



Instructions for authors, subscriptions and further details:

<http://redimat.hipatiapress.com>

Evaluating Mathematics Teachers' Professional Development Motivations and Needs

Mary C. Caddle¹, Alfredo Bautista², Bárbara M. Brizuela¹ and Sheree T. Sharpe³

1) Tufts University, United States of America

2) National Institute of Education, Nanyang Technological University, Singapore

3) University of New Hampshire, United States of America

Date of publication: June 24th 2016

Edition period: June 2016-October 2016

To cite this article: Caddle, M.C., Bautista, A., Brizuela, B.M. & Sharpe, S.T. (2016). Evaluating mathematics teachers' professional development motivations and needs. *REDIMAT*, 5(2), 112-134. doi: 10.4471/redimat.2016.2093

To link this article: <http://dx.doi.org/10.4471/redimat.2016.2093>

PLEASE SCROLL DOWN FOR ARTICLE

The terms and conditions of use are related to the Open Journal System and to [Creative Commons Attribution License](#) (CC-BY).

Evaluating Mathematics Teachers' Professional Development Motivations and Needs

Mary C. Caddle
Tufts University

Alfredo Bautista
Nanyang Technological University

Bárbara M. Brizuela
Tufts University

Sheree T. Sharpe
University of New Hampshire

(Received: 11 May 2016; Accepted: 16 June 2016; Published: 24 June 2016)

Abstract

While there is widespread agreement that one-size-fits-all professional development (PD) initiatives have limited potential to foster teacher learning, much existing PD is still designed without attention to teachers' motivations and needs. This paper shows that the strengths and weaknesses of middle school mathematics teachers that engage in PD may significantly vary. We present three representative cases that illustrate this diversity. The cases were selected from a cohort of 54 grades 5-9 mathematics teachers in the northeastern United States. The results show that: 1) these three teachers dramatically differed in their motivations and self-perceived needs regarding mathematical content, classroom instruction, and student thinking; 2) their perceptions were closely aligned with the results of our own assessments; and 3) the motivations and needs of these three teachers reflected the general trends identified in the cohort of 54 teachers. We conclude that "giving teachers voice" is essential when designing and implementing PD.

Keywords: Middle school mathematics teachers, teacher professional development, responsive PD, motivations, needs

Evaluando las Motivaciones y Necesidades de Desarrollo Profesional de Profesores de Matemáticas

Mary C. Caddle
Tufts University

Alfredo Bautista
Nanyang Technological University

Bárbara M. Brizuela
Tufts University

Sheree T. Sharpe
University of New Hampshire

(Recibido: 11 Mayo 2016; Aceptado: 16 Junio 2016; Publicado: 24 Junio 2016)

Resumen

Pese a existir un acuerdo generalizado en que las iniciativas de desarrollo profesional docente (DPD) del tipo "lo-mismo-para-todos" tienen un potencial limitado para promover el aprendizaje de los profesores, buena parte del DPD sigue todavía diseñándose sin prestar atención a las motivaciones y necesidades de los docentes. Este artículo muestra que las fortalezas y debilidades de los profesores de matemáticas que participan en DPD pueden variar de forma significativa. Se presentan tres casos representativos que ilustran esta diversidad. Los casos se seleccionaron de una cohorte de 54 profesores de matemáticas de escuelas medias (grados 5-9) en el noreste de Estados Unidos. Los resultados muestran que: (1) las tres profesoras difieren en sus motivaciones y necesidades percibidas respecto al contenido matemático, instrucción en el aula y pensamiento de los/as estudiantes; (2) sus percepciones están estrechamente alineadas con los resultados de nuestras propias evaluaciones; y (3) las motivaciones y necesidades de estas tres docentes reflejan las tendencias generales identificadas en la cohorte de 54 profesores. Concluimos que dar la voz a los docentes es esencial para diseñar e implementar DPD.

Palabras clave: Docentes de escuelas medias, desarrollo profesional docente, DPD diferenciado, motivaciones, necesidades

Much of the research on teacher professional development (PD) has yielded disappointing results regarding its effectiveness in helping teachers improve instructional practices and even more disappointing results regarding its impact on student learning and achievement (Desimone & Garet, 2015; Garet et al., 2011). While helping teachers broaden their subject-matter and pedagogical knowledge may seem simple, it is not, and improving their actual classroom practices has proven to be even more complicated (Borko, 2004). One common argument put forth to explain these difficulties is that PD might not attend and respond to the actual interests, desires, or demands of the teachers, or, in other words, that PD might not be ‘responsive’ (Włodkowski & Ginsberg, 1995). This idea is consistent with Desimone’s (2009) conceptual framework, according to which being *coherent* with a teacher’s own motivations and needs is one of the critical features for effective PD (see also Bautista, & Ortega-Ruiz, 2015).

In this paper, we address the question of how to consider the widely varying motivations and needs of middle school mathematics teachers as they engage in PD. We analyze what teachers stated as their goals, strengths, and weaknesses when they enrolled in our three-semester PD program, and how teachers’ statements compare to our assessment of their knowledge of mathematics content and student thinking. Based on this, we reflect on the resources that PD providers can use to determine teachers’ motivations and needs. We claim that it is crucial for PD providers to have a deep understanding of what teachers bring and what they seek to learn when they enroll in PD. Moreover, we claim that it is essential to systematically assess if teachers’ existing strengths and weaknesses are complemented by what PD can offer, and otherwise, consider how to vary offerings to meet their needs.

