Oligopoly Deregulation in General Equilibrium:
A Tax Neutralization Result

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Abstract

We examine the interplay between market structure and the form that commodity taxation should take in a general equilibrium model in which firms produce differentiated products and so are able to exert market power. Our analysis takes account of two important recent developments that affect market structure and so the appropriate design and effectiveness of commodity taxation: market deregulation and technological change. When market deregulation facilitates price discrimination, we find that tax policy is ineffective as a means to influence market structure. We further show that when tax rates are set optimally government is able to neutralize the potentially detrimental welfare impact of restrictive entry conditions in the differentiated product sector. Finally, we present conditions under which price discrimination is welfare improving.

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1. Introduction

In this paper we examine the interplay between market structure and the form that commodity taxation should take in a world in which firms produce differentiated products and so are able to exert some degree of market power. Our analysis takes explicit account of two important recent developments that carry significant implications for market structure and so for the appropriate design and effectiveness of commodity taxation: market deregulation and technological change. These developments considerably expand the willingness and ability of oligopolistic or monopolistically competitive firms to adopt discriminatory pricing practices. But discriminatory pricing can have a significant impact on market structure and so on consumer welfare. It follows that we should expect to find that the regulatory and technological regimes also affect the optimal design of commodity taxes. Put another way, taxes can conceivably serve as regulatory instruments to achieve desired market structure outcomes in imperfectly competitive environments but the appropriate design of such taxes must be sensitive to the specific context in which they are applied.

Our analysis is based on a variant of the Salop (1979) model, which has proved over the past two decades to be a particularly flexible and powerful model for the analysis of imperfectly competitive differentiated product markets. Kay and Keen (1983) were the first to recognize that the Salop model is also a powerful tool by which to investigate the appropriate design of commodity taxes in such markets. In particular, they showed that ad valorem taxes should be used to achieve desired product variety in the differentiated product market while specific taxes are used to set price optimally. Even if there were no need for distorting taxes for revenue raising purposes due to the existence of lump sum taxation, it still might be desirable to levy an
ad valorem tax to alter the equilibrium number of firms in the market. In other words, the ad valorem tax could be welfare enhancing over some range.

This brings us to the first important contribution of our analysis. The Kay and Keen analysis and, indeed, other more recent analyses of commodity taxation in oligopolistic, differentiated product settings (see, for example, Cremer and Thisse (1994)) have all been developed in a partial equilibrium framework. This runs the risk of ignoring potentially significant distortionary effects of commodity and labor taxes and of underestimating their deadweight losses since no account is taken of the interactions between the “inside”, imperfectly competitive differentiated product sector and potentially more competitive “outside” goods sectors. As a result, we take the novel step of embedding the Salop model in a general equilibrium framework.

Second, existing analysis pays no attention to the impact of potentially restrictive entry conditions in the differentiated product sector nor has there been any attempt to investigate the relationship between taxation policy and firms’ pricing policies. However, we know from the work of, for example, Eaton and Wooders (1985) and Norman and Thisse (1996) that free entry market structure in a differentiated product market can vary significantly, and so have very different welfare properties, depending upon entry conditions in this sector - what Norman and Thisse refer to as the degree of spatial contestability. As a result, we consider the connection between the degree of spatial contestability and the design of commodity taxes.

Finally, it is a familiar result that market structure and consumer welfare are significantly affected by whether firms adopt discriminatory or non-discriminatory prices. Our analysis, therefore, compares the design of commodity taxation in these two contrasting pricing regimes.
We present a number of startling results. First, we show that price discrimination leads to a situation in which tax policy has no ability to influence market structure in the monopolistically competitive, differentiated product sector. Second, we show that this same setting has an offsetting advantage in that government gains considerable freedom in setting labor and ad valorem tax rates. Third, whether or not firms price discriminate, and so whether or not commodity taxation can influence market structure, we show that government is able, through its choice of tax rates, to neutralize the potentially detrimental welfare impact of restrictive entry conditions to the differentiated product sector. In other words, and in sharp contrast to the partial equilibrium analysis, the degree of spatial contestability need not matter for welfare purposes provided that government can set tax rates optimally.

The rest of the paper is structured as follows. In the next section we provide some policy and analytical background for our analysis. Section 3 outlines the basic model on which our analysis is based. In section 4 we identify the impact of commodity taxes on market structure in the absence of price discrimination while in section 5 we allow for price discrimination. Our main conclusions are summarized in the final section.

2. Some Background

Until the early 1980s the predominant view in the United States and in Europe was that the public interest would be best served by regulating the pricing policies that firms could adopt. Norman and Thisse (1996) provide several examples of such regulations: airline pricing in the United States and Europe, the application of resale price maintenance, the consistent opposition to price discrimination by the Price Commission in the United Kingdom and a similar view in the United States by the Federal Trade Commission and embodied in the Robinson-Patman Act.
The last two decades have seen a significant relaxation, if not complete reversal of this policy stance. The Robinson-Patman Act has not been applied in the United States, the Price Commission was abolished in the United Kingdom, airlines have been broadly deregulated and, in general, firms have been allowed much more latitude in their choice of pricing policies. It should come as little surprise, therefore, that we now see the extensive implementation of discriminatory prices in a wide range of markets. Airline pricing is perhaps the most familiar example, but we also see this type of discrimination in cross-country pricing of pharmaceuticals, books and automobiles and in domestic prices in almost every aspect of economic activity.

