Reach For the Stars: A Strategic Bidding Game

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I. Introduction

The beginning of 1998 witnessed a spectacular bidding war for the right to broadcast games of the National Football League (NFL) on television. The conflict ended with CBS back in the football business after a four-year absence and NBC without professional football broadcasts for the first time in over thirty years. Perhaps even more noteworthy, though, was the value of the contracts reached. CBS paid $4 billion over eight years for the right to broadcast games in the American Football Conference. Fox Television paid $4.4 billion for similar rights to games in the National Football Conference. Disney/ABC paid a total of $4.8 billion for a package of prime time “Monday Night Football” and, through the cable unit, ESPN the entire cable TV rights for all NFL games. All these bids reflected increases of over 50% above the previous contracts signed only four years earlier. All of them also left many analysts wondering just how the broadcast companies would find the revenue to make such payments. In this respect, at least, the bidding for NFL broadcast rights has much in common with the bidding for NFL star players, as well as that for other sports and entertainment stars. All such cases often result in contracts that seem more likely to reduce a firm’s profit rather than to raise it.

In this paper, we offer a simple model designed to shed some light on bidding wars over scarce resources such as television rights and star talent. In this light, the paper may be viewed as a contribution to the economics literature on the compensation paid to “star” performers. This literature begins with the pioneering analysis of Rosen (1981), and includes a number of subsequent papers such as those by Lazear and Rosen (1981), O’Keefe, Viscusi, and Zeckhauser (1984), and, more recently, Frank and Cook (1995). The recent secular trend toward greater income inequality has only reinvigorated interest in understanding the market for star resources and the incredible gap in the remuneration between stars and lesser lights.

Our analysis of this topic differs fundamentally from previous work in that we explicitly address the strategic interaction between the firms that hire the scarce star input. Most of the prior research, including the papers cited above, has focused on the extra return to star talent as either reflecting the returns to increasing market size that
stars can command, or as the result of contests designed to elicit effort or to pool risks. Little attention is given to
the nature of competition in the downstream market in which the firms hiring the scare input actually compete.
Indeed, explicit consideration of the downstream market, usually assumed to be competitive, is generally suppressed
altogether in this earlier work. However star talent is typically not consumed directly by the public. Rather stars
are inputs that go into creating some product whether network program schedule, a film, a book, a sporting event,
or even a management team. Our model attempts to address this oversight. To be specific, we focus on how the
strategic interaction between the downstream firms affects the bidding for the star input upstream. We show that
the remuneration to the star is affected by the nature of the price competition between the firms in their downstream
market.

Our analysis is set in the context of a duopoly model. There are two firms who produce and market
differentiated downstream products and compete in price for customers. The talent or quality of the input that a
firm uses to produce or market its goods affects consumer demand for the good. However, star inputs are scarce.
In the context of a duopoly model, such scarcity is most easily captured by assuming that there is only one so-called
star in the pool of performers, e.g., one star program package over which the two firms bid. We show not only a
high marginal return to differences in talent, but also an interesting kind of prisoner’s dilemma outcome for the
firms--one that changes dramatically with the relationship between market demand and differences in the quality of
inputs. Our results help clarify the not infrequent popular claim that “ruinous competition” can emerge when
imperfectly competitive firms bid for scarce star inputs.

II. The Model

Each downstream firm $i$ sells a differentiated product whose demand, denoted by $q_i$, depends upon both its
own price and that of its rival’s, as well as on the quality, $z_i$, of a key, public good input. We call this input the
talent package. It may be a network’s program schedule. Alternatively, it may be a publisher’s book author, or an
actress hired by the producer of a film. Whatever the talent package is, the quality of this input is assumed to be
indivisible and, hence, to be embodied in each unit of the good produced. At the same time, even if both
downstream firms choose the same quality of talent package, their products would still remain differentiated. A
1940 Warner Brothers crime drama featuring James Cagney is different from a 1940 MGM musical also starring James Cagney. Similarly, the Fox Network broadcast of NFL games is different (in the eyes of viewers) from NBC’s broadcast of the same games. The above features are reflected by assuming a simple downstream demand for good \( i \) given by:

\[
q_i = 1 - p_i + \beta p_j + z_i - \alpha z_j; \quad i = 1,2; \quad j \neq i; \quad 0 < \alpha < 1, 0 < \beta < 1
\]  

(1)

Hiring decisions are made upstream. Thereafter, the talent package is used to produce the downstream good. Each firm \( i \) produces a quantity \( q_i \) of the downstream good at a constant unit cost of production which is assumed to be the same for each firm and, for simplicity, set equal to zero.

The upstream pool of talent packages from which a firm can hire its \( z \) is divided into two groups. There is one star package or performer for whom the quality level, \( z \), is normalized to equal 1. All the other performers are of a lesser quality, \( \rho \), where \( \rho \) satisfies \( 0 < \rho < 1 \). The quality level \( \rho \) can be interpreted as either an observable quality differential or, stochastically, as say the likelihood of an unknown performer being a star. These less talented, or less known, performers are sufficiently numerous that they can be hired at their reservation wage, \( \varpi \). Without loss of generality, \( \varpi \) is set equal to 0. In contrast, the star is unique, and therefore, unlike the “unknowns” has potential monopoly power. The star’s wage in alternative employment is denoted by \( W \) which may be greater than the reservation wage of an unknown.

The star package either is either sold to the downstream firm making the highest “offer”, or turns down the offers of both firms, and is sold elsewhere at the reservation wage, \( w \). When a firm bids for the star talent package, it offers a contract specifying a unique lump sum of money to be paid. However, both the owner of the star package (or, in the case of individual performers, the star herself) and the firm hiring that input can work out what that firm’s downstream profit level will be conditional on such a hiring, In turn, this means that every such lump sum can also be expressed as a unique fraction, \( \lambda \), of the firm’s downstream profit. That is, in our model, there is no difference between a regime in which offers to the star input are made in such explicit lump sum amounts and one in which, instead, the hiring firm promises to pay the star input a fraction, \( \lambda \), of the firm’s downstream profit. Since it turns out to be easier to work out our analysis by expressing upstream offers made to
the star in terms of $\lambda$, that is the approach we adopt here. But we again emphasize that our results would be identical if we instead focused on the dollar sums actually paid to the high quality input.

