klara, is an adult non-scientist. She was born and raised in China. She completed her primary, secondary, and undergraduate degrees in China. She completed high school mathematics, which includes algebra. She also had a year of high school physics (algebra-based) in 10th grade. She took a calculus course during the first year of college and earned good grades. After completing this coursework, she did not receive any other formal education in either mathematics or science. She earned her bachelor's degree in history and archaeology. At the time of the interview, she was 25 years old and studying museum education in graduate school at a private university in the United States. Klara is different from students assessed in previous studies. Participants in previous studies with adult learners were American college students [10,11] who were taking physics classes covering kinematics at the time of their assessment.

### **3.2. Setting**

The participant was interviewed in a non-school lab-like setting. Klara was seated at a desk on which an inclined track was placed (see [Figure 1](#)).

One book was placed under one end of the track to make it slightly inclined, and another book was later added in this same place, making the track incline steeper than before (see [Figure 2](#)). A cart was released from the upper end and slid down to the lower end. Klara was allowed to play with the cart and track by herself in addition to observing the interviewer's operation of it. We decided to interview Klara in this setting because the equipment roughly replicated what had been used in previous studies with adult students.[10,11] This way, the results of our study could be contrasted to those of previous studies. At the same time, our setting was different from that of previous studies in a couple of ways: participants in previous studies were all interviewed or assessed in school settings, whereas Klara was interviewed outside of school; while our setting was intended to be similar to previous

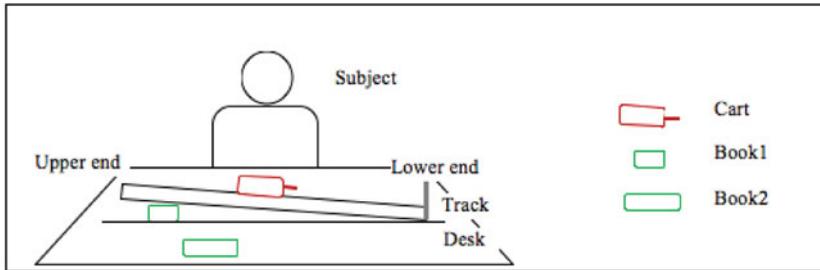


Figure 1. The setting for the interview with Klara.



Figure 2. Less inclined (less steep) track and more inclined (steeper) track.

studies, our interview protocol was purposely designed to be different from that used in previous studies, as described in the Section 3.3.

### 3.3. Interview protocol

During the interview, after presenting the sliding of the cart, the interviewer asked Klara questions such as: What did you notice? What do you mean by ... (a particular word she had used or a particular drawing she had just produced, etc.)? Could you show this idea in other ways?

As the interview progressed, the interviewer asked two specific questions: (1) When the cart slides, do you notice any difference between the beginning and the end? (This question was intended to probe Klara's ideas of non-constant speed in an accelerating motion.); (2) What is the difference between when I [the interviewer] put two books here [to make the track incline steeper] and when I [the interviewer] only put one book here [making the track less inclined] (see Figure 2)? This question was intended to probe Klara's ideas of acceleration, the rate of change of speed.

In previous studies, the interview questions were mostly initiated by asking a yes-or-no type of question (e.g., whether the two carts ever have the same speed) followed with questions asking students to explain their answer to that previous question (e.g., 'How could you tell?'). [10, p.1022] Interview questions in this study were more open-ended. The initial questions were intended to address the participant's focus (e.g., 'What did you notice?'), not to address the interviewer's focus. This difference will be shown more clearly in the transcript presented in Section 4.

### 3.4. Data collection and analysis

The entire interview lasted 90 minutes. It was videotaped and transcribed. This interview was carried out in Chinese and translated by the interviewer (the first author of this paper) into English. The interview transcript, notations, and drawings Klara made during the interview formed the corpus that was analysed for this study. During the first step in our data analysis, we labelled video clips that focused on Klara's oral descriptions and/or written work about the concept of speed as well as on her epistemologies.

Based on the clips that we identified in the first step of the analysis, in the second step we went into detail and flagged evidence for Klara's:

- (1) Conceptual resources in relation to her understandings of the concept of speed – for example, whether she used everyday experiences, intuitive understandings, mathematical resources such as recalled formulas or quantifications, etc.
- (2) Epistemologies and epistemological shifts – for example, evidence for whether she thought that her responses to the interview questions should look like a formula, a narrative description, or something else; evidence for whether she viewed the activity we were doing as a formal test, an informal test, or a casual chat between friends.

If she provided evidence for more than one kind of epistemology, we also examined whether the shifts were substantial or not. If considerable shifts were identified, then we would characterize her as activating epistemological resources, and we then looked into the triggers for those shifts; if, instead, few shifts were identified, then we would conclude that she held stable beliefs.

In the last step of our analysis, we examined the stability of her conceptual understandings and the mechanism of stabilization. To be more specific, we examined whether and, if so, how Klara's epistemologies helped to stabilize her understandings of the concept of speed.

The two authors of this paper were both involved in the data analysis. Since the interview was carried out in Chinese and translated into English, and the first author's first language is Chinese, we took turns and played different roles as we examined the data. For every round/step of data analysis, the first author examined the data according to our analytical protocol and annotated the transcript in relation to our research questions. Then, the second author read and commented on the annotation. Together, we then discussed the transcript, annotation, and comments. We repeated this cycle multiple times until we reached an agreement.