Considering the complex motivations and needs of mathematics teachers

Research has shown that effective mathematics teachers utilize many types of specialized knowledge. We draw on the work of Shulman (1986), who claims that teachers need a kind of pedagogical content knowledge (PCK), a knowledge of the subject matter that allows them to teach it. Further refinement of this theory has suggested that mathematics teachers’ PCK is part of a broader construct, mathematical knowledge for teaching, that

encompasses both subject matter knowledge and PCK, and which can be broken down further into additional specialized types of knowledge (Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008). As yet, there is no way to measure each of these types separately, to know if all the types have been captured, or to assess whether they are necessarily separable (Sherin, 2002). However, past qualitative research has demonstrated how widely teachers' knowledge profiles differ (Caddle, & Brizuela, 2014). That is, if we do try to identify the types of knowledge teachers use, we see that some teachers call most frequently on pure mathematical knowledge, others on knowledge of what their students tend to do with certain types of mathematical tasks, and so on. Our intent in calling on this framework is not to classify or measure our teachers' knowledge. Instead, we believe that breaking down teachers' specialized knowledge into smaller components has allowed the field to identify and examine what we hope teachers will understand. We take this past work as support for the argument that teachers may have varied strengths and weaknesses with regards to these different types of knowledge, and therefore varied motivations and needs.

There are only a few studies that have systematically investigated the varying motivations and needs of mathematics teachers, which is perhaps one of the reasons why much PD still tends to follow a one-size-fits-all approach (Darling-Hammond, Chung Wei, Andree, Richardson, & Orphanos, 2009). Beswick (2014) reviewed three projects aimed at identifying the self-reported mathematical content knowledge PD needs of different groups of mathematics teachers from Tasmania (Australia). Teachers of different grade levels had different mathematical backgrounds: primary teachers had mathematics curriculum units as part of their pre-service teaching qualifications, whereas most of the secondary teachers had taken mathematics courses during their undergraduate degrees. Despite these differences in mathematical background, both groups felt least confident about topics such as ratio and proportion and critical numeracy in the media. They also had little confidence in connecting numeracy across the curriculum, and in operations with fractions and decimals. Algebra (beyond year 8), problem solving, and decimals were also identified as problematic areas, even for teachers with high levels of training in mathematics.

Other studies have focused on exploring mathematics teachers' motivations and needs concerning both content and pedagogical elements. In a survey study, Chval, Abell, Pareja, Musikul, and Ritzka (2008) examined the PD experiences, expectations, and constraints of 241 middle and high

school mathematics and science teachers in the United States (US). With regards to content focus of PD, the 118 participating mathematics teachers expressed interest in learning about technology in mathematics, followed by topics from discrete mathematics, probability, statistics, and patterns and relationships. They also expressed a need for PD focused on critical thinking, problem solving strategies, student learning, making connections with the real world, and use of technology in teaching. The authors concluded that the PD experiences offered to these teachers were not responsive to their expressed needs, and thus ineffective. Similar conclusions were obtained in the large-scale survey study conducted by Bennison and Goos (2010), who investigated the PD experiences and needs of 400 secondary mathematics teachers.

Finally, recent research has suggested that the interests of middle- and high-school mathematics teachers tend to systematically differ. For example, the study by Matteson, Zientek, and Ozel (2013) has shown that middle school mathematics teachers tend to be more interested than secondary teachers in learning about new pedagogical resources for students, as well as in PD focusing on how to best meet the needs of diverse student populations (including low performing and students with learning disabilities). In contrast, mathematics teachers at the secondary level tend to exhibit more interest in topics such as pedagogical uses of technology. Interestingly, both groups of teachers valued learning from peers through the sharing of lessons. The goal of this paper is to illustrate the diversity of motivations and needs that middle school mathematics teachers have when engaging in PD programs. We present three representative cases that were selected from a cohort of 54 grades 5-9 mathematics teachers from nine school districts in the northeastern US. We analyze the written statements that these three teachers submitted when they initially enrolled in our three-semester PD program, as well as their scores on an assessment focused on mathematical content knowledge and student mathematical thinking. The cases are used to illustrate general tendencies within the larger cohort of teachers.

The present study differs from prior research in at least three ways. First, instead of basing our conclusions exclusively on teachers' self-reported data (e.g., surveys), we used multiple data sources that allowed us to compare the reports of the teachers with external measures of their knowledge. In particular, our "personal statement," described below, allowed us to collect self-reported information, which could then be compared with data from a written assessment of knowledge of mathematics and student thinking.

Second, the studies described above (Bennison & Goos, 2010; Beswick, 2014; Chval et al., 2008) have evaluated the needs of teachers with regards to specific types of knowledge; for example, only mathematical content or only pedagogical concerns. In contrast, the personal statement utilized in this study allowed teachers to freely describe their own strengths and weaknesses, regardless of whether these were specific to subject-matter knowledge, pedagogical knowledge, or other. Finally, our study attempts to investigate the extent to which certain teacher characteristics might be associated to specific PD motivations and needs. In particular, our study focuses on the variables “years of teaching experience” (YTE) and “educational background.”

The educational backgrounds of grades 5-9 mathematics teachers in the US vary widely, especially because the requirements for licensing mathematics teachers have shifted over time. Today, we might find two 6th grade teachers with different backgrounds in adjacent classrooms: one may hold a primary school license and have taken few to no post-secondary mathematics courses; the other may hold a middle school license and a master’s degree in mathematics. While there are differences in the current requirements across states in the US, in the three states represented in this study, licensure types overlap in the middle grades. For example, in one state included in this study, primary school teachers hold licenses for grades 1-6, while middle school teachers hold licenses for grades 5-8 and high school for grades 8-12. Each of these different licenses has different requirements for standardized testing prior to licensure and for educator preparation courses. As a result, in the band of grades 5 through 9 covered by this study, some teachers will have had little to no college-level mathematics coursework, while others may have four-year degrees in mathematics. Due to these widely differing requirements, teachers come to the classroom with different backgrounds as a matter of institutionalized processes, not just because of individual differences.