Denote by $\lambda$ the reservation profit share of the star input (the share of profit earned downstream by a firm hiring the star that yields a dollar payment equal to the star’s best alternative remuneration, $w$). If $\lambda$ is taken as exogenous, the game is modeled as a two-stage one. In the first stage, each firm $i$ makes a bid (expressed as a share of profit downstream) $\lambda_i$ for the star. The star is hired by the firm that makes the highest bid, provided the bid is greater than $\lambda$. In the case of a tie, the star talent package is assumed to be hired by firm 1, and firm 2 hires from the pool of talent packages with quality, $\rho$. If both bids ($\lambda_1, \lambda_2$) are less than $\lambda$, then each firm hires from this lower quality pool. After the talent inputs are hired, production takes place and, in stage three, the two downstream firms compete in prices, $(p_1, p_2)$, for consumers. We are interested in strategies consisting of upstream bids $(\lambda_1, \lambda_2)$, and downstream prices $(p_1, p_2)$ that yield a perfect equilibrium outcome. In the usual fashion, we work backwards from the final stage of the game.

**Stage 2:**

In stage 2 of the game the two downstream firms have hired some combination of talent packages quality ($z_1$, $z_2$) in the previous stage and now compete in prices for consumers of their respective downstream products. Recall that in stage 2 the unit cost of producing or replicating copies of the downstream good is constant and equal to zero for each firm.

Given downstream demand for product $i$, $q_i(p_i, p_j, z_i, z_j)$, $i = 1, 2, j \neq i$, described in (1), firm $i$ chooses price $p_i^*$ in stage 2 to maximize profit $\pi_i(p_i, p_j; z_i, z_j)$. The best response function for firm $i$ in stage 2 of the game is:

$$p_i^* = \left[1 + \beta p_j + z_i - \alpha z_j \right] / 2 ; \quad i = 1, 2 ; \quad j \neq i$$

(2)

The best response functions shown in equation (3) in turn imply that a Nash equilibrium in stage 2 of the game is a set of prices $(p_1^*, p_2^*)$ such that:

$$p_i^* = \left[ (2 - \alpha \beta) z_1 - (2 \alpha - \beta) z_2 + 2 + \beta \right] / \left[ 4 - \beta^2 \right]$$

(3)
Corresponding to these prices are equilibrium (gross) profits:

\[ \pi_1^*(z_1, z_2) = \left( \frac{(2 - \alpha \beta)z_1 - (2\alpha - \beta)z_2 + 2 + \beta}{4 - \beta^2} \right)^2 \]  

(4)

\[ \pi_2^*(z_1, z_2) = \left( \frac{(2 - \alpha \beta)z_2 - (2\alpha - \beta)z_1 + 2 + \beta}{4 - \beta^2} \right)^2 \]

A critical element in the expression in both equations (3) and (4) is the term, \((2\alpha-\beta)\). This term shows how, in equilibrium, the price chosen by one firm is affected by the quality of its rival’s talent package. If \((2\alpha-\beta) > 0\), then the rival’s choice to hire a more talented input in stage 1 induces a firm to lower its equilibrium price. That is, price competition between the two firms is *intensified*. The decision by one firm to employ the star talent input shifts the price response curves of both firms in such a manner that price competition is more aggressive in the final stage of the game. On the other hand, if \((2\alpha-\beta) < 0\), then the rival’s choice to hire a more talented input induces a firm to set a higher price in equilibrium. Here, the employment of the star talent input by one firm induces shifts in both firms’ response functions in such a way that the final equilibrium involves *softer* price competition. Because a firm’s decision to increase the quality of talent hired upstream can either toughen or soften the price competition downstream, we analyze the two cases separately in stage 1 of the game.

**Stage 1:**

*Case A: \((2\alpha - \beta) > 0\) and Quality Differences Upstream Toughen Price Competition Downstream*

In the first stage of the game the two firms hire their talent inputs. Because there is only one star input in this game, there are only three possible allocations of talent between the two firms. Either \(z_1 = 1\) and \(z_2 = \rho\), or \(z_1 = \rho\) and \(z_2 = 1\), or \(z_1 = \rho\) and \(z_2 = \rho\). For the allocations in which the star is hired by one of the firms, the gross profit of the firm employing the star is denoted by \(\pi^H(\rho)\) and the gross profit of the firm hiring an unknown is denoted by \(\pi^L(\rho)\). From (4) it follows that \(\pi^H(\rho)\) and \(\pi^L(\rho)\) are:
\[
\pi^H(\rho) = \left[ \frac{(2 - \alpha \beta) - (2 \alpha - \beta) \rho + 2 + \beta}{4 - \beta^2} \right]^2
\]

\[
\pi^L(\rho) = \left[ \frac{(2 - \alpha \beta) \rho - (2 \alpha - \beta) + 2 + \beta}{4 - \beta^2} \right]^2
\]

For the case when \textbf{neither} firm hires the star, and instead they both employ unknowns, the gross profit earned by each firm is denoted by \(\bar{\pi}(\rho)\), where \(\bar{\pi}(\rho)\) is given by:

\[
\bar{\pi}(\rho) = \left[ \frac{(2 - \alpha \beta) \rho - (2 \alpha - \beta) \rho + 2 + \beta}{4 - \beta^2} \right]^2
\]

Note that, for the case at hand where \((2 \alpha - \beta) > 0\), we have that \(\pi^H(\rho) > \bar{\pi}(\rho) > \pi^L(\rho)\).

The allocation of talent packages across firms in stage 1 depends upon the offers of profit shares, \((\lambda_1, \lambda_2)\), made by the two firms to the star input. Of course, bidding for the star package in stage 1 will depend upon \(\lambda\), the star’s reservation share of the downstream profit. If the pair of offers of profit shares, \((\lambda_1, \lambda_2)\), made by firms 1 and 2 are both less than \(\lambda\), then neither firm will hire the star.