Looking at the interview through Hammer et al.'s resources/beliefs framework, our analytical goal was to find out how the framework could potentially help explain Klara's ideas and performance during the interview.

### ***3.5. Episode selection***

Following the analysis process, we narrowed down the data to three focal episodes (which occurred at minutes 7, 29, and 32 of the interview) that illustrated Klara's ideas of (1) speed in general, (2) average speed, and (3) instant speed, and which helped us address our research questions. At the same time, we wanted the reader to have a sense of the whole interview. Therefore, we present the interview as five sections centred around the three focal episodes: pre-story, episode 1, episode 2, episode 3, and post-story.

## **4. Results**

In this section, we will describe the interview, provide selected transcripts, and highlight the relevant results related to this study's research questions.

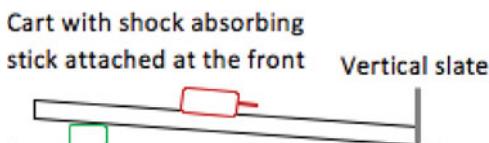


Figure 3. Vertical slate on the track and shock-absorbing stick on the cart.



Figure 4. Two successive sliding-down and bouncing-back movements.

#### 4.1. *Pre-story: Klara brought up the word speed*

At the beginning of the interview, the interviewer released the cart from the upper end of the inclined track (see Figure 1). The cart slid down and reached a vertical slate installed at the lower end of the track, so the cart bounced back and slid down again (see Figure 3). The cart slid down and bounced back for a few times before it rested at the lower end of the track. Then, the interviewer asked Klara: ‘What did you notice?’

She responded:

I noticed that the cart went down and bounced back. After several bounces, it stopped. I found that when the cart started from a higher point, it bounced back a longer distance. The next time, when it slid from a lower point, it bounced back not as far (see Figure 4).

The interviewer asked Klara how she would make sense of what she had noticed. She said there might be some strength driving the cart down, and when the cart collided with the vertical slate, there was a competing strength upward from the slate to bounce the cart back. After listening to her for a while, the interviewer noticed that Klara’s attention at that moment was on the phenomenon of bouncing and an idea of ‘strength.’ The goal of this interview was to probe her understanding of the concept of speed; therefore, we did not want her to spend too much time focusing on the phenomenon of bouncing, which at that moment the interviewer thought had more to do with forces. At the same time, the interviewer did not want to press her to simply drop her current thinking. The interviewer believed that by correcting Klara, she would become less confident in her own responses and instead guess answers she felt the interviewer wanted to hear. The interviewer decided to let her talk about the sliding, which relates to its speed, but steered her away from the idea of bouncing: ‘Okay, let’s save the bouncing stuff for later, now just think about the process of sliding down. What are the strengths you referred to, exactly?’

She murmured: ‘What are the strengths driving it down. . .’ Then, she said: ‘First there is gravity.’ The interviewer asked her: ‘What is gravity?’ She answered, ‘It is an attracting force from the center of the Earth. It drags things down. How much a thing weighs decides how much gravity it has.’ When she could not talk more about gravity, she brought up another word, inertia, and said: ‘Besides gravity, there might also be the inertia to keep the cart [moving] like this’ (she released the cart from the upper end and watched it slide down). The interviewer asked her to explain what inertia meant and she said:

Inertia might be when you do something, for example, when you are moving, you have kind of a strength because you want to keep moving. But I don’t know where the inertia comes from.

She continued talking about inertia:

When you drive and you brake, you still move forward for a while before you stop. I think all moving things have inertia. But I don't know whether inertia is a kind of force.

She paused to think about the meaning of inertia for quite some time. The interviewer waited patiently and then gently asked her to connect back to the phenomenon of the cart. Klara said:

Gravity makes the cart move down, and the inertia makes the cart move along this direction [finger gesturing a line along the track from the upper end to the lower end]. It will not move toward me or away from me [It will not move sideways].

She tried to bring up an example other than the track and cart that would express her understanding of inertia: 'Imagine you are walking along a slope.' She stopped and thought for a moment, and then said, '[You] still need to have a certain speed so that you can have inertia.'<sup>3</sup> This was the first time that Klara brought up the word *speed*.

During this part of the interview, starting with an observation of the cart-sliding phenomenon, Klara talked broadly and brought up a lot of words: *strength*, *gravity*, *inertia*, *force*, and *speed*. The interviewer did not force her to drop the ideas that were not the main focus of the interview, but showed interest in every word she brought up and allowed her to navigate the phenomenon. While explaining her ideas to the interviewer, Klara brought up the word *speed* on her own.

In retrospect, we interpret this section of the interview as the first steps of Klara's epistemological framing of the activities she was presented with. From the questions the interviewer asked, she was able to frame the activity as one that involved the interviewer's interest in her thinking and one in which the interviewer was not looking for specific answers.

#### 4.2. Episode 1: Klara's initial ideas about the concept of speed

At the end of the pre-story when Klara brought up the word *speed*, the interviewer followed up with, 'What is speed?' She responded:

Speed? It's running fast or slow? Meaning during the same period of time, if you run a longer distance, that means fast, the speed is fast; and during the same period of time, if the distance is shorter, the speed is slow.