The more relaxed policy stance has found justification, sometimes ex post, in modern developments in spatial analysis building on the seminal work of Hotelling (1929) and Salop (1979). The argument that has been advanced is that spatial price discrimination imposes tougher price discipline on firms and so may benefit consumers as compared with non-discriminatory pricing. Thisse and Vives (1988) provide one of the clearest statements of the idea that price discrimination is tougher for firms and so is pro-competitive:

denying a firm the right to meet the price of a competitor on a discriminatory basis provides the latter with some protection against price attacks. The effect is then to weaken competition, contrary to the belief of the proponents of naïve application of legislation prohibiting price discrimination like the Robinson-Patman Act, or similar recommendations of the Price Commission in the United Kingdom. (Thisse and Vives, 1988, p. 134)

Technological change over the same two decades has further enhanced, and in some cases forced firms to adopt discriminatory pricing policies. This period has seen the increasingly wide adoption of flexible manufacturing systems, defined as “a production unit capable of producing a range of discrete products with a minimum of manual intervention” (U.S. Office of Technology Assessment (1984), p. 60).
Flexible manufacturing technologies allow firms to switch product specifications easily, with the result that firms adopting this type of technology can customize their products to the specific requirements of their buyers at little or no cost penalty. It is worth noting that customized products have long been produced. For example, new home construction is an industry in which the product can often be customized based on the purchaser's specifications. What is important about flexible manufacturing, in other words, is not that it is a new idea, but rather that flexible systems are being adopted in an increasingly wide range of industries (see Mansfield (1993)). Applications range from ceramic tiles through heavy construction equipment to Levi Strauss, which offers “mass customization” of women’s jeans and Custom Foot which offers more than 10 million variants of footwear (Holusha (1996)). Other examples can be found in aerospace, automobiles, fashion garments and data warehousing. Developments in e-commerce have further extended the ability of firms to customize or “personalize” the services and products offered to customers based upon information gathered about each customer’s preferences.

“What this means in practice is that rather than display the same set of pages to every visitor, a Web site would present different information to each customer based on the person’s data profile.” (Stellin (2000))

What this also means is that not only do we get our very own customized products or personalized Web pages; we often also get our very own customized or personalized prices.

If discriminatory pricing is, indeed, pro-competitive it would appear that flexible manufacturing, which facilitates price discrimination, is also pro-competitive. More recent analysis, however, has questioned the benefits that have been claimed for discriminatory pricing (see, for example, Armstrong and Vickers (1993), Norman and Thisse (1996), Norman and Thisse (2000), and D’Aspremont and Motta (2000)). The common theme that emerges from
each of these analyses is that drawing a direct line from the view that discriminatory pricing is pro-competitive to the conclusion that it benefits consumers in lower prices may be wrong because this ignores important structural effects of changes in the regulatory and technological regime. Simply put, if discriminatory pricing is tougher for firms it will act as an entry deterrent. The short-run benefits gained from deregulation and technological change may then be more than offset by the longer-term losses from market exit.

In other words, it is essential to distinguish carefully between the short-run impact of changes in the pricing regime, when market structure can be taken as given, from the longer-run impact as market structure reacts to the changed competitive environment. By the same argument, the design of commodity taxes should also be sensitive to the regulatory and pricing regime. It is to this topic that we now turn.

3. The Model

We base our analysis on the Salop (1979) model that has become standard in the literature on horizontal product differentiation. A particular virtue of this model is that it explicitly allows us to identify the equilibrium number of firms and so to ascertain the connection between the regulatory and technological regimes, market structure and the impact – and so optimal design – of commodity taxes. As we noted above, an important innovative feature of our analysis is that we embed this model in a general equilibrium setting.

Our market is represented as a one-dimensional attribute space with support [0, L] which, to avoid end-point problems, we assume to be circular: we normalize $L = 1$ without loss of generality. We assume that there are three types of firm operating in this market. First, there is a group of $N$ firms that produce a horizontally differentiated product $z$ under monopolistically competitive conditions, where $N$ is to be determined endogenously by a free-entry condition.
Second, there is a group of firms that produce an outside good \( x \) under competitive conditions and third there is production of \( G \) units of a government commodity \( g \), with \( G \) being determined exogenously.

The differentiated product firms that enter this market are assumed to offer goods that are identical in all characteristics other than their locations in the attribute space. They may employ one of two technologies: a designated technology or a flexible technology.\(^1\) When firms employ the designated technology they each produce a single good with a defined characteristic that cannot be customized, so that the “location” of firm \( j \) is the product characteristic \( z_j \in [0, 1] \) that this firm offers. By contrast, with the flexible technology the producer starts with a ‘basic product’ and customizes or personalizes it to the precise specifications of particular consumers.

“This means that the firm now produces a band of horizontally differentiated products centered upon its basic product instead of a single product.” (Norman and Thisse (1996), p. 79)\(^2\)

In other words, with flexible manufacturing the “location” of firm \( j \) is the attribute of the basic product \( z_j \in [0, 1] \) on which the range of customized products offered by firm \( j \) is centered.

The only input to production is assumed to be labor and we take the gross wage rate \( w \) as the numeraire with \( w = 1 \). Production costs for the \( z \) firms are assumed to be identical and to exhibit economies of scale. The labor input for firm \( j \) is assumed to be

\[
C_j(Q) = f + c.Q \tag{1}
\]

where \( c \) is (constant) marginal labor input and \( f \) is a fixed labor requirement.\(^3\) Without loss of generality, we normalize \( c = 0 \). If the \( z \) firms operate the flexible technology, they also incur

\(^1\) We do not model firms’ choice of technologies but rather assume that either they all employ the designated technology or they all employ the flexible technology.

\(^2\) See also Eaton and Schmitt (1994) for a discussion of this model as a model of flexible manufacturing.

\(^3\) We do not compare designated and flexible equilibria with each other and so do not distinguish their fixed costs.
customization costs. The labor cost of customizing basic product $z_j$ to attribute $z$ is assumed to be

$$c_j(z) = r|z - z_j|$$

(2)

The differentiated product firms face an ad valorem tax rate $\tau_v$ and a unit or specific tax rate $\tau_s$.

The outside good $x$ is produced under competitive conditions, each unit of $x$ requiring one unit of labor input. Each unit of the government good also requires one unit of labor input.