The threat by an owner of a star input to decline employment at a downstream firm if she earns less than \(\lambda \pi^H(\rho)\) is only credible to the firms if the remuneration that the star input can earn in alternative employment is such that \(w \geq \lambda \pi^H(\rho)\). For example, if the star input does not have any alternative means by which it may exploit its talent, then the owner of that talent has no credible reservation wage except that the other, lower quality talent inputs, i.e., \(w = 0\). In that case, the only credible reservation share announcement in stage 1 is \(\lambda = 0\). This case may be relevant to bidding over NFL broadcast rights, for example. There is little that the NFL can do with its television rights if no broadcast network purchases them.

Alternatively, \(w\) may be greater than zero because star inputs often do have alternative options superior to competitors of lower quality. An athlete may have opportunities in a variety of professional sports, or even in film and TV. A famous film star may have opportunities on Broadway far more lucrative than those available to her
unknown screen competitors. Moreover, when star status is the result of prior performance and reputation it
provides a certain bargaining power of its own. Here again, a star talent will have opportunities not available to
unknowns. For our purposes, whether the difference between a star input and an unknown reflects a differential in
talent or a difference in celebrity status is not material. Unlike MacDonald (1986), we are not concerned with the
mechanism behind the creation of a star. What is relevant is that such star inputs exist and that, when they do, the
star talent can credibly announce a non-zero reservation wage.

It is important now to note that the gross profit, \( \pi^H(\rho) \), earned by a downstream firm in stage 2 when it
employs the star talent package is not the net profit earned by that firm. Because the firm must share its
downstream profit with the star, its net profit is \( (1-\lambda)\pi^H(\rho) \), for any given value of \( \lambda \geq \bar{\lambda} \). In contrast, for the rival
firm who hires an unknown or lower quality talent input, net profit is in fact equal to gross profit \( \pi^L(\rho) \) because we
have set the reservation wage, \( \bar{\pi} \), of the unknown talent equal to zero.

To work out a pair of best responses \((\lambda_1^*, \lambda_2^*)\) for stage two of the game it is useful to define two critical
shares of gross profit \( \pi^H(\rho) \), denoted by \( \lambda^* \) and \( \bar{\lambda} \), which satisfy the following equations:

\[
\pi^H(\rho) (1-\bar{\lambda}) = \bar{\pi}(\rho) \\
\pi^H(\rho) (1-\lambda^*) = \pi^L(\rho).
\] (7)

The share of profit \( \bar{\lambda} \) is such that the net profit earned by the firm hiring the star package is exactly equal to the
profit the firm would have earned in the case that neither firm hired this high talent input. Similarly, \( \lambda^* \) is the profit
share such that the net profit of the firm hiring the star input is exactly equal to the profit of the rival firm hiring a
lower quality talent package. Since we are assuming \( (2\alpha - \beta) > 0 \) and \( \pi^H(\rho) > \bar{\pi}(\rho) > \pi^L(\rho) \), it follows that the
profit share \( \lambda^* > \bar{\lambda} \).

**Proposition A1**: If the star talent package has a reservation share \( \bar{\lambda} \), such that \( 0 \leq \bar{\lambda} \leq \bar{\lambda} \), then the
unique Nash equilibrium outcome to bidding for the star input in stage 2 is \( \lambda_1 = \lambda^*, \lambda_2 = \lambda^* \), where, given
our tie-breaking rule, firm 1 hires the star package and firm 2 does not.
**Proof:** The proof that this combination is a Nash equilibrium is straightforward. When $\lambda_1 = \lambda^*, \lambda_2 = \lambda^*$, firm 1 hires the star input and earns a net profit, $\pi^H(\rho) (1-\lambda^*)$, equal to $\pi^I(\rho)$, the profit earned by firm 2 who hires a lower quality talent. Firm 2 does not have a profit incentive to deviate from its offer of $\lambda_2 = \lambda^*$. Offering a lower $\lambda_2$ will leave firm 2 in the same outcome of hiring an unknown and earning profit equal to $\pi^I(\rho)$. An offer of $\lambda_2 > \lambda^*$ will win the star input for firm 2, but at the cost of driving firm 2’s net profit below $\pi^I(\rho)$. Clearly, neither deviation improves net profits. Similarly, firm 1 has no profitable deviation. A higher offer $\lambda_1 > \lambda^*$ is less profitable for firm 1, and a lower offer of $\lambda_1 < \lambda^*$ has the star input going instead to firm 2 and firm 1 hiring an unknown. Firm 1 would then earn a net profit of $\pi^I(\rho)$ which is no greater than the net profit the firm currently earns.

To see that the solution $\lambda_1 = \lambda^*, \lambda_2 = \lambda^*$ is also a unique Nash equilibrium outcome when $0 \leq \underline{\lambda} \leq \lambda^*$, consider first any pair of offers $(\lambda_1, \lambda_2)$ where $\underline{\lambda} \leq \lambda_1 < \lambda_2 < \lambda^*$. Clearly this cannot be an equilibrium outcome because in this case firm 1 has an incentive to raise its offer above that of firm 2 to hire the star talent and thereby increase its downstream net profit. The same reasoning holds for firm 2 for any pair of offers $(\lambda_1, \lambda_2)$, where $\lambda_1 < \lambda_2 \leq \underline{\lambda}$. For offer pairs $(\lambda_1, \lambda_2)$ where $\lambda_1 < \lambda_2 \leq \underline{\lambda}$, firm 1 has an incentive to increase its offer to $\underline{\lambda}$ and hire the star. By doing so, the firm would earn net profit $\pi^H(\rho) (1-\underline{\lambda}) > \pi^I(\rho)$ since $\underline{\lambda} < \lambda^*$. Similarly, for pairs of offers $(\lambda_1, \lambda_2)$, where $\lambda_2 \leq \lambda_1 \leq \underline{\lambda}$, there is an incentive for firm 2 to deviate and offer $\lambda_1$ and increase its profit.

**Proposition A2:** If in stage 1 the star input’s reservation share $\lambda$ satisfies $\underline{\lambda} < \lambda < \lambda^*$, then there are multiple Nash equilibria in stage 2. One equilibrium is the offer pair, $\lambda_1 = \lambda^*, \lambda_2 = \lambda^*$, in which firm 1 hires the star and firm 2 does not. In addition, there is also a second set of (multiple) equilibria consisting of all offer pairs, $(\lambda_1, \lambda_2)$ satisfying $\lambda_1 < \underline{\lambda}$ and $\lambda_2 < \lambda$, such that neither firm hires the star.