She suddenly sat straight (before she had been drawing and talking, leaning towards the table) and repeated the question in a rising tone 'Speed?' Before this question, the interviewer had requested explanations for other concepts she had brought up, such as 'What is gravity?' and 'What is inertia?' She did not look surprised by these questions. However, when the interviewer asked, 'What is speed?', her reaction seemed to show that she thought this was a silly question, perhaps because she thought everyone knew what speed was.

After she finished speaking, the interviewer moved on: 'When the cart slides, did you notice any difference between when it [pointing to the cart] was at the beginning and when it was at the end [of the track]?' Once the interviewer asked this question, Klara released the cart from the top and watched it slide twice, and then said: 'I feel the speed is getting faster when the cart is going down.' She slid the cart a third time, and added:

同样时间，距离远一些。  
同样距离，时间少一些。

Figure 5. The description of speed that Klara wrote. The first line reads ‘same time, longer distance,’ and the second line reads ‘same distance, shorter time.’

I feel when it [the cart] is at the higher part [of the track], the speed is, um, to cover the same distance, it takes a longer time. For example, if we mark the middle point of the track [pointing at the middle], the distances for each half are roughly the same. Look, the time passing this part [the higher half] is shorter than that of passing this part [the lower half].

When she was explaining this idea to the interviewer, she slid the cart once more.

As she elaborated her understanding of the quantitative aspects of speed, she drew upon resources from everyday experience (running), perception (watching the cart slide), and her ability to quantify (naming variables like distance and time, and quantifying with longer, same, and shorter, for instance). In her first statement about speed, answering the interviewer’s question of ‘What is speed?’, she claimed that if during the same period of time, one runner covers a longer distance than another runner, then the first runner’s speed is faster. In her answer, she brought up her understanding of the quantitative relationship among speed, distance, and time. Clarifying this relationship, she kept the time variable constant and compared the distance variable to decide which speed was faster. Later on, when she responded to the interviewer’s question about the difference between when the cart was at the upper and lower ends of the track, she provided further evidence of her understanding of the quantitative relationship among speed, distance, and time. She kept the distance variable constant (by marking the half point along the track) and compared the time variable: a shorter time spent on the upper half of the track corresponding to a faster speed, and a longer time spent on the lower half of the track corresponding to a slower speed.

Her description did not allude to a formula she learned in school. Instead, she was making sense of speed, not gathering information from an external authority. This is evidence of her epistemology with respect to learning as a sense-making activity.

In order to probe Klara’s ideas further, the interviewer asked her to show her idea on a piece of paper.<sup>4</sup> She started to write and murmured that she remembered having learned this before. She wrote down two lines of words, in Chinese (see Figure 5), while saying:

I sort of remember, I might have learned before, that speed is, during the same period of time, if you run a longer distance, then your speed is faster; or for the same distance, if you spend shorter time, then your speed is faster.

The interviewer asked Klara if these two lines of words were expressing the same meaning. She said yes. The interviewer followed up by asking why these two sentences

Figure 6 shows handwritten Chinese characters for speed (速度), distance (距离), and time (时间) at the top. Below them is a diagram of a horizontal line with a curved path above it, and three points labeled A, B, and C. Underneath the diagram are two mathematical formulas:  $A \times C = B$  and  $A = \frac{B}{C}$ .

Figure 6. Formula of speed that Klara wrote.

about distance and time were actually talking about speed. Klara pondered for a while and said: 'I feel speed is the distance you run in a unit time.' The interviewer asked her to explain what she meant by a unit of time. She answered, 'A stipulated period of time. A standard to compare our speeds. It could be an hour, a minute, but [it] has to be the same for both of us.' She further clarified her idea by providing an example: 'If you and I are running and comparing our speeds, each of us is given one minute. If I run farther, then my speed is faster than yours.' Because she had consistently brought up verbal descriptions of the quantitative relationship among distance, time, and speed, orally and in writing, the interviewer pushed her to show her idea in ways other than in Figure 5. When making this request, the interviewer wrote down the Chinese characters speed, distance, and time (see the top of Figure 6).

Klara paused for a while, and then added the letter S under the word speed. She hesitated when picking a letter for distance, and then wrote down a D. The interviewer saw her struggling over which letters to use and suggested that she use arbitrary letters, like A, B, and C. Klara immediately accepted the interviewer's suggestion and wrote A for speed, B for distance, and C for time, and then wrote an equation. While she was writing it, she explained: 'Speed multiplied by time is distance.' She wrote down the formula, A times C equals B (see the middle of Figure 6). The interviewer asked her: 'How can this [formula] explain these two sentences [the verbal expressions in Figure 5]?' Klara answered while modifying the formula to a division at the bottom of Figure 6:

So look, if we were going to compare speed. Speed is A [circling A in the formula]. A equals B divided by C [writing the division formula]. Then you only need to compare B and C. Oh, to compare my B and C with your B and C. So then if time is the same, time is B, oh no, time is C, if C is the same, then a bigger B makes a bigger A, and a smaller B means a smaller A. If distance. . . distance is B, if our Bs are the same, then it is obvious that a smaller C makes a bigger A.

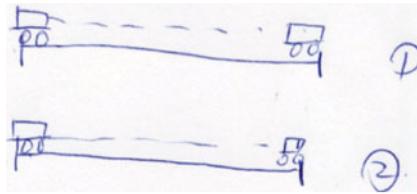


Figure 7. Klara's drawing about comparing speeds.