Consumers are distributed over the attribute space at density $M$ and are each endowed with one unit of time that they can either consume as leisure or supply as labor in the production of $z$, $x$ and $g$ at the net wage rate $1 - \tau_l$, where $\tau_l$ is the tax rate on labor. Consumer $i$’s “address” is defined as $z_i \in [0, 1]$, where $z_i$ denotes this consumer’s most preferred differentiated product attribute. The indirect utility of consumer $i$ from consuming a differentiated product with attribute $z$ at price $p(z)$, the outside good at price $p_x$ and leisure is assumed to be separable in the differentiated product and given by

$$V_i(z, x, l) = V - \psi|z - z_i| + \Phi(p_x, p(z), \tau_l, \pi_l)$$

(3)

The parameter $\psi$ is a taste parameter that measures the valuation consumers place on the utility they lose from having to consume other than their ideal product. As a result, $V - \psi|z - z_i|$ is consumer $i$’s utility derived from consuming $z_i$; we assume $V$ is sufficiently high that each consumer always purchases exactly one unit of the differentiated product. Utility over the differentiated good is strongly separable from utility over leisure ($l_i$) and the outside good ($x_i$). Indirect utility over these two goods ($\Phi(p_x, p(z), \tau_l, \pi_l)$) is defined as

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4 Treating $\psi$ as a taste parameter rather than a transport cost offers two advantages. First, we do not have to model an explicit transport sector and second we do not have to consider the impact of prices on consumers’ location decisions.
where $\pi_i$ is consumer $i$'s share of profits from ownership of the differentiated product firms. We assume that consumers wholly own the differentiated product firms and share equally in their aggregate profits. If we denote the profit of firm $j$ as $\pi_j$ then:

$$\pi_i = \sum_{j=1}^{N} \pi_j / M$$

In the analysis below we assume for ease of exposition that the utility function in (4) is Cobb-Douglas:

$$\phi(x, l) = x^a l^{1-a}$$

First, consider equilibrium in the goods markets. Since the outside good is produced competitively we have:

$$\tilde{p}_x = w = 1$$

where $\tilde{p}_x$ is the producer price of $x$. If the outside good is subject to the same set of commodity taxes as the inside good, then the consumer price of $x$, $p_x$, is given by

$$p_x = \frac{1 + \tau_x}{1 - \tau_x}.$$  

Standard analysis then yields the individual demand functions and indirect utility function over leisure and the outside good. After incorporating equations (7) and (8), we can write them as:

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5 We have also performed the analysis on the assumption that the utility function in (4) is CES. While more complex, the results are qualitatively unaltered. In particular, our core tax neutralization result applies in the CES version of the model. Details are available from the authors on request.
\[ x(p(z), \tau) = \frac{a(1 - \tau_s)(1 - \tau_v - p(z) + \pi_i)}{1 + \tau_s} \]

\[ l(p(z), \tau) = \frac{(1-a)(1 - \tau_v - p(z) + \pi_i)}{1 - \tau_v} \]

\[ \Phi(p(z), \tau) = A(1 - \tau_v)^a (1 + \tau_s)^{-a} (1 - \tau_v)^{-(1-a)} (1 - \tau_v - p(z) + \pi_i) \]

where \( \tau = (\tau_s, \tau_v, \tau_l) \) and \( A = a^a (1-a)^{1-a} \).

The flexible manufacturing technology has two characteristics. First \( 0 \leq r \leq \psi \) and second, each consumer obtains the same utility from basic product \( z_j \) customized to attribute \( z \) as from a basic product \( z \) if they are offered at the same price.\(^6\)

By contrast, the differentiated product firms compete in a two-stage game. In the first stage firms (simultaneously) decide whether to enter and the locations, or attributes of their (basic) products. In the second stage they compete in prices à la Bertrand. We confine our attention to two, exogenously determined, pricing policies. With non-discriminatory pricing firm \( j \) sets one price available to all customers. We use the notation \( m(z) \) to refer to this uniform consumer price. With discriminatory pricing firm \( j \) may set an individual price for a consumer at location \( z \). We use the notation \( p_j(z) \) to refer to this consumer price. The only restriction imposed on firm \( j \)'s price \( p_j(z) \) is that the firm never prices below marginal cost, including commodity taxes.\(^7\)

In order to keep the analysis tractable we confine our attention throughout to symmetric locations of the entrant firms. This is actually not particularly restrictive in that it can be shown that a symmetric configuration of products is a location equilibrium in this type of model (see, for example, Novshek (1980) or Kats (1995)). We still need to identify, however, what is meant

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\(^6\) The implicit assumption with the designated technology is that, while customization might be possible, it can be implemented only at a “distance” cost \( r_d >> \psi \) and so will be rejected by consumers.
by equilibrium in the entry subgame. Entry takes place to the point where no additional entrant expects to break even but this leaves a potentially wide range of potential equilibria. Norman and Thisse (1999) show that this range is determined by the costs that incumbent firms incur in re-anchoring their (basic) products. We concentrate throughout on two polar cases:

(i) **spatial contestability (SC)** in which re-anchoring costs are zero; and

(ii) **spatial non-contestability (SNC)** in which re-anchoring costs are prohibitive.

With SC, the only candidate equilibrium is the maximum packing configuration in which all incumbent firms just break even. With SNC, the candidate equilibria range from this maximum packing equilibrium to the minimum packing equilibrium in which a sophisticated entrant just fails to break even given the (symmetric) locations of the incumbents. By a sophisticated entrant we mean an entrant that can “foresee the price equilibrium that would prevail if they were to enter” (Eaton and Wooders, 1985, p. 283). We confine our attention in the SNC case to the minimum packing equilibrium.