**Proof:** The proof that $\lambda_1 = \lambda^*, \lambda_2 = \lambda^*$ is an equilibrium is the same as that given for Proposition A1. Given that firm 1 hires the star and firm 2 does not, but firm 2 would if firm 1 lowered its offer, neither firm has an incentive to deviate and either raise or lower its offer. That all offer pairs where both $\lambda_1$ and $\lambda_2$ are less than $\underline{\lambda}$ are also Nash equilibria is also readily verified. Clearly, in each such case, no firm has an incentive to lower its offer.
No firm has an incentive to raise its offer either. Doing so can only alter the equilibrium outcome when the offer is raised to meet the star’s reservation share, \( \lambda \). However, the only result that this will achieve is to lower the profit of the bidding firm from \( \pi (\rho) \) to \( \pi^H(\rho) (1-\lambda) \). Obviously, this is not desirable either. Observe that both firms earn higher net profits in the equilibrium outcome in which neither firm hire the stars than in the equilibrium outcome in which one firm hires and the other does not; that is, \( \bar{\pi} (\rho) > \pi^H(\rho) (1-\lambda^*) = \pi^L(\rho) \).

**Proposition A3:** If \( w > \lambda^* \pi^H (\rho) \), then there are many Nash equilibria, all of which are characterized by a pair of offers satisfying \( \lambda_1 < \lambda \) and \( \lambda_2 < \lambda \). In this case, the star in this case has no incentive to work for either downstream firm. The firms do not bid up the star’s payment and she is not hired in this industry.

**Proof:** The proof of this last proposition is self-evident. Hiring the star requires offering a profit share \( \lambda \) above \( \lambda^* \), and that will unequivocally lower the profit of the firm making such an offer. Hence, neither firm has any incentive to make such an offer.

The various types of equilibria in the stage 2 bidding game for the case in which \( (2 \alpha - \beta) > 0 \), carry a number of interesting implications. First, note that in any equilibrium in which the star talent package is hired by one of the two duopolists, it earns a profit share \( \lambda^* \). This outcome has the intriguing feature that it results in a net profit for each firm of \( \pi^H(\rho) (1-\lambda^*) = \pi^L(\rho) \), which is less than \( \bar{\pi} (\rho) \). In other words, the successful recruitment of the star input yields a market outcome with the “prisoners’ dilemma” aspect in which both firms earn lower profit than they would be if the star never existed. Such an outcome is certain when the star’s alternative remuneration is relatively low and \( 0 \leq \lambda \leq \lambda^* \). It may also occur even when that alternative remuneration is somewhat higher so that \( \lambda^* < \lambda \leq \lambda^* \). While “a star is born” is the stuff of Hollywood legends, the firms that employ such talent will wish that such a birth had never occurred.

In addition, the equilibria just discussed also suggest that a star talent with a relatively low opportunity cost, specifically, one such that \( 0 \leq \lambda \leq \lambda^* \), may actually do better than a star input in which the next best alternative is relatively high-paying, i.e. those for whom \( \lambda^* < \lambda \leq \lambda^* \). In the first case, the bidding will unquestionably produce a contract in which the star input earns the share \( \lambda^* \) of its downstream employer’s profit. In the second case, in
which it is widely recognized that the star input already has a relatively good opportunity outside the industry in question, the two duopolists may simply decline to match that reservation wage altogether. For example, the bidding by professional football teams for a gifted athlete who also has excellent opportunities in professional baseball or a fairly lucrative television and film market, may be less intense than the bidding for an equally super talented one whose skills are only appropriate for the gridiron. To take another example, consider a parcel of land overlooking a scenic river that may be costlessly enjoyed by any number of houses built on the land. The owner of such a plot may be better off with a restriction against commercial use of the land so as to insure a bidding war by residential developers that will guarantee a payment equal to $\lambda^*$ times the winning bidder’s profit.

The possibility that the talent input may be better off with a lower reservation wage also suggests that in cases in which that reservation wage is private information, the star talent has a strategic interest in announcing her reservation share $\tilde{\lambda}$. Specifically, owners of a star talent package in which $\tilde{\lambda} < \lambda < \lambda^*$, will wish to pool with those whose reservation profit share is less than $\tilde{\lambda}$ by announcing a reservation share in this lower range, i.e., below that share which corresponds to her earnings in the alternative employment, $w$. This really implies a three-stage game, in which the first stage is now the strategic announcement by the owner of the star input of its reservation share, $\tilde{\lambda}$, followed by the second and third stages of noncooperative bidding and downstream price competition, respectively. The structure of the model is such that when the reservation wage of the star input is known only to that input’s owner, the downstream firms have no ability to discern whether an announcement of a reservation profit share below $\tilde{\lambda}$ reflects an honest statement or strategic shading. As a result, all equilibria in which the star input is hired will be ones characterized by the offer pair $\lambda_1 = \lambda^*$, $\lambda_2 = \lambda^*$, where, again, firm 1 hires the star package and firm 2 does not. ($\rho \geq w$). The star strategically announces a reservation share $\tilde{\lambda}$ less than her opportunity cost share and invites the firms to bid up her share of downstream profit in stage 2 of the game.

In short, when $(2\alpha - \beta) > 0$ and the star’s reservation wages $w$ lies in the range $0 \leq w \leq \lambda^* \pi^H (\rho)$, the star can generally secure employment in a downstream firm in stage 2 with remuneration equal to $\lambda^* \pi^H (\rho)$. This will certainly be the case when $0 \leq w \leq \tilde{\lambda} \pi^H (\rho)$. It may also be the case when the star input has even more lucrative
alternative opportunities such that $\widetilde{\lambda} \pi^H (\rho) \leq w \leq \lambda^* \pi^H (\rho)$, especially if the value of the star input’s alternative income is private information. Of course, whenever $w > \lambda^* \pi^H (\rho)$, neither duopolist recruits the star package. Yet, for those cases in which one of the downstream firms does acquire the star talent, it is typically the case that the noncooperative bidding that yields this outcome also results in both firms being less profitable than they would be if the star input did not exist.