So far, Klara did not have much trouble with the questions about speed. Overall, she talked calmly and emphatically. Although she occasionally struggled a little with a particular question, such as what is a unit of time, she soon found a way to articulate her thinking and showed confidence in her explanations. The mathematical expression of speed in terms of distance and time made sense to her. She started to reflect upon the relationship between her intuitive understanding and the mathematical expression when answering the interviewer's questions. Her tone of speech showed that she was comfortable making a connection between her experience and the mathematical formula.

After Klara worked out the formula for speed, the interviewer asked her how she felt about her verbal expression for speed (see Figure 5) and the formula for speed (see Figure 6). She said that the formula was clearer and more accurate than the verbal expression. She also said that speed had always been a pretty tangible thing to her, but when she was asked about the formula, it became more abstract. She added that if the interviewer gave her a problem to solve, she would 'promptly grab this one [the formula].' But, as she said, because 'here we are just chatting,' she would not do that.

Klara framed the interview as 'just chatting.' Although she was aware of other ways to approach a physics problem, she chose a sense-making approach. She started describing speed using her common sense, and then gradually formulated the mathematical expression in response to the interviewer's questions. She did not show any trouble with using arbitrary letters to represent words after the interviewer made this suggestion.

#### 4.3. Episode 2: Klara's understanding of the concept of average speed

The interviewer kept asking Klara to show her ideas in other possible ways. In order to make that request clearer, the interviewer suggested, 'For example, you may draw.' Klara then drew the picture shown in Figure 7 and said:

Well, two same distances. And two carts are running. When they reach the ends [she drew two parallel solid lines in Figure 7, each with a cart on the left end], read the time it takes. Like run run run. . .run to here [drew the top dashed line and the right-end cart]. Run run run. . .run to here [drew the bottom dashed line and the right-end cart]. If this one [the upper cart 1] took more time, then it was slower. This one [the lower cart 2] took less time, so it was faster.

Since the interviewer thought Klara did not expand upon her ideas through her drawings, the interviewer asked Klara to observe the following. The interviewer held the cart at the top of the track and let it go. Klara watched it very carefully. Then, the interviewer added a second book under the first book (see Figures 1 and 2), lifting the upper end of the track even higher. The interviewer explicitly told Klara that the track was lifted. Then, the interviewer released the cart on the lifted track as Klara watched.

The interviewer asked Klara, ‘What difference did you notice between the cart sliding down this time [on the more inclined track] and the cart sliding down the previous time [on the less inclined track]?’ Klara answered immediately: ‘Definitely this higher [steeper] one is faster. It’s obvious.’ The interviewer asked: ‘Where exactly is it faster?’ In retrospect, we realize that this was not a clear question. Klara did not answer the question immediately, but put the cart back at the top of the steeper track, let it go, and watched. Then she said: ‘Overall, I feel it is much faster.’ The interviewer asked: ‘By faster you mean. . .?’ Klara answered: ‘Meaning it [the cart] spent less time from here [the upper end] to here [the lower end] than it did before [on the less inclined track].’ She repeatedly released the cart from the top and carefully watched it sliding several times, and then she confirmed: ‘Right. Overall it is faster.’ She repeatedly manipulated and observed the cart, and said: ‘The overall time is shorter.’ Then she added: ‘And I feel the average speed is faster.’ The interviewer followed up by asking: ‘What is an average speed?’

When the interviewer asked the question, Klara could not help laughing, probably because she realized that she had introduced another term that the interviewer would certainly ask her to explain. The following exchange took place:

Klara: Because I think. . .like what I just said about this one [pointing to her drawing shown in [Figure 7](#)]. . .If this road is very long, if we only compare the time they come to the end, then I can first do fast, and then do slowly for a while [playfully twisting and swaying her body to act out a fast and a slow move]. And this one [cart 2] may first be fast, even faster than that one [cart 1]. But ultimately the two. . .Or let’s put it this way: two things, at the same time, um, set off, and they arrive at the end at the same time. You cannot say that they have the same speed, right?

Interviewer: Why not?

Klara: Oh. You can only say, overall, they have the same average speed. . .I feel

I should add the average speed kind of description.

Interviewer: The average speed you are saying is. . .

Klara: It is that I can only say, during a chunk of time [she gestured a segment in the air using both hands]. . .Average speed. . .It’s so hard! I can’t really say it clearly. . .I know it [speed] is distance divided by time, but to describe what average speed is. . .Meaning I don’t consider the details in the middle, that sometimes I go fast, sometimes I go slowly. I just say, overall, let’s compare our speeds.

Interviewer: You mean if we compare the overall speed, that speed is the average speed?

Klara: Right. Meaning not considering details.

During this part of the conversation, Klara explained the concept of average speed to describe the ‘overall’ speed, ignoring the variation of speed during a given time interval. As she elaborated on her understanding, Klara coordinated all the resources she had access to at that point: her understanding of the ‘overall’ speed of the two situations, her understanding of a time interval, her understanding of the varying speed during the course of moving, etc. More specifically, she linked her explanation to the quantitative relations among speed, distance, and time that she had established during episode 1 and added a more detailed description by referring to time intervals: a chunk of time, a unit of time, a stipulated period of time, etc. She tried to build a coherent explanation that included all of them.