Three further conditions are necessary to close the model. First, there is the market clearing condition in the labor market, which is

\[
N.f + M.x(p(z),\tau) + G = M.\lambda(p(z),\tau)
\]

(10)

The left-hand side is labor demand and the right-hand side is labor supply. Second, there is the government revenue constraint:

\[
R_v + R_s + R_l = G
\]

(11)

where \(R_r\) is government revenue from taxes of type \(r\) (ad valorem, specific and labor taxes). We define these more completely below. Third, we have a social welfare objective in which

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7 The reader will note that we have used the notation \(p(z)\) previously. We will continue to use this notation in place of either \(m(z)\) or \(p(z)\) in statements that apply in either the price discrimination or non-discrimination environment. Note too that we occasionally abbreviate the pricing notation to \(m_j\) or \(p_j\) for convenience.

8 See Norman and Thisse (1996).
government sets the tax rates to maximize social welfare. In doing so we assume that the
government can correctly anticipate the impact that tax policy has on equilibrium prices and on
product variety in the differentiated product sector.

3. **Tax Policy in the Absence of Price Discrimination**

In this section, we consider whether tax policy can be used to maximize social welfare
when the differentiated product firms employ the designated technology and are unable to price
discriminate.

Suppose that there are $N$ active firms uniformly distributed over the market. Consider
firm $j$, which sets consumer price $m_j$ under the assumption that all other firms have set the
consumer price $m$. Then the consumer who is indifferent between buying from firm $j$ and the
nearest neighbor to firm $j$ is located at $z$ such that

$$m_j + \psi z = m + \psi \left( \frac{1}{N} - z \right) \Rightarrow z = \frac{1}{2N} + \frac{m - m_j}{2\psi}$$

Demand to firm $j$ is $2Nz$ or

$$D_j(m_j; m) = M\left( \frac{1}{N} + \frac{m - m_j}{\psi} \right)$$

and profit to firm $j$ is

$$\pi_j = \left( (1 - \tau_v) m_j - \tau_v \right) D_j(m_j; m) - f.$$  

Taking the derivative with respect to $m_j$, setting this to zero and solving for $m_j$ with the symmetry
assumption that in equilibrium $m = m_j$ gives the equilibrium consumer price for the inside good

$$m(N) = \frac{\psi}{N} + \frac{\tau_v}{1 - \tau_v}.$$  

Equilibrium profits for the firm are given by

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9 This is the equilibrium considered by Kay and Keen (op. cit.)
\( \pi_j = \frac{(1 - \tau_v) \psi M}{N^2} - f. \)

When the market is SC free entry to the differentiated product sector drives profit to zero, as a result of which (16) gives the equilibrium condition on the degree of product variety

\( N^{sc}(\tau) = \sqrt{\frac{(1 - \tau_v) \psi M}{f}}. \)

Note that the equilibrium degree of product variety is affected only by the ad valorem tax rate, implying that any attempt by government to influence product variety in the sector can be effected solely by use of ad valorem taxes.

Now suppose that the market is SNC. Then the relevant equilibrium condition is that an entrant just fails to break even given that the incumbents do not relocate in response to entry (because re-anchoring costs are prohibitively high) but are expected by the entrant to change their prices optimally in response to entry. We must first, therefore, identify the Nash equilibrium prices that the entrant expects to prevail post-entry.

It is a familiar result that an entrant should locate midway between some pair of incumbents. So suppose that the entrant, denoted 0, locates midway between two incumbents, denoted –1 and 1. The incumbents will change their prices in response to entry and this price change will affect their nearest neighbors, 2 and –2 who can be expected to change their prices, affecting their neighbors 3 and –3 and so on. A chain reaction is set up in the post-entry mill prices of the incumbent firms.

Suppose that there are \( N \) firms pre-entry. Then we have the following:\(^{10}\)

**Theorem 1:** The post-entry consumer prices charged by the entrant \( (m_0) \) and the incumbent firms \( (m_i, \ i = \pm 1, \pm 2, \ldots) \) are:
\[ m_0(N, \tau_s, \tau_v) = \frac{\tau_s}{1 - \tau_v} + \frac{3(1 + \sqrt{3}) \psi}{3 + 2\sqrt{3} 2N} \]
\[ m_{\pm 1}(N, \tau_s, \tau_v) = \frac{\tau_s}{1 - \tau_v} + \frac{3 + 4\sqrt{3} \psi}{3 + 2\sqrt{3} 2N} \]
\[ m_{\pm 2}(N, \tau_s, \tau_v) = \frac{\tau_s}{1 - \tau_v} + \frac{7\sqrt{3} \psi}{3 + 2\sqrt{3} 2N} \]
\[ m_i(N, \tau_s, \tau_v) = \frac{\tau_s}{1 - \tau_v} + \left( 2 - \frac{3(2 - \sqrt{3})}{3 + 2\sqrt{3}} \right) \frac{\psi}{2N} \quad (i = 3, -3; 4, -4; \ldots) \]

Using the prices from Theorem 1 to calculate the profits of the entrant and setting these to zero gives the equilibrium degree of product variety with SNC

\[ N^{SNC} (\tau) = \sqrt{\frac{3(1 - \tau_v)\psi M}{2(2 + \sqrt{3})f}} \]

As with the SC case, the equilibrium product price \( m(N^{SNC}) \) is obtained by substituting (19) into (15). Unlike the SC case, firms now earn positive profits:

\[ \pi^{SNC} = \left( 1 + 2\sqrt{3} \right)f/3 \]

The government’s objective is to choose tax rates in order to maximize utility subject to the revenue constraint (8) and subject to the labor-market clearing condition (10). The government's tax instruments are an ad valorem wage tax, an ad valorem tax on commodities \((x, z)\) and a specific tax on commodities \((x, z)\).\(^{11}\) More specifically, aggregate utility is:

\[ U_d(\tau) = M \left( V - \frac{\psi}{4N_c} + A_c \frac{(1 - \tau_v)^x}{(1 - \tau_i)^{x-a}(1 + \tau_i)^x} \left( 1 - m(N_c) + \pi_i - \tau_i \right) \right) \quad c \in \{SC, SNC\} \]

and the government budget constraint is:

\(^{10}\) The proof follows Eaton and Wooders (1985) and can be obtained from the authors on request. See also Norman and Thisse (1996).