Method

Participants

This study is based on a cohort of 54 mathematics teachers who taught grades 5 through 9 (students from 11 to 15 years of age) in nine school districts in the northeastern US. These teachers were applying to participate

in a grant-funded PD program for mathematics teachers (Teixidor-i-Bigas, Schliemann, & Carraher, 2013). Teachers from partner districts were invited to participate through the mathematics coordinator in their district. The program did not have any application criteria, other than teaching the appropriate grade level in a partner district. There were 48 female teachers and 6 male teachers, ranging from 24 to 64 years of age (average = 41.1, standard deviation = 10.657). Their professional experience as mathematics teachers ranged from 0 years (2 months) to 25 years (average = 9.1, standard deviation = 6.198).

When they enrolled, teachers were told that the PD program would focus on algebra and the mathematics of functions as they relate to the middle school curriculum, and that they would explore the multiple perspectives on mathematics employed by mathematicians, scientists, teachers, and students. They were told that the goal of the program was to improve students' deep understanding and enthusiasm for mathematics by involving their teachers in an intellectual community. They were also told that the program was designed to offer a broad, unified framework from which to view the mathematics they currently taught.

Data Sources

Prior to starting the PD program, we asked teachers to complete the following two items:

Application and personal statement

Teachers were asked to complete an online application to provide us with information about their educational background, teaching experience, and biographical data. In addition, they were asked to upload a personal statement of no more than 1,000 words, including information about their motivations and needs (“*Who or what influenced your decision to apply to the [PD] program?*”), goals (“*In what ways do you hope that participation in the [PD program] will impact you personally and professionally?*”), and mathematical biography, including their strengths and weaknesses (“*What math do you find most interesting or enjoyable? What math do you find particularly easy or challenging to teach? What math do you find your students enjoy most? What math seems most challenging for them? What math are you hoping to learn more about in these courses?*”).

Assessment of teachers' knowledge of mathematics and students' mathematical thinking

In addition, teachers completed a mathematics assessment online. Its purpose was to evaluate changes and progress in the teachers' mathematical knowledge as a result of participating in the PD. Thus, teachers took the assessment at the beginning of the first course (between two weeks prior to and two weeks after the first day of the course) and again after completing the three semester-long courses. Since this study focuses on teachers' initial motivations and needs and does not look at their changes over time, we look only at the results of the initial assessment. The assessment was designed by the research team to cover mathematical content relevant to the goals of the program. Specifically, items were chosen that drew upon understanding of algebraic relations, functions, and their representations. This included being able to generalize mathematically and to use and operate on an unknown, as well as to work with rational numbers, the real line, and the coordinate plane. In addition, items were chosen to cover a variety of written representations, including tables, graphs, pictures, algebraic notation, and written language. Where available, the research team selected assessment items from existing sources with past performance data [e.g., Trends in International Mathematics and Science Study [TIMSS], Foy & Arora (2009); National Assessment of Educational Progress [NAEP], US Department of Education (2007)]. If no existing items were found to cover an area relevant to the program, the research team designed new items. The items designed by project researchers account for 16 (out of 47 possible) points on the assessment.

Some of the items had been used with students on a prior project. As a result, and because of our interest in how teachers understand students' mathematical thinking, we also included samples of student work, and asked teachers to interpret and respond to the student productions.

Analysis

The 54 personal statements submitted by the participating teachers as part of their online application were analyzed through lexicometry (Lebart, Salem, & Bècuc, 2000). The software used was DtmVic (version 5.6), which is available online (visit: http://www.dtmvic.com/05_SoftwareE.html). Among

other functionalities, lexicometry allows the investigator to: a) study the existence of lexical differences in the verbal/written productions of several groups of participants (in this study, teachers grouped according to different variables, as described below), and b) rank the participants within each group according to how representative the individual is of the group, based on the lexicon used, from most to least representative. Regarding teachers' mathematical content knowledge, the scores in the assessments before participating in the PD were analysed using an analysis of variance (ANOVA). IBM SPSS Statistics (version 23) software was used to analyse the data. Finally, teachers' responses to the item focusing on students' mathematical thinking were analysed qualitatively.

Case selection

The three cases selected for this study, Marissa, Judy, and Katherine (all pseudonyms), were identified on the basis of the lexicometrical analysis of the personal statements. Following the taxonomy used by Ghaith and Shaaban (1999), we split the 54 participating teachers into three groups based on their amount of prior teaching experience: beginning teachers (less than five years of teaching experience [YTE]), experienced teachers (between five and 15 YTE), and highly experienced teachers (more than 15 YTE). Marissa, Judy, and Katherine were the most representative participants from each of these three groups, respectively, when we compared the statements according to the variable YTE. By "most representative" we mean that each one was the person within their YTE group who most frequently used the words and phrases that were statistically associated with that group.

The correlation between YTE and teacher's age was significant, $r(51) = .49, p < .001$. In other words, the older teachers are, the more YTE they tend to have. Further, these two variables (YTE and age) were also associated with the variable educational background. The participants in our project had a variety of educational backgrounds, which we grouped into two broad categories:

- *Mathematics*: Teachers who earned their bachelor's or master's degree in disciplines that involve significant study of mathematics, such as Mathematics, Mathematics Education, Physics, Engineering, Biology, or Chemistry (21 teachers).