It is worth noting that until this point, from her tone, her face, and her gestures, Klara appeared to be confident and comfortable. Also, her explanations thus far about speed and average speed aligned with the mathematical expression she had written (see [Figure 6](#)).

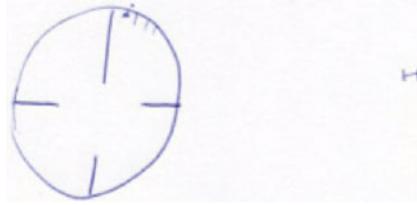


Figure 8. Klara's drawing about comparing speeds.

We would argue that the epistemologies she expressed in episode 2 were consistent with those she expressed in episode 1: that is, to Klara, her ideas about the concept of speed are coherent, and her mathematical expression is consistent with her everyday experiences. She was able to coordinate resources to explain average speed, which contrasts with the confusion and frustration that emerged, as we will see later, when she could not coordinate resources to explore the concept of instant speed.

#### 4.4. Episode 3: Klara's understanding of the concept of instant speed

Right after Klara said that average speed meant a speed 'not considering details,' the interviewer pushed her on this point.

Interviewer: What about the speed that does consider details?

Klara: That speed I feel is talking about. . . I feel the time [for that speed] is cut into slimmer [slots]. No, that slimmer time can still have average speed. Mm. . . You are asking what the difference is between average speed and that speed. That one is called instant speed, right?

Interviewer: It doesn't matter what it is called.

Klara: Oh, oh, I just feel. . . that is a 'real-time' speed. It's just, at one moment, I compare my speed with yours, at a specific point. I feel time is constituted by many, many points. No. It should be. . . time can be cut by many points. (She gestured with her hands effusively.)

Interviewer: You may draw it if you want.

The interviewer expected that Klara might draw a time line, but she did not. Instead, she drew the clock as shown in [Figure 8](#).

Klara: Okay. It's like. . . If we have a clock [drawing the clock in [Figure 8](#)], then we know time. We can specify. . . I decide [to look at] that just at this point, what about our speeds? Wait. No. Nope.

Interviewer: What nope?

Klara: If it's just a [time] point, I don't even run, how come we compare speed? There still has to be a span [draws a short line segment on the right in [Figure 8](#), puts down the pen, and gestures a small bit with her fingers], right? At a point. . . Why does it become more complicated when I think more? I feel that at a point. . . At a specific time point, we still can have a speed. I feel that time can be divided into infinitely slim [slots]. Because there is light speed, right? Traveling that much far just in a second. So. . . Oh my! I'm so confused. It is I feel instant speed is. . . Okay. Just put it this way: the instant speed is slimmer than the average speed.

Interviewer: What is slimmer?

Klara: The time to compare [speeds] is slimmer.

Interviewer: How slim is slim enough, do you think?

Klara: How slim is enough? Mm. . . Uh. . . I'm so confused. How slim is enough? Average speed. . . Instant speed. . . [murmuring]

During this episode, Klara tried to describe the specific speed at a specific instant, or ‘time point’ as she called it. She brought up the concept of instant speed but had a very hard time reconciling this intuitive idea with the algebraic formula for speed, which demands a time interval. She had taken calculus in college, but the derivative expression of instantaneous velocity, which represents a velocity at an instant, did not occur to her during the interview. Even if it had, she might have remained dissatisfied since the calculus formula would not answer the question of ‘At a time instant, I don’t even run, how come we compare speed?’ Although she had a verbal explanation about what instant speed means and evidence from everyday experience about speed at an instant, she kept trying to figure out a formula that was coherent with other aspects of her understanding. It seems as if, for Klara, a mathematical formula was indispensable for her to explain the concept of speed.

She explained the difference between average speed and instant speed, as well as the difference between an instant and an interval. She said that the time span for an instant speed is ‘slimmer’ than that for an average speed. But when the interviewer asked her: ‘How slim is slim enough?’, she could not determine whether this slimmer time span was still a span, although very short, or an exact point.

Klara had been focused on the topic of instant speed for several minutes. The interviewer did not respond to Klara’s questions about instant speed when she was thinking out aloud, but just listened. Klara emphasized an instant speed for any moving object. She pointed to the speedometer as evidence of instant speed: ‘Just like driving, at every single moment, the speedometer tells you the instant speed.’ When the interviewer asked whether the cart had an instant speed, she immediately answered: ‘Of course it has. Everything has an instant speed.’ She moved the cart back to the top and let it go several times, showing it to the interviewer: ‘See. This cart’s instant speed is the speed at every single point. At every single point how fast it runs. Just like the speedometer, it tells you instantly that you are driving 80 miles [per hour] or 100 miles [per hour].’

Although she could not make all her ideas coherent enough to satisfy herself, she never doubted that her ideas should be coherent. From these episodes, we have evidence that Klara’s epistemologies were all her ideas should be consistent with one another, and she needed to coordinate all the pieces on her own if she wanted to understand the phenomena thoroughly. These epistemologies were pretty robust throughout the interview even when she had great difficulty coordinating the pieces into a coherent structure. Instead of shifting in her epistemological position, she doubted her knowledge of the concept by asking the interviewer, ‘What is speed after all?’