\(^{11}\) We restrict the taxes to be equivalent for \(x\) and \(z\). Otherwise, it will turn out that the specific tax on the differentiated commodity serves as a lump sum tax. In that case, the optimal taxes on \(x\) and labor are zero, the ad valorem tax on \(z\) is set to effect the socially optimal number of firms, and all residual revenue is collected with the specific tax on \(z\).
The three terms on the left-hand side of (22) are respectively revenues from labor, unit and ad valorem taxes. We show in the Appendix that our model is closed and consistent, in that any set of tax rates that satisfy the government revenue constraint also satisfy the labor market clearing constraint. The government’s objective is, therefore

\[
\max_{\tau_l, \tau_u, \tau_v} U_d(\tau) \text{ subject to (22)}
\]

When the market is SC there are the further constraints (7), (8), (9), (15), (17) and \( \pi_i = 0 \), while if the market is SNC there are the constraints (7), (8), (9), (15), (19) and \( \pi_i = \pi^{SNC}.N^{SNC}/M \).

This program is too complex to allow of simple analytical solution. Extensive numerical investigation, however, generates the following result:

**Lemma 1**: When utility from consumption of the outside good and leisure is Cobb-Douglas, given by equation (6), optimal commodity tax rates satisfy:

\[
\tau_l^* + \tau_u^* + \tau_v^* - \tau_l^* \tau_v^* = 0
\]

whether the market is SC or SNC.

Lemma 1 considerably simplifies the analysis. Substituting (24) into the revenue constraint (22), solving for the labor tax, substituting into the objective function and solving for the optimal ad valorem tax gives:

**Theorem 2**: When utility from consumption of the outside good and leisure is Cobb-Douglas, given by (6), and firms do not price discriminate, the optimal commodity tax rates are:

When the market is SC
\[ \tau^v_{SC} = 1 - 1/4a^a \left( 1 - a \right)^{1-a} \]

(25)

\[ \tau^i_{SC} = \frac{G.N^SC \left( \tau^SC \right) \left( 1 - \tau^v_{SC} \right) + M \left( N^SC \left( \tau^SC \right) - \psi \left( 1 - \tau^v_{SC} \right) \right)}{G.N^SC \left( \tau^SC \right) - M \left( N^SC \left( \tau^SC \right) - \psi \left( 1 - \tau^v_{SC} \right) \right) \left( 1 - \tau^v_{SC} \right)} \]

\[ \tau^x_{SC} = \frac{G.N^SC \left( \tau^SC \right)}{M \left( N^SC \left( \tau^SC \right) - \psi \left( 1 - \tau^v_{SC} \right) \right) - G.N^SC \left( \tau^SC \right)} \]

When the market is SNC:

\[ \tau^v_{SNC} = 1 - \left( 2 + \sqrt{3} \right)/\left( 6a^a \left( 1 - a \right)^{1-a} \right) \]

(26)

\[ \tau^i_{SNC} = \frac{2 + 2\sqrt{3}}{2 \left( 2 + 2\sqrt{3} \right) N^SNC \left( \tau^SNC \right)} \left( 1 - \tau^v_{SNC} \right) + M \left( N^SNC \left( \tau^SNC \right) - \psi \left( 1 - \tau^v_{SNC} \right) \right) \tau^v_{SNC} \]

\[ \tau^x_{SNC} = \frac{6 + \sqrt{3}}{2 \left( 2 + 2\sqrt{3} \right) N^SNC \left( \tau^SNC \right) - 3\sqrt{3} \psi \left( 1 - \tau^v_{SNC} \right) - 2 \left( 2 + 2\sqrt{3} \right) G.N^SNC \left( \tau^SNC \right)} \]

Table 1 describes some of the comparative statics of the equilibrium tax rates (25) and (26). All three tax rates are symmetric about \( a = 0.5 \) with the labor and unit taxes being similarly affected by changes in \( a \). By contrast, these taxes move in opposite directions in response to changes in \( G/M, \psi \) and \( f/M \).

(Tables 1 near here)

Note that the optimal ad valorem tax rate is determined solely by the Cobb-Douglas coefficient \( a \), being minimized when \( a = 1/2 \) and maximized when \( a = 0 \) or \( 1 \). This provides an interesting comparison with the optimal tax rates that have been derived from partial equilibrium analysis. From Kay and Keen (op. cit.) and Metcalf and Norman (2002) the partial equilibrium optimal ad valorem tax rate – which is the tax rate that minimizes costs in the differentiated product sector – is 75% when the market is SC and 37.8% when it is SNC. From (25) and (26) it is easy to see that the optimal general equilibrium ad valorem tax rates are always less than the partial equilibrium rates provided only that \( a \neq 0 \) or \( 1 \). In other words, once account is taken of the interaction between the inside and outside products and of the full distortionary impacts of
commodity and labor taxes, government prefers to see a higher level of activity in the
differentiated product sector than the level that minimizes costs in this sector. Indeed, when the
market is SNC we find that for \( a \in [0.182, 0.818] \) the optimal ad valorem tax rate is actually
negative: government chooses to subsidize activity in the differentiated product sector.