- *Non-mathematics*: Teachers who earned their bachelor’s or master’s degree in disciplines that do not involve significant study of mathematics, such as History, English, Special Education, Theology, or Literature (33 teachers).

As shown in Table 1, most teachers in the “Less than 5 YTE” group belonged in the “Mathematics” educational background group, whereas most teachers of the two other YTE groups belonged in the “Non-mathematics” educational background group. In particular, note that only one teacher with “More than 15 YTE” belonged in the “Mathematics” group, whereas 12 belonged in the “Non-mathematics” group.

Table 1
Relationship between teachers’ Years of Teaching Experience (YTE) and their Educational Background

		Educational Background		
		Mathematics	Non-Mathematics	Total
Years of teaching experience (YTE) in Mathematics	Less than 5 YTE	10	7	17
	Between 5 and 15 YTE	10	14	24
	More than 15 YTE	1	12	13
Total		21	33	54

A chi-square test on the two-way contingency table above was conducted to evaluate the differences in the proportions of Mathematics to Non-mathematics across the three levels of YTE. The proportions were found to be significantly related, Pearson χ^2 (2, N=54) = 8.244, p = .016. The two pairwise differences that were significant were between “more than 15” and the other two levels. The three cases featured below belonged in the cells highlighted in bold, which had the highest numbers of participants for the variable YTE.

Results

Application and Personal Statement: Teachers' Declared Motivations and Needs

Case 1, Marissa: “I want more ideas on how to create an active role for the students within my classroom.”

Marissa was a high school teacher born in 1986. She earned a B.S in Mathematics in 2008 (with a minor in Secondary Education), and a Masters in Education in 2010 (with an emphasis on Secondary Education). Thus, she belonged in the group Mathematics described in the ‘Case selection’ section. When Marissa wrote her personal statement, she was teaching Algebra I and II and had held a full-time teaching position as a grade 9-11 mathematics teacher for one year. Prior to that, Marissa worked as a substitute teacher for one year and had several months of experience as a mathematics teacher intern. Overall, she focused on describing how teaching and learning should occur in an ideal scenario, but claimed that she needed new strategies to bring these ideas into the classroom.

Marissa’s statement contained many references to students and to the processes of teaching and learning. The main goals she expressed were twofold. First, she wanted to better motivate her students to learn mathematics more deeply and to be more active and engaged in the classroom (e.g., “*My goal is to encourage students to ask questions; I request that my students enter the classroom prepared to be challenged and willing to struggle with a concept in order to understand it more; I want more ideas on how to create an active role for the students within my classroom*”). Her second goal was to improve her teaching strategies by incorporating new activities and projects in her teaching (“*My hope is that the [PD program] will provide me with more strategies in inspiring my students and making mathematics more accessible to them; I hope to gain more strategies and insight on how to teach algebraic topics more effectively*”).

In contrast, Marissa talked very little about mathematics and did not mention any need to improve her mathematical knowledge through our PD program. As mentioned above, Marissa had a bachelor’s degree in mathematics, and her statement implicitly conveyed her perception that she had the mathematical content knowledge required, and now she just needed

to improve her pedagogical knowledge and skills. Perhaps because she had only been teaching for a few years, Marissa did not describe much about the way she taught. This radically differs from the two teachers featured below, Judy and Katherine, who provided a wealth of details about their teaching approaches. Instead, Marissa repeatedly mentioned that she needed to, hoped to, or wanted to learn new teaching skills during the PD program (e.g., “*By collaborating with professionals from [name of institution], I hope to be able to design lessons that engage and introduce new mathematics topics as familiar and related to their world; I hope to learn more skills that will allow me to help students who are not getting material right way to eventually be competent and confident in using new math skills*”). As can be seen in these examples, the teaching skills were described in a rather general way, without reference to specific elements (e.g., “*design lessons that engage and introduce new mathematics topics as familiar and related to their world*”).

Based on the lexicon used, Marissa’s statement was automatically selected as the most representative of the “Less than 5 YTE” group. As can be observed in the quotes presented above, words such as *Student(s)*, *Teach(er)*, *Teaching*, *Think(ing)*, and *Classroom* commonly appeared in the statements written by these beginning teachers (significantly more than in the statements of the other two groups). The statements of beginning teachers tended to be student-centered. For example, some of the most commonly repeated segments (i.e., chains of words) in these statements were “students have difficulty,” “students struggle with,” or “to help my students.” In addition, these teachers used the terms *Understand* and *Understanding* significantly more than the other two groups (e.g., “understanding of mathematics,” “a deeper understanding,” “a solid understanding of,” “my understanding of,” “their understanding of”). The statements had a significantly higher proportion of sentences formulated in the first person singular (*I*, *Me*, *My*) and in future tense (e.g., “will allow me to,” “will be able to,” “will help me,” “will help me better”). The idea of internal agency was prominent (e.g., “I want to be,” “I want to learn,” “I need to,” “I will”).

Case 2, Judy: “I want my students to see the deeper mathematical thinking so that they can be more successful in standardized tests.”