#### **4.5. *Post-story: Klara’s description of the concept of acceleration***

After Klara’s struggle with instant speed, the interviewer thought that she had exhausted her thoughts about speed and wanted to move on. So she started to probe Klara’s understandings about acceleration. Trying to avoid saying acceleration explicitly, the interviewer asked Klara: ‘Mm. Alright. My question now is this: You just said that when I put only one book here [pointing to the position under the track where the books were placed], and when I put two books here, the carts’ speeds were different. I wanted to ask you: are they always different, right from the beginning, or are they just becoming different while moving?’

After hearing the interviewer’s question, Klara released the cart and watched it over and over again and then concluded that the cart that started in the higher position had been faster than the cart that started in the lower position. She had also agreed that when the

cart stayed at the top, the speed was zero, for both situations. Following her statements, the interviewer asked her why the cart started with the same speed, zero, and ended up with different speeds when the track had different inclinations.

She grappled with this question for a long time. First, she said, ‘Because this one [steeper track] is higher.’ The interviewer asked why higher matters and Klara answered that gravity affected speed. The interviewer then asked how gravity in the two situations was different, and she figured out that gravity should be the same. Then she claimed that the higher track was shorter but soon realized that it was always the same track with the same length. She thought about it really hard and got very confused and tired, and so she took a half-hour break. When the interview resumed, the interviewer set up a motion detector (equipment to collect the data of the cart’s positions over time and graph position–time and velocity–time on a computer screen) and showed Klara the graphs of the cart’s motion in the two situations. She worked on it for a short moment, but the graphs shown on the screen did not seem helpful to her. So she soon stopped and said, ‘Let’s not look at this.’

In order to assist Klara a little bit, the interviewer asked her to compare the speeds at the middle point of the track for the two situations. After this moment, little by little Klara formulated the description of the cart moving on the less inclined track as ‘speeding up slower.’ After clarifying this idea of ‘speeding up slower,’ Klara came up with the description of the cart moving on the steeper track: ‘It is getting fast faster’ and explained this description by stating, in a slow, steady, and decisive voice, ‘During the same period of time, for the higher cart, the speed it picked up is bigger than what the lower cart did.’ Thus, she found that one difference between the cart on tracks with two different inclinations was ‘the speed of speeding up.’ This idea of ‘the speed of speeding up’ is very close to the canonical concept of acceleration.

The interviewer followed up by asking her to express this concept in a mathematical expression, like she had done for speed in [Figure 6](#). She wrote down:

$$\frac{S_2 - S_1}{t}$$

and added a clear explanation: the difference between speed 2 and speed 1 divided by the time elapsed. When she referred to speed 2 and speed 1, each representing a speed at a certain time instant, she was again confused about a time instant and an instant speed. Not able to explain instant speed clearly enough to be satisfied, she said that she did not even know what speed was:

What is speed after all? How to measure an instant speed? If it [speed] is distance divided by time, then I don’t have a distance [for an instant speed]. And at that time point. . . What is the time [interval] you use to [divide]. . . An instant. . . Does that count as a time [interval]? There is no time span [for an instant]. There is no. . . Right? How to measure an instant speed? Is speed truly distance divided by time? I’m so confused. . . . At that point, you don’t have a time [interval] to divide. You don’t have a distance, either. I’m so confused. I don’t know what the instant speed is. This is torture. The more I think, the more I get confused. I don’t know what speed is. If you are just talking [about speed], it’s a feeling of moving fast or slowly. But to make it a thing over another thing [the division formula], speed, time, distance. . .

The post-story confirmed that Klara’s epistemologies were robust and consistent throughout the interview. She was satisfied with her description of the concept of acceleration in her own words, ‘the speed of speeding up,’ which did not conflict with other

pieces of knowledge she had activated. But she was not satisfied with her explanation of instant speed because she perceived it as incoherent with some of her other ideas. She started to doubt the algebraic formula of speed near the end of the interview by saying, ‘Is speed really distance divided by time?’ She considered that she ‘really do not know what speed is,’ which starkly contrasts with her reaction at the beginning of episode 1 when she responded to the interviewer’s question ‘What is speed?’ In episode 1, she thought she certainly knew what speed was and thought the question was silly.

## 5. Discussion

### 5.1. Klara’s epistemological beliefs and resources

We argue that during this interview, the grain size of Klara’s epistemologies could be considered both beliefs and resources. On the one hand, she exhibited no considerable shifts. On the other hand, the interviewer did not examine her epistemologies in any context other than this single interview. Therefore, we could view Klara’s epistemologies either as stable and unitary beliefs or as sets of resources stably structured.

According to Hammer’s [1] framework of epistemological beliefs, Klara’s can be characterized as *Coherence–Concept–Independent* along the three dimensions of the structure of physics, the content of physics, and the learning of physics, respectively. To elaborate, Klara appeared to hold beliefs including ‘the explanations for the concept of speed should be coherent’ (*Coherence*); ‘the mathematical formalism should not conflict with intuitive understandings’ (*Concept*); and ‘my re-explaining of the meaning of the concept is indispensable’ (*Independent*). Examples that demonstrated these beliefs can be found throughout the interview. She first answered the question ‘What is speed?’ using common sense: ‘Speed is running fast or slow.’ She formalized the mathematical expression of speed in terms of distance and time while referring to everyday examples. She repeatedly operated the cart herself and watched it very carefully, making sure that her explanation of speed was valid not only in terms of her everyday experiences but also in this lab-like setting. She kept trying to coordinate her intuitive understandings with the algebraic formula when more nuanced aspects of speed (average speed, instant speed) were introduced in the interview, even and especially when she found that this coordination was challenging.