The results in Theorem 2 allow us to compute equilibrium product variety in the
differentiated product sector, aggregate welfare and aggregate labor supply. We obtain the
following:

**Theorem 3:** When the market is SC (SNC), the differentiated product firms do not price
discriminate, and commodity taxes satisfy (25)((26)), the socially optimal number of
differentiated product firms is

\[
N_d^0 = \frac{1}{2} \sqrt{\frac{M \psi}{f \cdot a^a (1 - a)^{1-a}}}
\]

aggregate utility is:

\[
U_d^0 = MV + a^a (1 - a)^{1-a} (M - G) - \sqrt{M \cdot f \cdot \psi \cdot a^a (1 - a)^{1-a}}
\]

and aggregate labor supply is:

\[
L_d^0 = G + a(M - G) + f(1 - a)N_d^0
\]

This is one of our core results that we show in the next section also extends to the case
where firms apply discriminatory pricing. Theorem 3 states first, that government can, through
its choice of tax rates, totally neutralize the potentially detrimental impact of restrictive entry
conditions in the differentiated product sector.\(^{12}\) In other words, provided that government sets
tax rates optimally, commodity taxation renders the degree of spatial contestability irrelevant for

\(^{12}\) While the precise equations in Theorem 3 derive from our Cobb-Douglas assumption, we obtain the same
“neutralizing” results if we assume that consumer utility in (4) is CES.
welfare purposes. Second, tax policy can neutralize the effect that positive pure profits generated by spatial non-contestability would otherwise have on the labor market through their impact on entry to the differentiated product sector and on individual’s consumption/leisure choices.

4. Tax Policy with Price Discrimination

As we pointed out in the section 2, a trend towards weaker regulation of pricing policies has occurred both in the United States and Europe. In this section, we consider how price discrimination influences the ability of tax policy to affect market structure. This analysis is particularly relevant given that, as we also noted in section 2, firms are increasingly able to adopt flexible technologies. Recall that this means that the firms can customize their products to the precise requirements of their customers: in a purely spatial context it is equivalent to the firm controlling and potentially charging for delivery of the product to its consumers. It also means that, in the absence of regulations to the contrary, firms are likely to be forced to adopt discriminatory pricing. In other words, flexible manufacturing leads to a very different price equilibrium for the differentiated product producers than does a less flexible, designated technology.

An essential feature of flexible manufacturing is that it allows firms to customize their products to the precise specifications demanded by consumers. Consider a consumer whose most preferred product attribute lies between firm $j$ and $j+1$ “distance” $z$ from firm $j$ such that this consumer is located nearer to firm $j$ than firm $j+1$. Both firms offer this consumer a customized product that meets her exact requirements, as a result of which she will buy from the

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13 It might be thought that since government expenditure reduces aggregate welfare, taxation must also reduce welfare. This is, however, not the case. We can always set tax rates according to (25) or (26) even if $G = 0$. While the labor tax rate "undoes" the ad valorem tax rate in the consumer budget constraint so that this constraint is identical to a constraint with zero taxes, it preserves the ad valorem tax rate ability to influence market structure.
firm offering such a customized product at the lowest price. Competition between firms \( j \) and \( j+1 \) for this consumer drives the price down to the point at which firm \( j+1 \) cannot break even. If firm \( j+1 \) quotes a price \( p_{j+1}(z) \) to this consumer then, given an ad valorem tax rate of \( \tau_v \) and a specific tax of \( \tau_s \), firm \( j+1 \) receives post-tax revenues of \( p_{j+1}(z)(1 - \tau_v) - \tau_s \) and this must be no less than costs \( r(1/N - z) \). The Nash equilibrium price that firm \( j \) charges to consumer \( z \) with Bertrand competition is, therefore, the minimum price firm \( j+1 \) can quote, giving firm \( j \)'s equilibrium price schedule for consumers between firms \( j \) and \( j+1 \) as:

\[
(30) \quad p_j(N,z) = \frac{\tau_s + r}{1 - \tau_v} \left( \frac{1}{N} - z \right) \quad z \in \left[ 0, \frac{1}{2N} \right]; j = 1, ..., N
\]

It follows that firm \( j \)'s profit is

\[
(31) \quad \pi_j(N) = 2M \int_0^{1/2N} \left( p(N,z)(1 - \tau_v) - \tau_s - r \right) ds = -f = \frac{M \cdot r}{2N^2} - f.
\]

This is precisely the same as the profit that firm \( i \) earns when all tax rates are zero. It follows that when firms operate flexible technologies and are allowed to price discriminate all taxes are passed on in full to consumers. In other words, since the producer price at each consumer location is \( \bar{p}_j(z) = \tau_s + r(1/N - z) \) and \( N \) is independent of \( \tau_v \), ad valorem taxes have no direct or indirect effect on producer prices in the differentiated product sector.

What about the equilibrium number of basic products that will be established? By the same argument as in section 3 we know that with spatial contestability the equilibrium location configuration is such that all incumbents just break even. By contrast, with spatial non-contestability it must be that an entrant, who gains a market share \( 1/2N \) and profits of one-quarter those of the incumbents, just fails to break even. It follows from (31) that the equilibrium number of product variants in these two cases is
With SNC each differentiated product firm earns profit

\[ \pi_f^{SNC} = 3f \]

An immediate and important implication of equations (32) and (33) is that tax policy can no longer be used as a policy instrument to affect product diversity in the differentiated product sector. Some other instrument will be needed but the options available to policy makers are limited. Equilibrium product variety is a function solely of customization costs, fixed costs and, implicitly, re-anchoring costs.

It is the case, however, that tax policy affects consumer prices in the differentiated product sector and consumption decisions with respect to the outside good \( x \) and leisure, with the result that government is not necessarily without power. In other words, we can still investigate optimal tax policy in our general equilibrium setting. As in the previous section, we assume that government sets tax rates to maximize total surplus subject to a budget constraint.