Judy was a middle school teacher born in 1963. She earned a B.S in Dental Hygiene in 1985, and Professional Teacher Certification in 2004 (Elementary Education Certification). She was included in the Non-Mathematics group. When Judy wrote her personal statement, she was about to start her seventh year as a mathematics teacher in a middle school. She was teaching 6th grade at the time of enrollment in our PD program. Overall, she did not provide many details regarding her teaching philosophy and instead shared more about her experience as a teacher. She emphasized her experience and knowledge of students, but expressed concern about getting students to think more deeply about mathematics, as illustrated in the last part of the following quote: *“I am far enough along in my teaching career that I can create a relationship that makes my students want to learn for me. I am missing the piece that allows my students [to] access the understanding.”*

Judy explained that she needed to improve her teaching strategies to deal with students’ fears, to help them learn more and better, and specifically, to help them with tests and to improve their scores. She stated that, *“I need to gain more understanding about the ways in which we measure students competency in these content areas and how I can better help my students understand.”* Her statement was at times pragmatic and focused on students “getting it right.” Judy also explained that she needed to improve her mathematical knowledge of certain topics that were particularly difficult or problematic for her to teach.

Her main goals were to improve her teaching strategies, and to a lesser extent to improve her mathematics knowledge. Her statements were often success-oriented: *“I find success with my average and above average students but I was failing my under-resourced students and my English language learners. Other schools are finding success in these areas, what are they doing that I was not able to do?”* Similarly, she wrote: *“My realistic goal is to see a significant increase in the number of students meeting the standard and seeing the number of students partially meeting the standard shrink. I believe we should be able to add 25% of our students currently partially meeting the standard to the percentage of students meeting the standard. [...] if I could be able to make a difference in the percent of students able to access their math skills I would feel that I had met personal*

and professional success.” Judy also perceived weaknesses in her ability to access students’ deep mathematical thinking, e.g., *“The kids delight in the game playing but trying to get them to see the deeper mathematical thinking is very difficult for me;”* *“probability is the most challenging topic for me to teach.”*

Judy’s statement was automatically selected as the most representative of the group of teachers with between 5 and 15 YTE. The lexicon of her statement reflected the general lexical trend of the group. Words such as *Improve* (e.g., “I can improve my,” “improve my teaching,” “to improve my”), *Teaching* (e.g., “teaching of mathematics,” “improve my teaching,” “in my teaching”), and *Learning* (e.g., “learning more about,” “my students are learning”) were frequently repeated in these statements, reflecting the concern of this group for improving pedagogical practices in order to raise student achievement. In addition, the words *Mathematics* and *Mathematical* were frequently identified in the statements, which indicates the motivation of these teachers to improve their content knowledge (e.g., “my understanding of mathematics,” “middle school mathematics,” “teaching of mathematics”).

Case 3, Katherine: “We need help with the math.”

Katherine was a middle school teacher born in 1968. She earned a B.S in Elementary Education, and a masters of arts in teaching degree in 2004. She held Professional Certification as a grades 1-6 teacher. She was also coded in the Non-Mathematics group. The year she enrolled in our PD program she was teaching mathematics in 5th grade. She had been teaching Mathematics for 19 years. In the past, she taught grades 2, 4, 5, and 6 as a general educator, as well as English and social studies to grades 6-8 students. The main theme in Katherine’s statement was her need to improve her mathematics, as illustrated in the following: *“My math knowledge is very limited because mathematics is challenging -- for me and for many other people, including math teachers! WE need to learn more mathematics (algebra, geometry, proofs, etc., etc., etc.). The [PD program] is a great opportunity for us to collaborate and work with other teachers!!”*

Katherine frequently reiterated that mathematics was difficult for her: *“Mathematics for many is not their favorite subject or it just does not come easily for them. I am one of those people. And, yes, I am a math teacher.”* Katherine described her history with mathematics as a challenging process:

“Growing up I struggled with math. I will never forget my freshman year in high school and algebra I. I worked so hard. I appreciated that my teacher gave partial credit on tests because he could see that you at least understood part of it.” Katherine was frank about her shortcomings, both in her statement and with her students: “As a math teacher, I am honest with my students; they know I struggled and want them to succeed. I let them know that there are things that are a bit difficult, and then there are the fun topics like graphs and geometry.” Katherine’s ultimate goal was to know more mathematics to teach better: “In order to be an effective teacher I need to continue to be a student. Each class and discussion helps me to have a deeper understanding of the content I am teaching. Deeper understanding leads to better teaching.”

Katherine’s statement was the most representative among the group of highly experienced teachers, with more than 15 YTE. As can be observed in the quotes presented above, this group of teachers tended to use mathematics specific terms such as *Math*, *Mathematics*, *Algebra*, and *Geometry*, showing their interest in furthering their content knowledge. Other words that were significantly more frequent in these statements were *Work*, *Opportunity*, *Skills*, *Teachers*, *Time*, and *Years* (e.g., “years I have”). An interesting adjective frequently identified in these statements was *challenging*, which alluded to these teachers’ difficulties with the mathematical content knowledge itself. These statements had a significantly higher proportion of sentences formulated in the first plural person (*Our*, *We*). This plural phrasing was particularly common in the context of teachers’ references to the struggles and difficulties with the content knowledge.

Assessment on Teachers’ Knowledge of Mathematics and Student Mathematical Thinking

Table 2 shows the results of the 54 participating teachers in the pre-assessment of their mathematical content knowledge, with the corresponding break down for the variable YTE. As can be observed, the teachers with the fewest YTE obtained the highest mean score on the teacher assessment, whereas the teachers with the most years of teaching experience obtained the lowest mean score. The differences in the mean pre-assessment scores when considered with YTE as a categorical variable were not significant under ANOVA ($p = .063$). However, the correlation between years of teaching

experience and teachers' pre-assessment score was significant, $r(51) = -.32$, $p = .019$.