In the latter part of the interview, no matter how hard it was for her to reconcile the inconsistency between the algebraic formula and her intuitions about instant speed, she never gave up on the belief that ‘different ways of explaining should be consistent.’

### 5.2. Klara’s epistemological framing

Klara’s epistemological framing reflected her understandings of the activity she was engaged in with the interviewer. This is another aspect of epistemology that the literature has discussed.[2] Based on what Klara said and did in the interview, we claim that she framed the interview as ‘just chatting’ and making sense of the phenomena at hand. The triggers for her to frame the activity this way were many: the relation between the interviewer and the interviewee (they were friends), the ways that the interviewer framed the interview (the interviewer was not looking for any correct answers, and was interested in Klara’s genuine thinking), the questions asked and the responses provided by the interviewer, Klara’s own background in education, etc. She knew that there were other ways to approach a physics problem (e.g., by ‘promptly grab[bing] a formula’), but she chose not to do so.

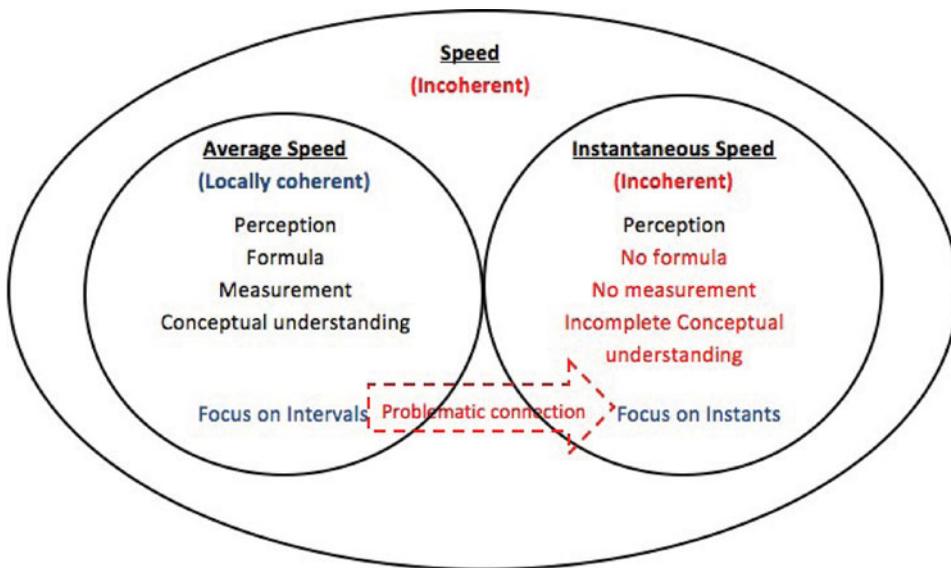


Figure 9. Klara's understandings of the concept of speed.

Compared with the use-formula-solve-problem framing, the explain-phenomena-in-everyday-language framing that Klara adopted in the interview is considered by education researchers as more productive.[2]

### 5.3. Stability of Klara's understandings of speed

In terms of how Klara's epistemologies affected her conceptual understanding of speed, we focus on the issue of cognitive stabilization.[2] We will, in this section, summarize Klara's understandings of the concept of speed and evaluate its stability. Then, in the next section, we will address whether or not her epistemologies played a role in the process of stabilizing her understanding.

When Klara was talking about speed in general in episode 1, and about average speed in episode 2, she relied on (1) her perceptions, (2) a mathematical expression, and (3) measurement: the distance and time intervals. When she was talking about instant speed in episode 3, she only had her perception, with no mathematical formula, nor viable measurement. In the meantime, she parsed a time interval into smaller and smaller slices in order to explain the connection between average speed and instant speed but had a very hard time figuring out whether this kind of parsing could finally transform the time interval into a time instant. We model Klara's understandings of speed in Figure 9.

Klara's performance, on the one hand, exemplified what previous study claimed that '[the] interpretation of instantaneous velocity as a number referring to a *single instant* is a real conceptual hurdle for many students.' [10,p.1024] On the other hand, her case was an exception of previous study's claim that 'students frequently do not relate their intuition of how fast an object is going to the ratio of the distance travelled to the elapsed time or to the idea of velocity *at* an instant.' [10,p.1027] As shown in the transcript, Klara was always trying to connect her intuition to the ratio expression of speed and to the idea of velocity at an instant.

Therefore, we argue that Klara's understanding of speed was on a productive path towards stabilization, based on the assumption that a structural comprehension becomes stable more easily than a loosely or disconnected collection of ideas. Klara's understanding of speed tends toward structured (see in Figure 9). Her multiple pieces of ideas about speed connected to each other, directly or indirectly. She constantly built the connections while she explained her ideas to the interviewer. She had a solid notion of what made sense to her, what did not, and the ways in which the former and latter diverged. At one point, she asked the interviewer to tell her 'What is speed after all?' We interpret this request as looking for extra information to make her argument sound and clear. Perhaps, she was ready to pick up and make sense of calculus.