More specifically, aggregate utility is:

\[ U_f(\tau) = M \left( V + 2AN^{c} \cdot \frac{(1 - \tau_r)^a}{(1-\tau_r)^{1-a}(1+\tau_r)^a} \int_{0}^{2N^{c}} (1 - p(N^{c}, z) + \pi_i - \tau_j)dz \right) c \in \{SC, SNC\} \]

and the government budget constraint is:

\[ M\tau_i \left( 1 - 2N^{c} \frac{(1 - a)}{(1-\tau_r)} \int_{0}^{2N^{c}} (1 - p(N^{c}, z) + \pi_i - \tau_j)dz \right) + \]

\[ M\tau_s \left( 1 - 2N^{c} \frac{a(1 - \tau_r)}{(1+\tau_r)} \int_{0}^{2N^{c}} (1 - p(N^{c}, z) + \pi_i - \tau_j)dz \right) + \]

\[ 2MN^{c}\tau_s \int_{0}^{2N^{c}} (p(N^{c}, z) + a(1 - p(N^{c}, z) + \pi_i - \tau_j))dz = G \]
The government objective is then to set tax rates to solve the program:

\[(36) \quad \max_{\tau, \pi_c, \pi_r} U_j(\tau) \text{ subject to } (35)\]

In addition, we have the labor market clearing condition and the relevant set of equations (7) – (9) and (30) – (33), determined by whether the market is SC or SNC. As in the no-price-discrimination constraint, we show in the Appendix that our model is consistent and closed in that any set of tax rates that satisfy the budget constraint also satisfy the labor market clearing condition.

Once again this program is too complex to admit of direct analytical solution. However, numerical simulation confirms that Lemma 1 holds, so that we have \(\tau^*_l + \tau^*_v + \tau^*_l - \tau^*_v \tau^*_r = 0\). We use this as before to solve the program (36), giving:

**Theorem 4:** When utility from consumption of the outside good and leisure is Cobb-Douglas, given by (6), and firms price discriminate, the optimal commodity tax rates are:

1. **When the market is SC**

\[
\begin{align*}
\tau^*_l^{\text{SC}} &= \frac{4G}{4(M - G) - 3\sqrt{2Mrf} \\
\tau^*_v^{\text{SC}} &= -\frac{4G - (4(M - G) - 3\sqrt{2Mrf})\tau^*_l^{\text{SC}}}{4(M - G) - 3\sqrt{2Mrf}(1 - \tau^*_l^{\text{SC}})}
\end{align*}
\]

2. **When the market is SNC**

\[
\begin{align*}
\tau^*_l^{\text{SNC}} &= \frac{4G(1 - \tau^*_l^{\text{SNC}}) + 3\tau^*_l^{\text{SNC}}\sqrt{2Mrf}}{4(M - G)(1 - \tau^*_l^{\text{SNC}}) - 3(2 - \tau^*_l^{\text{SNC}})\sqrt{2Mrf}} \\
\tau^*_v^{\text{SNC}} &= \frac{-4G - (4(M - G) - 3\sqrt{2Mrf})\tau^*_l^{\text{SNC}}}{4(M - G)(1 - \tau^*_l^{\text{SNC}}) - 3(2 - \tau^*_l^{\text{SNC}})\sqrt{2Mrf}}
\end{align*}
\]

Note that the condition in equation (24) was used to derive the equations in either (37) or (38) and is not an independent equation. In other words, we have three tax rates defined by two equations. It might be argued that this is but another example of a “redundancy” property in tax
policy: that setting three tax rates to maximize social welfare is one tax instrument too many.

There are two reasons, however, for our preferring the alternative interpretation, that this indeterminacy provides budgetary flexibility to the government in setting tax rates. First, it can be seen from (37) and (38) that in the SC case government is free to set either the labor or ad valorem tax rate but the unit tax rate is fixed by the model parameters. By contrast, with SNC any one of the three tax rates can be set arbitrarily. Moreover, the “redundancy” interpretation typically assumes that there are zero (untaxed) pure profits and yet we have just seen that flexibility in rate setting increases in the SNC case, which is precisely the case where there are positive pure profits.

The comparative static properties of the equilibrium tax rates are as in Table 1 with the exception that these tax rates are independent of the demand parameter.

Substituting (37) and (38) into the utility function gives:

**Theorem 5:** When the market is SC( SNC), the differentiated product firms price discriminate and commodity taxes satisfy (37)((38)), aggregate utility is:

\[ U^0_j = MV + (M - G)a^a(1 - a)^{1-a} - 3\sqrt{2Mr} a^a(1 - a)^{1-a} / 4 \]  

and the aggregate supply of labor is:

\[ L^0_j = aM + (1 - a)(G + 3\sqrt{2Mr} / 4) \]

Theorems 4 and 5 have three important implications. First, despite the fact that commodity taxes have no influence on equilibrium product variety in the differentiated product sector, government can use the price effects of commodity taxes to neutralize the welfare impact of differences in the degree of spatial contestability in the differentiated product sector. Second, tax policy also neutralizes the labor market impact of positive pure profit when the market is

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14 As in the no-price-discrimination case, exactly the same implications hold if the utility function in (4) is CES.
SNC. Third, government gains a reasonably considerable degree of freedom in the setting of ad valorem and labor taxes. Any set of ad valorem and labor tax rates that satisfies (37 b) or (38 b) also meets the relevant utility maximizing conditions.

The final question we can investigate is the impact of price discrimination on aggregate welfare and on product variety in the differentiated product sector, given that government sets tax rates optimally. Suppose that \( r = \rho \varphi \) where, given our assumptions on the nature of flexible manufacturing, we have \( \rho \leq 1 \). Then comparison of (28) and (39) gives:

**Theorem 5:** Suppose that \( \varphi \rho = r \). Given that the utility function in (4) is Cobb-Douglas and that commodity taxes and labor taxes are set optimally.

1. Price discrimination always gives a smaller number of differentiated product firms when the market is SNC and gives a smaller number of differentiated product firms when the market is SC if \( \rho < 1/2a^a(1-a)^{1-a} \).

2. Price discrimination gives greater aggregate utility than non price discrimination provided that

\[
\rho < 8/9a^a(1-a)^{1-a}
\]

Sufficient conditions for price discrimination to increase utility are either that \( a \in [0.025, 0.975] \) or that \( \rho < 8/9 \).