Table 2
Pre-assessment scores by YTE

	N	Mean	Min	Max	Standard deviation
All teachers	54	36.26	21	46	7.138
Less than 5 YTE	17	38.82	22	46	5.681
Between 5 and 15 YTE	24	36.37	24	46	6.639
Greater than 15 YTE	13	32.69	21	43	8.625

A one-way analysis of variance was conducted to evaluate the relationship between teacher educational background and their pre-assessment score. The ANOVA was significant, $F(1, 53) = 5.50$, $p = .023$ (see Table 3).

Table 3
Pre-assessment scores by background

	N	Mean	Min	Max	Standard deviation
All teachers	54	36.26	21	46	7.138
Mathematics	21	39.00	22	46	6.488
Non-Mathematics	33	34.52	21	43	7.072

For the three teachers described in the prior section, we can look in more detail at their scores and the details of their responses on the written assessment. As mentioned above, the teachers completed this assessment prior to participating in the PD program, as was the case with the statements analyzed above. As shown in Table 4, the three representative teachers followed the general pattern seen across the groups. That is, Marissa, with less than five YTE, had the highest pre-assessment score of the three teachers, and Katherine, with more than fifteen YTE, the lowest. It is of note that Katherine's score was much lower than the mean score for her group.

Table 4

Teacher assessment scores for three selected teachers

Teacher Name	Pre-Assessment	Group mean
Marissa (< 5 YTE)	41	38.82
Judy (5-15 YTE)	31	36.37
Katherine (> 15 YTE)	22	32.69

However, what is most telling from this data is that their scores on the assessment accurately reflect their own self-assessment of their PD needs in terms of mathematical content knowledge. Marissa, having not addressed mathematical content knowledge, as a PD need at all, demonstrated competence by getting a high score on the assessment (the highest score in the cohort of 54 teachers was 46). Judy and Katherine's scores similarly reflect their perception of their own mathematical skills as evidenced in their written statements.

To expand on this connection, we examined one of the problems from the assessment, shown in Figure 1. The initial part of the question (the diagram and the first question, "How many sides would be in the 25th figure?") is taken from the NAEP (US Department of Education, 2007, identifier 2007-8M7 #14). We had extended this problem in prior work with students, adding the question, "What will be the perimeter of the n^{th} figure in the pattern?" because we wanted to examine students' generalization to the n^{th} case. In the assessment for this project, we asked teachers to first respond to the questions themselves, and then (after their own response) to examine a sample student response taken from the prior project; the student work is also shown in Figure 1. Note that the teachers' own correct or incorrect responses (102 cm and $4n+2$ cm) each counted for one point in the numeric assessment score above; their responses to the student work are not accounted for in the numeric scoring.

We selected this problem for this analysis because it included the teachers' own mathematical work as well as examination, interpretation, and response to student work, which we consider to be an important task of teaching (see [Ball et al., 2008](#)).

Each figure in the pattern below is made of hexagons that measure 1 centimeter on each side.



Figure 1
Perimeter = 6 cm



Figure 2
Perimeter = 10 cm

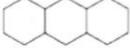


Figure 3
Perimeter = 14 cm

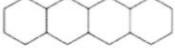


Figure 4
Perimeter = 18 cm

If the pattern of adding one hexagon to each figure is continued, what will be the perimeter of the 25th figure in the pattern?

102 cm

Show how you found your answer.



$10 +$
 $4 \cdot 23 = 102$

What will be the perimeter of the n^{th} figure in the pattern?

$n \cdot 4 + 10$

Show how you found your answer.

the ends are
5 and the inside
hexagons are 4.

Figure 1. Item asking teachers to interpret a student’s mathematical thinking

In Marissa’s response, she got both the numerical case (102 cm) and the algebraic expression ($4n+2$) correct. She wrote that she used a table (which she refers to as an “input/output chart”). In her response to the student work, she seemed to recognize the student’s strategy and addressed precisely how the student’s formula could be corrected by replacing n with $n-2$. She said that the student recognized the pattern and knew to make an equation, though they stumbled on expressing “figure number minus 2 algebraically.” She also pointed out that the student forgot to add the 10 in the first part. There were no statements that appeared to be unsupported by evidence from the student work. In terms of instructional support, Marissa suggested bringing the student’s attention to where the correct expression is written for the numeric case, and using that to have the student identify each piece and explain where the “23 came from,” using the 23 to make the connection to $n-$

2. She also mentioned a second strategy to help the student notice the “double counting.”

Judy also correctly responded to both the numerical case (102 cm) and the algebraic expression $(4n+2)$, stating that she used “algebraic pattern recognition.” Her meaning is not definitively clear, although it suggests that she was focused on the recursive, or increasing by 4, aspect of the problem. In her response to student work, Judy saw many positive elements of the student’s understanding, mentioning that the student understood patterns and knew to use multiplication. She also recognized that the student was making an “exception” for the end hexagons, elaborating that while she was not sure, perhaps the 23 was a way to consider only interior hexagons, and if so that it wasn’t reflected in the formula. This suggests that Judy did recognize the trouble with the formula. In terms of working with the student, Judy suggested having the student “test his formula” and “look more deeply.” There was nothing incorrect in this response, but the actions suggested were general and not targeted specifically to the student’s response.

In her own response, Katherine got the numerical case (102 cm) correct and the algebraic expression incorrect (writing “ $6+4(n)$ ”). She recognized the pattern and used that (“I noticed that with each additional hexagon the perimeter increased by 4 centimeters”), also stating that she “multiplied the number of additional hexagons times 4 then added 6 for the initial hexagon.” It seems from her statement that she was able to extend the pattern to correctly get 102 without necessarily writing out each consecutive term, but not to generalize to the n th term using algebraic notation. In her response to student work, Katherine mentioned that the student forgot to add the 10, and also noted that the student understands perimeter. She also seemed to comprehend the strategy the student was using with interior/end hexagons: “The student also recognizes that he can multiply the number of interior hexagons by 4 to get the perimeter of the inner hexagons and then add the 10 for the hexagons on the end.” For a response to the student, Katherine suggested having the student explain and then use manipulatives to show the problem, although she did not elaborate what the student could do with the manipulatives or how this might impact his or her work. As with Judy’s response, there was nothing incorrect in Katherine’s response to student work, but the suggestions were very general.