#### 5.4. Stabilizing mechanism

As for the mechanism that stabilizes Klara's understanding, we argue that her case fits the second mechanism described by Hammer et al. [2]: *deliberate*. Compared with the other two mechanisms, *contextual* and *structural*, the main feature of a *deliberate* mechanism is that the learner examines the coherence of different aspects of their understanding and always tries to make it globally coherent.

During the interview, Klara actively coordinated her ideas. When she was explaining new ideas, such as those of average speed and instant speed, she frequently returned to what she had already written earlier: the algebraic expression of speed, for example. She checked the formula she had written, which had worked well for average speed, and found that it did not work for instant speed. She also repeated over and over again the incoherence between the algebraic formula of speed and her intuition about instant speed, struggling to mitigate the incoherence, and found that the key to it lay in the relation between the concepts of time interval and time instant.

From the evidence above, we can see that Klara was explicitly and frequently monitoring her reasoning when she provided explanations. By doing so, Klara connected every piece of her understanding and structured her comprehension of speed, represented in Figure 9; a structure that moved towards stabilization. Klara illustrates that seeking coherence can help to stabilize a set of resources, a mechanism that Hammer defines as *deliberate*. [2]

### 6. Implications for instruction

Klara's knowledge of physics at the moment of the interview was largely equivalent to that of an adult novice. Thus, her case has implications for physics education at the introductory college and high school levels. We consider Klara as a learner in this interview, and a productive learner because she was thinking thoroughly and towards a promising direction of comprehending the concepts involved.

Klara's case indicates that it is important for an instructor to prompt students to activate productive epistemologies in a learning activity. In Hammer's work, [12] he suggests that at times an instructor can provide epistemological prompts that help students to transition to a productive epistemology. One such example presented by Hammer [12]: a student came to the physics instructor and complained that he had studied very hard and done many exercises in the textbook but still could not do well on the exam. The instructor suggested to this student that he should try to explain a concept to a 10-year-old. [12] The epistemology that the instructor alluded to in his suggestion was that learning science is making sense of phenomena. Holding epistemologies such as this one, students stand a better chance at

understanding mathematics and physics, approaching concepts and phenomena in sense-making ways. Klara's case illustrates that an epistemology-prompting pedagogy such as the one suggested by Hammer [12] could be used with learners with a wide range of backgrounds.

Finding ways to provide students feedback that helps to support their productive reasoning could improve instruction. A couple of examples of this were apparent in our interview with Klara. When Klara was asked to formalize the quantitative relationships among speed, distance, and time, which she had verbally described to a certain extent, she could not remember the canonical letters for speed, distance, and time, and hesitated. The interviewer suggested to Klara that she just arbitrarily use A, B, and C, thus relieving her from having to recall conventional symbolism. Also, when Klara tried to describe instant speed but could not remember the conventional name (instantaneous velocity) but instead called it 'instant speed' and 'a real-time speed,' the interviewer encouraged her to use whatever language was sensible to her by saying, 'It doesn't matter what it is called.' At the same time, the interviewer also sought productive moments when follow-up questions could prompt Klara to think more deeply. Some examples include asking questions such as 'What is that speed that does consider the details?' and 'How slim is slim enough?'

## 7. Closing thoughts

This study of a young adult non-scientist's conception of speed has provided additional evidence in support of the theories about learners' epistemologies, especially about the perspectives of epistemological beliefs and resources.[1,2] The learner's characteristics in this study are different from subjects of previous studies. Thus, we extended the explanatory scale of these theories, leading us to suggest that they could be more widely applied.

It is not possible to conclude how Hammer and his colleagues' theories about learners' epistemologies could be extended and/or adjusted based on one single interview. Nevertheless, Klara's case suggests that these theories could apply to individuals regardless of the learner's academic background, language, education experience, and cultural background. More cases should be examined with learners from different academic backgrounds and different cultures.

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No potential conflict of interest was reported by the authors.

## Notes

1. In Chinese, velocity and speed are the same word; therefore, there is no verbal difference when the subject says speed (an everyday word describing how fast an object is moving) and when she says velocity (a physics term for the vector describing how fast and in what direction an object is moving). When translating the interview, carried out in Chinese, into English for this paper,

we use the word speed whenever Klara spoke. By doing this, we are assuming that because of her novice background in physics it is unlikely that she would have used the physics term.

2. In his work, Hammer [1] also found some intermediate beliefs that cannot be definitively categorized into any of the six aforementioned groups along the three dimensions. For example, *Weak Coherence/Weak Conceptual* represents the belief that physics should be a coherent knowledge system, and the formulas should be conceptual meaningful, but the responsibility to make the system coherent and the formulas meaningful lies with the experts, not with the learners themselves.[1]
3. What Klara called inertia was in some sense similar to the canonical meaning of inertia in physics; that is, the attribute of an object that keeps it moving (or at rest) at the same velocity. But she also believed, although not very confidently, that inertia was a force or strength. This made the inertia she was talking about also like the naïve physics concept of impetus, an inert agent in the direction of motion that drives the moving object. Finally, she mentioned speed in relation to inertia, so what she called inertia was also similar to the concept of momentum.
4. Each time the interviewer asked Klara to show her idea on a piece of paper, the interviewer was expecting that Klara would draw a graph. However, throughout the interview Klara never came up with a graph.

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