Once again we have an important contrast with the partial equilibrium analysis. In the latter setting, Norman and Thisse (1996, 2000) show that the degree of spatial contestability “matters” in determining the welfare impact of price discrimination as a result of the interplay between the pro-competitive and market structure effects of price discrimination. In our general equilibrium setting, the same interplay is at work but there is now an additional countervailing force since government tax policy neutralizes the effects of spatial contestability no matter the
firms’ pricing policies. As a result, what matters most is the pro-competitive effects of price discrimination.

5. Conclusion

In this paper we have examined the interplay between market structure and tax policy in a general equilibrium model in which firms produce differentiated products and so are able to exert market power. We have paid particular attention to two recent developments in the analysis of imperfectly competitive markets. First, there is a theoretical development in that we now have a greater understanding of the welfare and structural properties of such markets when potential entrants face more or less restrictive entry conditions: referred to in the spatial analog that we have presented in this paper as the degree of spatial contestability. Second, there is a market development in that developments in both real and virtual technologies have greatly enhanced the ability of firms to price discriminate, with the consequent need for government to determine the optimal response to price discrimination.

This paper provides a number of new findings. First, embedding a differentiated product model within a general equilibrium model leads to lower optimal commodity tax rates than those implied by partial equilibrium analysis. Second, if firms are able to price discriminate, the government is no longer able to influence market structure through the use of tax policy. Third, a tax neutralization result holds in that commodity tax policy can undo the welfare impact of spatial contestability. In other words, the potentially adverse welfare effect of incumbents’ ability to exploit potentially restrictive entry conditions facing potential entrants can be neutralized through tax policy. Simply put, the welfare of a representative agent will be unaffected by the extent of non-contestability if government sets tax rates optimally. Fourth, there is a plausible range of parameters over which welfare is higher when government allows
price discrimination than when it prohibits price discrimination. In contrast to prior work in a partial equilibrium setting, this result holds regardless of the degree of spatial contestability.

Our results raise other possibilities that we leave to subsequent research. In particular, we have assumed throughout our analysis that the choice of pricing policy is exogenous: firms either choose to price discriminate or not. We can justify this on the argument that employing a flexible technology undermines a firm’s commitment not to price discriminate but this could be argued to beg the larger question of why the flexible technology was chosen in the first place. The endogenous choice of (spatial) price policy has been analyzed in Thisse and Vives (1988) in a world without commodity taxes and where the underlying technology choice is only implicit. A natural extension to their analysis would be to consider the role and design of commodity taxes in a world in which firms endogenously choose their pricing policies, perhaps by endogenously choosing their technologies.
Table 1: Equilibrium Tax Rates: No Price Discrimination

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\tau_l$</th>
<th>$\tau_s$</th>
<th>$\tau_v$</th>
</tr>
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<tbody>
<tr>
<td>$a &lt; 0.5$</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$a &gt; 0.5$</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$G/M$</td>
<td>-</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>$\psi$</td>
<td>+</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>$f/M$</td>
<td>-</td>
<td>+</td>
<td>0</td>
</tr>
</tbody>
</table>
Assume that there are $N$ uniformly distributed differentiated product firms.

### A.1 No Price Discrimination

Assume that each differentiated product firm charges price $m$. Aggregate demand for labor is:

\[ AD_i = N.f + G + \frac{M.a(1-m-\tau_i + \pi_i)(1-\tau_v)}{(1+\tau_s)} \]  

Aggregate supply of labor is:

\[ AS_i = M - \frac{M(1-a)}{(1-\tau_s)}(1-m-\tau_i + \pi_i) \]

The excess demand for labor is then

\[ ED_i = AD_i - AS_i \]

The government revenue constraint is:

\[ G = \tau_i AS_i + \tau_i M \left( \frac{1+a(1-m-\tau_i + \pi_i)(1-\tau_v)}{(1+\tau_s)} \right) + \tau_v \left( m + a(1-m-\tau_i + \pi_i) \right) \]

Substituting (A.1), (A.2) and (A.4) into (A.3) and simplifying gives

\[ ED_i = N.f + M \left( \pi_i + \tau_i - m(1-\tau_v) \right) \]

The profit of each firm is $\pi_i = \frac{M}{N}(m(1-\tau_v) - \tau_s) - f$ and each consumer’s profit share is $\pi_i = \frac{N\pi_i}{M}$. Substituting into (A.5) gives $ED_i = 0$ as required.

### A.2 Price Discrimination

Each differentiated product firm charges prices $p(z)$ given by equation (33). Aggregate demand for labor is:

\[ AD_i = N.f + 2M.N \int_{0}^{\frac{1}{2N}} \frac{a(1-p(z)-\tau_i + \pi_i)(1-\tau_v)}{(1+\tau_s)} dz + 2M.N \int_{0}^{\frac{1}{2N}} r.dz \]
Aggregate supply of labor is:

(A.7) \[ AS_i = M = 2M \cdot N \int_{0}^{\frac{1}{2N}} \left( \frac{1-a}{1-\tau_i} \right) (1-m-\tau_i + \pi_i) dz \]

The government revenue constraint is:

\[ G = 2M \cdot N \cdot \tau_i \int_{0}^{\frac{1}{2N}} \left( p(z) + a(1-p(z)-\tau_i + \pi_i) \right) dz + \tau_i AS_i + \]

(A.8) \[ M \cdot \tau_i \left( 1 + 2N \int_{0}^{\frac{1}{2N}} \frac{a(1-p(z)-\tau_i + \pi_i)(1-\pi_i)}{(1+\pi_i)} dz \right) \]

Substituting for \( p(z) \), evaluating the integrals and substituting (A.8) into (A.6) gives the excess demand for labor:

(A.9) \[ ED_i = N \cdot f + M \left( \pi_i - \frac{r}{2N} \right) \]

We know that the profit of each firm is \( \pi_j = M \cdot r / 2N^2 - f \). Evaluating \( \pi_i \) and substituting into (A.9) confirms once again that \( ED_i = 0 \).
References


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<td>FULLERTON, Don and Gilbert METCALF</td>
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