The responses to this sample item can deepen the picture we already have from the teachers’ statements and overall assessment scores. Marissa’s statement reflected her concern about learning more about students and

pedagogy. However, at least in this isolated case, she is well able to parse the student's mathematical thinking. In addition to that, she also offers the most specific and targeted ideas for addressing the problems in the student's response. Her suggestions focus on this one case of student work, not on working on this problem with a general audience. In Judy's response, we see that she was able to handle the mathematical content, although she only tentatively identifies the problem with the student's formula for the n th case. This uncertainty may be connected to the generality of her suggestions for working with the student. Similarly, Katherine's case suggests that she was able to understand the student's reasoning. However, she wasn't initially able to offer any specific suggestions as to how to help the student. Noting that she wasn't able to correctly find the algebraic expression herself, perhaps she was not able to act as a guide here. It is also possible that she didn't find it necessary to base her recommendation on what the student had already done.

Discussion and Conclusions

While PD in mathematics is generally designed and implemented with the best of intentions, the research cited above demonstrates that one-size-fits-all PD has had limited success in promoting teacher learning (Darling-Hammond et al., 2009). Through the cases and data presented here, we have helped to fill in the picture about why this might occur. The vast differences in teachers' mathematical backgrounds and experience, and in their motivations and needs, indicate that in order to support teachers better, we need to *meet them where they are*. That is, we need to be able to find the right fit in PD programs in order to complement existing strengths and facilitate improvement in other areas. This is not straightforward, and we claim that the analysis provided here constitutes a useful first step. To summarize, we will argue that (1) teachers' needs and motivations vary widely, as shown by the three cases; (2) the combination of data sources used here supports giving teachers a voice in selecting their PD; and (3) we, as a field, need to explore various ways to determine teachers' motivations and needs accurately.

Regarding the first point, we described three cases. Katherine's case is perhaps the clearest in terms of showing motivations and needs that are well defined and aligned. She was specific in her request for help with mathematics content, and her assessment reflects this need. In this way, she

was also consistent with the teachers surveyed in the research above who report needing help with content (Chval et al., 2008). Other teachers, like Judy, may have needs that are harder to determine. Her score on the mathematical content assessment was not so low as to suggest an urgent need for support in this area, nor did she report in her written statement a significant need for help with content. However, she demonstrated a strong motivation to improve student test scores and stated that she had trouble getting students to access “deep understanding.” Considering these elements, together with the fact that she was not as mathematically precise as Marissa in explaining the difficulty in the student work and how to address it, we conjecture that Judy would be especially motivated to participate in PD focused on how students are thinking about challenging mathematics, and how to help address specific misunderstandings. PD focusing on generic ideas related to mathematics teaching and learning might be, therefore, not suitable for teachers like her.

Marissa represented a group that addressed mathematics content infrequently in their personal statements, and, both in Marissa’s case and the overall group, high scores on the content assessment support the omission. We also know that Marissa was strong at interpreting students’ paper and pencil mathematical work. While we don’t know if she was typical of the <5 YTE group in being able to parse the student thinking, it is worth considering how this aligns with mathematical content knowledge when planning PD. For example, in a PD program where teachers are asked to plan pedagogical supports, would teachers who cannot easily parse students’ thinking need more time and support prior to engaging in planning interventions? Also of note is the contrast between Marissa and the teachers cited in the research above (Beswick, 2014) who needed more support in mathematical content. This emphasizes the importance of recognizing different teacher motivations and needs; for instance, enrollment in PD focused on mathematical content knowledge would not be a productive use of time for Marissa and those with a similar profile.

The point of revisiting these cases is to show how vast the differences between teachers are. Prior studies have investigated teachers’ motivations or needs, but we know very little about how teachers might aggregate into groups with different profiles. The lexicometry analysis conducted on the personal statements showed that groups of teachers with varying mathematics backgrounds and YTE seem to have different PD motivations and needs. By looking at the written assessment, both in total scores and in

student work, we can support the teachers' self-reported data. Our assessment shows that, at least in some ways, teachers were accurate in assessing their own strengths and weaknesses. We see that teachers, including Katherine, who claim to need help with mathematical content knowledge, are (as a group) self-aware and able to identify this need. This is particularly salient because the assessment data shows that variables such as YTE and mathematical *versus* non-mathematical background are associated with different levels of performance on the mathematical content. However, we do not claim that all teachers in each of these groups have the same motivations and needs. Instead, we argue that coherence between the data sources used here, the self-reported statement and the assessment, supports giving teachers a voice in selecting the focus of PD. This demonstrates the importance of identifying teachers' own motivations and needs prior to the design and implementation of the PD initiative itself (Bautista, & Ortega, 2015; Desimone & Garet, 2015).