**Stage 1:**

**Case B: $(2\alpha - \beta) < 0$ Quality Differences Upstream Soften Price Competition Downstream**

When the term $(2\alpha - \beta) < 0$, price competition is different than in the previous case because now an increase in the quality of performer hired by one firm softens price competition between both firms downstream. One can readily verify from equation (4) that the downstream gross profit in stage 2 is now such that $\pi^H (\rho) > \pi^L (\rho) > \widetilde{\pi} (\rho)$.

If one firm hires the star talent, the gross profit of the rival hiring an unknown *exceeds* the gross profit earned by each firm when neither hires the star. As in Case A, we define profit shares $\widetilde{\lambda}$ and $\lambda^*$, such that $\pi^H (\rho) (1-\widetilde{\lambda}) = \widetilde{\pi} (\rho)$, and $\pi^H (\rho) (1-\lambda^*) = \pi^L (\rho)$. Note though that since now $\pi^H (\rho) > \pi^L (\rho) > \widetilde{\pi} (\rho)$, it is also the case that $\widetilde{\lambda} > \lambda^*$.

**Proposition B1:** If the star input has a minimum reservation share $\lambda$ such that $0 \leq \lambda \leq \lambda^*$, then there is a unique Nash equilibrium outcome in stage 2 described by the pair of offers $\lambda_1 = \lambda^*$, $\lambda_2 = \lambda^*$, where again, given our tie-breaking rule, firm 1 hires the star and firm 2 does not.

**Proof:** The proof is straightforward. When $\lambda_1 = \lambda^*$, $\lambda_2 = \lambda^*$, firm 1 hires the star package and earns a net profit, $\pi^H (\rho) (1-\lambda^*) = \pi^L (\rho)$, the profit earned by firm 2 who hires an unknown or low-quality talent package. Firm 2 does not have a profit incentive to deviate from its offer of $\lambda_2 = \lambda^*$. Offering a lower $\lambda_2$ will leave firm 2 in the same outcome of hiring an unknown and earning profit equal to $\pi^L (\rho)$. A higher offer of $\lambda_2 > \lambda^*$ will allow firm 2 to hire the star resource, but at the cost of driving firm 2’s net profit below $\pi^L (\rho)$. Clearly such a deviation is not profitable. Similarly, there is no profitable deviation for firm 1. A higher offer of $\lambda_1 > \lambda^*$ is less profitable for firm 1, and a lower offer to the star input of $\lambda_1 < \lambda^*$ has the star input going instead to firm 2 offer so that firm 1 hires the low-quality talent, which does not raise its profit.
That the solution $\lambda_1 = \lambda^*$, $\lambda_2 = \lambda^*$ is a unique Nash equilibrium outcome when $0 \leq \lambda \leq \lambda^*$ is shown by the following argument. Consider first any pair of offers $(\lambda_1, \lambda_2)$ where $\lambda_2 \leq \lambda_1 < \lambda_2 < \lambda^*$. Clearly this cannot be an equilibrium outcome because in this case firm 1 has an incentive to raise its offer above that of firm 2 and hire the star package. As long as $\lambda_1 < \lambda^*$, firm 1’s net profits will be increased by this action. A similar argument holds for firm 2 where $\lambda_1 < \lambda_2 < \lambda^*$. For pairs of offers $(\lambda_1, \lambda_2)$ where $\lambda_1 < \lambda_2 < \lambda$, or where $\lambda_2 \leq \lambda_1 < \lambda$, neither firm hires the star resource. As a result, they both face tougher price competition and earn profit $\bar{\pi}(\rho)$. In such a situation, at least one firm has an incentive to increase its offer to $\lambda_-$ and hire the star talent. By changing its offer, such a firm would earn net profit $\pi^H(\rho)(1-\lambda) > \bar{\pi}(\rho)$ since $\lambda < \lambda$.

**Proposition B2:** If the star talent has a reservation share $\lambda$ satisfying $\lambda^* < \lambda \leq \tilde{\lambda}$, then any pair of offers $(\lambda_1, \lambda_2)$ where either $\lambda_1 = \lambda$ and $\lambda_2 < \lambda$, or $\lambda_1 < \lambda$ and $\lambda_2 = \lambda$, constitutes a Nash equilibrium to stage 1.

**Proof:** The firm that hires the star talent has no incentive to lower its offer because doing so simply means that its net profit would decrease to $\bar{\pi}(\rho)$. Raising its offer also will also reduce the firm’s net profit by transferring more of it to the owner of the star talent. Similarly, the firm that hires an unknown or low quality input cannot gain by lowering its offer, and can only attract the star input by raising its offer above $\lambda$, which will reduce its profit.

**Proposition B3:** If the owner of the star input has a reservation share in stage 1 so high that $\lambda > \tilde{\lambda}$, then there are multiple equilibria consisting of offer pairs $(\lambda_1, \lambda_2)$ satisfying $\lambda_1 < \lambda$ and $\lambda_2 < \lambda$, and in all of which neither firm hires the star talent.

**Proof:** Clearly, no firm can do better by lowering its offer. Neither can any firm do better by raising its offer. Doing so only alters the profit outcome if the raised offer now meets the star’s demand. Yet if $\lambda > \tilde{\lambda}$, meeting the star’s demand implies that the firm hiring the star will earn less profit than if the star is hired by neither firm. Hence, it cannot be in the interest of any firm to do so. When $\lambda > \tilde{\lambda}$, neither firm will hire the star.
For this case in which \((2\alpha - \beta) < 0\), the equilibrium outcome of particular interest is that described by Proposition B2. What makes this outcome so intriguing is its “winner’s curse” feature. Both firms are better off than either would be if no star talent existed, i.e., \(\pi^H(\rho) (1-\lambda) \geq \bar{\pi}(\rho)\) and \(\pi^L(\rho) > \bar{\pi}(\rho)\). Recall however that for the case at hand in which \((2\alpha - \beta) < 0\), it is also the case that \(\pi^H(\rho) > \pi^L(\rho) > \bar{\pi}(\rho)\). Thus, the firm hiring the star package here earns a net profit \(\pi^H(\rho) (1-\lambda)\) which is less than the net profit of its rival employing the lower quality talent, \(\pi^L\). If neither firm has met the reservation share of the star input, \(\lambda\), then each will earn \(\bar{\pi}\) the level achieved. Yet in such a situation, at least one firm will find it profitable to raise its offer to the star input and thereby increase its profit above \(\bar{\pi}\). Unfortunately, in so doing, the firm that successfully hires the star talent thereby sufficiently softens price competition in the downstream market that the net profit of its rival increases even more.\(^5\)

The equilibria just derived for Case B, where \((2\alpha - \beta) < 0\), would be unchanged even if the value of the alternative income available to the star talent were private information, and the game were changed to become a three-stage one. This is because in this case the star input now has no strategic incentive to lower its minimum reservation offer in stage 1 below that corresponding to its true opportunity cost, (as she did in Case A). Now the star talent can simply announce in stage 1, a minimum offer \(\lambda\) equal to \(w/\pi^H(\rho)\), which is the true opportunity cost of accepting employment at one of the two duopolists. When the value of the outside earnings \(w\) lies in the range \(0 \leq w/\pi^H(\rho) < \lambda^*\), this announcement in stage 1 will lead to a remuneration being for the star input above its opportunity cost in stage 2 of the game. When the value of the income from alternative employment is such that \(\lambda^* \leq w/\pi^H(\rho) \leq \bar{\lambda}\), the stage 1 strategy of announcing \(\lambda = w/\pi^H\) will lead to a remuneration to the star input exactly equal to the reservation \(w\). However, in this second case, no alternative announcement will yield a superior outcome. Finally when \(w/\pi^H(\rho) > \bar{\lambda}\), it is again optimal for the star to make an honest announcement in stage 1 of a minimum reservation share of \(\lambda = w/\pi^H(\rho)\). Here again, the star talent will end up earning precisely its reservation wage, \(w\), but will now be employed in the alternative industry. In this case it is not desirable for either the owner of the star talent or the rival duopolists that the star input is hired in the downstream market.
III. The Returns to Scarce Talent Inputs and Downstream Price Competition

We noted at the outset, that the bidding for a scarce public-good input such as NFL football programming, has much in common with the bidding for NFL stars themselves and, indeed, is similar to the bidding for star talent in many professional sports and entertainment industries. It is this phenomenon that lies behind the superstar literature that began with Rosen (1981). An important feature in this literature is the skewed distribution between income and relative talent. We now briefly consider the implications of our model for this same issue.

Let \( Z = 1-\rho \) define the talent of a star performer relative to that of an unknown or lower-quality one. When hired, the star’s remuneration, denoted here by \( R(Z) \), is equal to the star’s share of the downstream firm’s gross profit. We showed earlier that, for the case in which price competition is tough, i.e., if \((2\alpha - \beta) > 0\), \( R(Z) = \lambda^* \pi^H(\rho) \), where \( \rho = 1 - Z \), in all cases in which one of the duopolist firms actually hires the star input. It will also be the case that \( R(Z) = \lambda^* \pi^H(\rho) \) when price competition is soft, so long as \( 0 \leq \lambda \leq \lambda^* \). In turn, for all such cases in which \( R(Z) = \lambda^* \pi^H(\rho) \), we can solve for \( R(Z) \) as:

\[
R(Z) = \frac{Z^2(Z - 2) + 2\alpha - Z + 4}{4 - \beta^2} \quad (8)
\]

A quick inspection of equation (8) then reveals that:

\[
R'(Z) = \frac{2(1 + \alpha)(2 - Z - \alpha(1 - Z))}{4 - \beta^2} > 0; \\
R''(Z) = \frac{2(\alpha - 1)(1 + \alpha)}{4 - \beta^2} < 0. \quad (9)
\]

In other words, the star’s remuneration \( R(Z) \) is concave in \( Z \). Such concavity may or may not be interpreted as reflecting some of the skewness observed in superstars’ relative salaries. On the one hand, concavity means that the relative payment to scarce star talent does not grow without bound as it would if \( R(Z) \) were convex. On the other hand, depending on the degree of concavity, small differences in relative talent or \( Z \) can still imply large differences in remuneration. A common measure of the degree of concavity is the ratio: \( \frac{-R''(Z)}{R'(Z)} = \frac{(1 - \alpha)}{2 - Z - \alpha(1 - Z)}. \)
The point we wish to make at this juncture is that the nature of the downstream competition and, specifically, the value of \( \alpha \) or the effect of quality differences on downstream demand, is an important influence on just how large the superstar premium will be. To see this, differentiate both \( R'(Z) \) and the degree of concavity with respect to \( \alpha \). Such an exercise quickly reveals that the first of these derivatives is positive while the second is negative. In other words, as \( \alpha \) increases and the quality of the talent input hired by one’s rival becomes a more powerful influence on one’s own demand, the \( R(Z) \) function becomes both steeper and less concave. This implies that the premium earned by so-called superstars is likely to be particularly large in high \( \alpha \) markets, or those where quality differences critically affect a firm’s demand. Figure 1 shows the \( R(Z) \) function for two different values of \( \alpha \).

**Figure 1**
Effects of Changes in \( \alpha \) on Star’s Income

![Graph showing the effect of changes in \( \alpha \) on Star’s Income](image)

Clearly, for a given value of \( \beta \), the remuneration described by \( R(Z) \) will be greater under the tough price competition case than it will be under the soft price competition case. That is, when \( 2\alpha - \beta > 0 \), star inputs tend to do especially well and the firms competing to hire such an input, tend to do relatively poorly. Indeed, such firms are less profitable than they would if the star talent simply did not exist. Caught in a tough price competition game in which inter-firm differences in the quality of the talent input matter greatly, the firm employing the star input is forced to share a very large portion of its profit with that input’s owner.