Finally, we argue that as a field we need to explore other ways to find out how to align PD with teachers' motivations and needs. Although we show here that teachers were accurate in assessing their needs in mathematical content knowledge and to some extent in interpreting student thinking, one limitation is that these measures have not demonstrated the accuracy of their self-assessment in other areas. For example, Marissa claimed to need help with pedagogical strategies (e.g., "*My hope is that the [PD program] will provide me with more strategies in inspiring my students and making mathematics more accessible to them*"). With our available data sources we do not know if her statement was accurate, or if we could assess Marissa's PD needs better by visiting her classroom or using some other metric. Similarly, Katherine did not report difficulty with interpreting student thinking, but she had trouble being specific about the problem with the student work in the assessment (see Figure 1). We do not know if this was an isolated instance, if her focus on content-related needs overshadowed other needs that she would be aware of, or if she was not able to accurately self-assess in this particular area.

Our intent in this paper is not to make a universal statement about the value of our own measures, but to show the importance of using multiple ways of finding out what teachers' needs and motivations are. Indeed, other measures may also be helpful, and may complement this work to generate a broader picture of PD possibilities. As a field, this analysis should act as a starting point for thinking about what kinds of information we could collect

in order to design more tailored and useful PD. Both researchers and PD providers should be creative and investigative in order to be responsive to and supportive of our teachers.

Agradecimientos

This study was funded by NSF MSP Grant #0962863, “The Poincaré Institute: A Partnership for Mathematics Education.” The ideas expressed herein are those of the authors and do not necessarily reflect the ideas of the funding agency.

References

- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407. doi: [10.1177/0022487108324554](https://doi.org/10.1177/0022487108324554)
- Bautista, A. & Ortega-Ruíz, R. (2015). Teacher professional development: International perspectives and approaches. *Psychology, Society and Education*, 7(3), 240-251.
- Bennison, A., & Goos, M. (2010). Learning to Teach Mathematics with Technology: A Survey of Professional Development Needs, Experiences, and Impacts. *Mathematics Education Research Journal*, 22(1), 31-56. doi: [10.1007/BF03217558](https://doi.org/10.1007/BF03217558)
- Beswick, K. (2014). What teachers’ want: Identifying mathematics teachers’ professional learning needs. *The Mathematics Enthusiast*, 11(1), 83-108.
- Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher*, 33(8), 3-15. doi: [10.3102/0013189X033008003](https://doi.org/10.3102/0013189X033008003)
- Caddle, M.C., & Brizuela, B.M. (2014). *Using interviews to explore teacher knowledge profiles in the area of permutations*. Paper presented at the Joint 38th Conference of the International Group for the Psychology of Mathematics Education [PME] and the 36th Conference of the North American Chapter for the Psychology of Mathematics Education [PME-NA], Vancouver, Canada.
- Chval, K., Abell, S., Pareja, E., Musikul, K., & Ritzka, G. (2008). Science and Mathematics Teachers’ Experiences, Needs, and Expectations

- Regarding Professional Development. *Eurasia Journal of Mathematics, Science & Technology Education*, 4(1), 31-43.
- Darling-Hammond, L., Chung Wei, R., Andree, A., Richardson, N., & Orphanos, S. (2009). *Professional learning in the learning profession: A status report on teacher development in the United States and abroad*. Stanford University, CA: National Staff Development Council.
- Department of Education, U. S. (2007). *Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment*. Retrieved from <http://nces.ed.gov/NationsReportCard/nqt/>.
- Desimone, L. M., & Garet, M. S. (2015). Best practices in teachers' professional development in the United States. *Psychology, Society and Education*, 7(3), 252-263.
- Foy, P., & Arora, A. (2009). *TIMSS Advanced 2008 user guide for the international database: Released items advanced mathematics*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.
- Garet, M. S., Wayne, A. J., Stancavage, F., Taylor, J., Eaton, M., Walters, K., & Doolittle, F. (2011). *Middle school mathematics professional development impact study: Findings after the second year of implementation*. (NCEE 2011-4025). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, US Department of Education.
- Ghaith, G., & Shaaban, K. (1999). The relationship between perceptions of teaching concerns, teacher efficacy, and selected teacher characteristics. *Teaching and Teacher Education*, 15, 487-496. doi:10.1016/S0742-051X(99)00009-8
- Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39(4), 372-400.
- Lebart, L., Salem, A., & Bècuc, M. (2000). *Análisis estadístico de textos*. Lleida: Milenio.
- Matteson, S., Zientek, L. R., & Ozel, S. (2013). Identifying What In-Service Teachers Want in Professional Development Experiences. *Teacher Education and Practice*, 26(3), 569-580.

- Sherin, M. G. (2002). When teaching becomes learning. *Cognition and Instruction, 20*(2), 119-150. doi: [10.1207/S1532690XCI2002_1](https://doi.org/10.1207/S1532690XCI2002_1)
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.
- Teixidor-i-Bigas, M., Schliemann, A. D., & Carraher, D. (2013). Integrating disciplinary perspectives: The Poincaré Institute for Mathematics Education. *The Mathematics Enthusiast, 10*(3), 519-561.
- Wlodkowski, R. J., & Ginsberg, M. B. (1995). A framework for culturally responsive teaching. *Educational Leadership, 53*(1), 17-21.

Mary C. Caddle is a project director and instructor in the Department of Education, Tufts University, United States of America.

Alfredo Bautista is research scientist and lecturer at the National Institute of Education, Nanyang Technological University, Singapore.

Bárbara M. Brizuela is professor of mathematics education in the Department of Education and a Dean of Academic Affairs for Arts & Sciences, Tufts University, United States of America.

Sheree T. Sharpe is an assistant professor in the Department of Mathematics & Statistics, University of New Hampshire, United States of America.

Contact Address: Direct correspondence concerning this article, should be addressed to Mary C. Caddle. Postal address: Poincaré Institute, Eaton Hall, 5 The Green, Tufts University, Medford MA 02155. **Email:** Mary.Caddle@tufts.edu