As noted in Proposition B2, above, the soft price competition setting does offer the star input the possibility of earning more than \( \lambda^* \pi^H(\rho) \), if its alternative opportunity is to earn \( w \) satisfying \( \lambda^* \pi^H(\rho) < w \leq \bar{\lambda} \pi^H(\rho) \). While
the “winner’s curse” aspect of this case is interesting, it is in fact one in which the star talent simply earns its opportunity cost and nothing more. For the star talent to earn a premium not only relative to lower quality inputs but also above its next best alternative, it must be the case that \( w < \lambda^* \pi^H(\rho) \). In such settings, the star input does particularly well, and the downstream firms do particularly badly, when \( 2\alpha - \beta > 0 \), and the nature of the downstream price competition is tough.

The results of our analysis are summarized in Table 1, below.

**TABLE 1**

**Case 1:** \((2\alpha - \beta) > 0\) Tough Price Competition \( \pi^H(\rho) > \bar{\pi}(\rho) > \pi^L(\rho) \) and \( \lambda^* > \bar{\lambda} \)

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>( 0 \leq w \leq \lambda^* \pi^H(\rho) )</td>
<td>( 0 \leq \lambda \leq \bar{\lambda} )</td>
<td>( \pi^H(\rho) - \pi^L(\rho) )</td>
<td>( \pi^H(\rho) (1 - \lambda^*) = \pi^L(\rho) &lt; \bar{\pi}(\rho) )</td>
</tr>
</tbody>
</table>

**Case 2:** \((2\alpha - \beta) < 0\) Soft Price Competition \( \pi^H(\rho) > \pi^L(\rho) > \bar{\pi}(\rho) \) and \( \bar{\lambda} > \lambda^* \)

<table>
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<tbody>
<tr>
<td>( \lambda^* \pi^H(\rho) \leq w \leq \bar{\lambda} \pi^H(\rho) )</td>
<td>( \lambda = w/\pi^H(\rho) )</td>
<td>( \pi^H(\rho) - \pi^L(\rho) )</td>
<td>( \pi^H(\rho) (1 - \lambda^*) = \pi^L(\rho) &lt; \bar{\pi}(\rho) )</td>
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V. **The NFL Package Once Again**

We wish briefly now to revisit the anecdotal case of the bidding for the NFL TV broadcast rights noted at the outset. In many ways, the available facts of the case seem to fit very well with the tough price competition regime, Case A. The input in question—television programming—is very much of the quality input we have been discussing. Once purchased, NFL broadcasts affect the quality of viewing for every member of the network’s audience. The number of firms bidding for the contracts were small. Indeed, while the bidding war involved five firms in total, there were typically only two firms engaged in the bidding for any one contract. Thus, CBS vied with NBC for the rights to the American Football Conference (AFC) broadcasts; NBC then bid against Disney/ABC for the rights to Monday Night Football; and Disney/ESPN and Turner were the two firms competing for the cable broadcast rights. The results of this noncooperative bidding were, as noted above, dramatic. The
NFL will receive nearly $18 billion in for all the contracts combined over the next eight years. Moreover, it seems clear from the bidding for both the AFC broadcasts and those of Monday Night Football that a major motivation of the television networks was the fear that without such “high quality” programming, their ability to collect advertising revenue and fees from local affiliates would be diminished. In fact, CBS officials were quite blunt regarding this issue in defending their $500 million per year offer for the AFC package. They acknowledged that such revenues had declined after CBS lost the National Football Conference broadcast rights to Fox four years earlier—especially in key cities such as Boston and New York. It was clear to observers that CBS now hoped to increase substantially its advertising rates. NBC also recognized this fact. Because the contracts were awarded sequentially, NBC found out that it had lost the AFC contract before bidding for Monday Night Football was concluded. It was at this point that NBC began to bid aggressively for the Monday Night program. In fact, the initial NBC offer of $500 million was more than double the $230 million that Disney/ABC had previously paid for Monday night rights. Further, when it became clear to NBC that it had not only lost the AFC contract but that Disney/ABC was going to fight to keep its Monday night franchise, NBC sweetened its offer still more to an estimated value of $540 million per year. In short it appears that, just as in our Case A, this is a market in which the equilibrium price for a firm’s product declines significantly as the quality its rival’s product rises. The implication is that the broadcast networks would be more profitable if the NFL did not exist—or if the bidding for the broadcast rights could be made cooperative. While we explicitly cannot test this proposition, it does seem consistent with both the fantastic sum paid to the NFL and the comments of many outside observers who viewed the outcome as evidence that television profit is “shifting from networks to content providers.”

VI. Concluding Remarks

The enormous sums paid to the owners of scarce star inputs have been a recurring topic in both popular and professional media over the past several years. While economists have not ignored this topic of “the economics of superstars”, the analysis presented thus far has effectively suppressed all consideration of the downstream market in which the firms hiring star inputs actually compete. In this paper, we have presented a model that explicitly addresses this issue. We have shown that the outcome of noncooperative bidding for scarce star talent depends
critically as to how differences in their respective quality inputs affect price competition between firms. On the one hand, a firm without a star may be so disadvantaged relative to a rival employing the star that the non-star firm opts to cut price aggressively, thereby, intensifying the downstream price competition. On the other hand, when one firm has a star and its rival does not, the products of the two firms become further differentiated which has the effect of weakening downstream price competition.

We have examined the market outcomes for both cases. In general, the owner of the scarce star input does quite well, always earning a substantial premium relative to the income of owners of non-star inputs and, typically, a premium well above the payment that the star input itself could receive in its next best alternative. Perhaps more interesting is the fact that each case has rather unfortunate “prisoner’s dilemma” type results for the firm. When downstream price competition is tough, the bidding for the star input and the subsequent price game results in all downstream firms earning less profit than they would have done if no star input had existed. When downstream competition is soft, the hiring firms do better than they would have done had the star not existed but, paradoxically, the firm that wins the bidding for the star input may wind up with the lowest profit. We also find that the premium paid to star talent over that of lesser quality inputs is likely to be greatest in the first of these cases, that is, in markets in which quality differences induce fierce price competition. We believe that our analysis is consistent with much of what is observed in the real world, including the bidding for television programming, and book authors, CEO’s, and film stars.
Notes

*We thank George Norman, Daniel Spulber, and Robert Tamura for helpful comments.


3. Occasionally, one does observe a player “sitting out” a season or part of a season in a salary dispute. Such one-period actions serve to establish credibility in a game lasting several periods. However, the relevant season of our game is the last stage and there is only one such period here. One does not observe stars sitting out their entire careers! Hence, credibility in this setting can exist if the star’s alternative salary $w \geq \lambda \pi^H(\rho)$.

4. The tie-breaking rule that we have adopted does not affect the equilibrium strategies. We could have just as well chosen a tie-breaking rule that has the star hired by firm 1 with probability one half.

5. This outcome suggests a possible mixed strategy in which each firm meets the stars reservation share offer with, say, probability one half. The essentials of the outcome, however, would be the same. The firm hiring the star is worse off than the firm hiring the unknown is.

6. The pioneering paper on this topic is Rosen (1981). Frank and Cook (1995) is a more recent analysis of the same phenomenon.

References


Demand in the Downstream Market

\[ q_i = 1 - p_i + \beta p_j + z_i - \alpha z_j; \quad i = 1, 2; \quad j \neq i; \quad 0 < \alpha < 1, 0 < \beta < 1 \]

Stage 3: Downstream Competition

Downstream Response Function (Bertrand Competition)

\[ p_i = \left[ \frac{(2 - \alpha \beta) z_i - (2 \alpha - \beta) z_j + 2 + \beta}{4 - \beta^2} \right] ; \quad i = 1, 2 ; \quad j \neq i \]

Nash Equilibrium Prices and Profits

Prices

\[ p_1^* = \left[ \frac{(2 - \alpha \beta) z_1 - (2 \alpha - \beta) z_2 + 2 + \beta}{4 - \beta^2} \right] \]

\[ p_2^* = \left[ \frac{(2 - \alpha \beta) z_2 - (2 \alpha - \beta) z_1 + 2 + \beta}{4 - \beta^2} \right] . \]

Profits

\[ \pi_1^*(z_1, z_2) = \left[ \frac{(2 - \alpha \beta) z_1 - (2 \alpha - \beta) z_2 + 2 + \beta}{4 - \beta^2} \right]^2 \] (4)

\[ \pi_2^*(z_1, z_2) = \left[ \frac{(2 - \alpha \beta) z_2 - (2 \alpha - \beta) z_1 + 2 + \beta}{4 - \beta^2} \right]^2 \]

Stage 2: Competition for the Services of the Star
Case A: \((2\alpha - \beta) > 0\) Quality Differences Toughen Price Competition Downstream

Either \(z_1 = 1\) and \(z_2 = \rho\), or \(z_1 = \rho\) and \(z_2 = 1\), or \(z_1 = \rho\) and \(z_2 = \rho\). IF the star is hired by one of the firms, gross profits \(\pi^H(\rho)\) and \(\pi^L(\rho)\) are:

\[
\pi^H(\rho) = \left[ \frac{(2-\alpha\beta) - (2\alpha - \beta)\rho + 2 + \beta}{4 - \beta^2} \right]^2
\]

(5)

\[
\pi^L(\rho) = \left[ \frac{(2-\alpha\beta)\rho - (2\alpha - \beta) + 2 + \beta}{4 - \beta^2} \right]^2
\]

IF neither firm hires the star, gross profits are:

\[
\bar{\pi}(\rho) = \left[ \frac{(2-\alpha\beta)\rho - (2\alpha - \beta) + 2 + \beta}{4 - \beta^2} \right]^2
\]

Star’s payment in terms of share, \(\lambda\), of downstream employer’s gross profits. Two critical values of \(\lambda\), are

\[
\pi^H(\rho) (1 - \lambda) = \bar{\pi}(\rho) \\
\pi^H(\rho) (1 - \lambda^*) = \pi^L(\rho).
\]

Nash Equilibrium:

\[
\text{If } 0 \leq \lambda \leq \bar{\lambda} \text{ Then } \lambda_1 = \lambda^*, \lambda_2 = \lambda^* \\
\text{If } \bar{\lambda} < \lambda \leq \lambda^* \text{ Then } \lambda_1 < \lambda \text{ and } \lambda_2 < \lambda
\]

Case B: \((2\alpha - \beta) < 0\) Quality Differences Soften Price Competition Downstream

Nash Equilibrium

\[
\text{If } 0 \leq \lambda \leq \lambda^* \text{ Then } \lambda_1 = \lambda^*, \lambda_2 = \lambda^* \\
\text{If } \lambda^* < \lambda \leq \bar{\lambda} \text{ Then Either } \lambda_1 = \lambda \text{ and } \lambda_2 < \lambda, \text{ or } \lambda_1 < \lambda \text{ and } \lambda_2 = \lambda
\]

Stage 3: The Star’s Reservation Demand for Compensation

If \(w\) satisfies \(\lambda^* \pi^H(\rho) \geq w \geq \bar{\lambda} \pi^H(\rho)\), then star announces \(\bar{\lambda}\) satisfying \(0 \leq \bar{\lambda} \leq \bar{\lambda}\).
If \( \bar{\lambda} \pi^H (\rho) > \mathcal{W} \geq 0 \) then star can announces \( \bar{\lambda} = \mathcal{W}/\pi^H (\rho) \)

If \( \mathcal{W} > \lambda^* \pi^H (\rho) \) then

### Summary of Equilibrium and Profits

**Case 1:** \( (2\alpha - \beta) > 0 \)  **Tough Price Competition** \( \pi^H(\rho) > \bar{\pi}(\rho) > \pi^L(\rho) \) and \( \lambda^* > \bar{\lambda} \)

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**Case 2:** \( (2\alpha - \beta) < 0 \)  **Soft Price Competition** \( \pi^H(\rho) > \pi^L(\rho) > \bar{\pi}(\rho) \) and \( \bar{\lambda} > \lambda^* \)

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<td>( \pi^H(\rho) (1 - \lambda^*) = \pi^L(\rho) &gt; \bar{\pi}(\rho) )</td>
</tr>
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</table>

| \( \lambda^* \pi^H(\rho) \leq \mathcal{W} \leq \bar{\lambda} \pi^H(\rho) \) | \( \bar{\lambda} = \mathcal{W}/\pi^H(\rho) \) | \( \mathcal{W} \) | \( \pi^H(\rho) (1 - \bar{\lambda}) < \pi^L(\rho) > \bar{\pi}(\rho) \) |