Endogenous Technology Adoption and R&D as Sources of Business Cycle Persistence*

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Abstract

We examine the hypothesis that the slowdown in productivity following the Great Recession was in significant part an endogenous response to the contraction in demand that induced the downturn. We first present some descriptive evidence in support of our approach. We then augment a workhorse New Keynesian DSGE model with an endogenous TFP mechanism that allows for both costly development and adoption of new technologies. We estimate the model and use it to assess the sources of the productivity slowdown. We find that a significant fraction of the post-Great Recession fall in productivity was an endogenous phenomenon. The endogenous productivity mechanism also helps account for the slowdown in productivity prior to the Great Recession, though for this period shocks to the effectiveness of R&D expenditures are critical. Overall, the results are consistent with the view that demand factors have played a role in the slowdown of capacity growth since the onset of the recent crisis. More generally, they provide insight into why recoveries from financial crises may be so slow.

JEL: JEL Classification: E3, O3

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1 Introduction

One of the great challenges for macroeconomists is explaining the slow recovery from major financial crises (see, e.g. Reinhart and Rogoff (2009)). This phenomenon is only partly accounted for by existing theories. A large literature has suggested that demand shortfalls can account for slow growth following the financial crisis and Great Recession. The evidence for this channel is certainly compelling: the deleveraging process was likely an important cause of persistent reduced spending by borrowers as they saved to lower debt. Constraints on macroeconomic policy likely also contributed to sluggish demand: the zero lower bound on the nominal interest rate limited the ability of monetary policy to stimulate the economy and the political fight over the national debt ceiling effectively removed fiscal policy as a source of stimulus.

While these demand side factors have undoubtedly played a central role, it is unlikely that they alone can account for the extraordinarily sluggish movement of the economy back to the pre-crisis trend. This has led a number of authors to explore the contribution of supply-side factors. Both Hall (2014) and Reifschneider et al. (2015) have argued that the huge contraction in economic activity induced by the financial crisis in turn led to an endogenous decline in capacity growth. Hall (2014) emphasizes how the collapse in business investment during the recession brought about a non-trivial drop in the capital stock. Reifschneider et al. (2015) emphasize not only this factor but also the sustained drop in productivity. They make the case that the drop in productivity may be the result of a decline in productivity-enhancing investments, and thus an endogenous response to the recession.

Indeed, sustained drops in productivity appear to be a feature of major financial crises. This has been the case for the U.S. and Europe in the wake of the Great Recession. The same phenomenon holds broadly for financial crises in emerging markets: in a sample of East Asian countries that experienced a financial crisis during the 1990s, Queralto (2015) finds a sustained drop in labor productivity in each case to go along with the sustained decline in output.

What accounts for reduced productivity growth following financial crises? There are two candidate hypotheses: bad luck versus an endogenous response. Fernald (2014) makes a compelling case for the bad luck hypothesis. As he emphasizes, the productivity slowdown began prior to the Great Recession, raising questions on whether the crisis itself can be a causal factor. Figure 1 illustrates the argument. The figure plots detrended total factor

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1See, for example Christiano et al. (2015) for a taxonomy of possible explanations for the slow recovery.
Figure 1: Detrended Capacity Adjusted TFP and Labor Productivity

All series are log-linearly detrended. Labor productivity is GDP divided by hours worked (see Appendix A.1 for data sources). TFP is Utilization-Adjusted Total Factor Productivity (available at http://www.frbsf.org/economic-research/total-factor-productivity-tpfp/; see Fernald (2012) for details).

productivity, specifically Fernald’s utilization corrected measure, along with labor productivity. Both measures show a sustained decline relative to trend in the years after the Great Recession, but the decline appears to begin around 2004-05.

There are several different theories of how the productivity slowdown could reflect an endogenous response to the crisis. The one on which we focus involves endogenous growth considerations. Specifically, to the extent that the crisis induced a large drop in expenditures on research and development as well as technology adoption, the subsequent decline in productivity could be an endogenous outcome. We provide evidence of a fall in R&D and adoption expenditure during the Great Recession in Section 2. There was also a sharp

2An alternate approach stresses misallocation of productive inputs following a financial crisis. See for example Garcia-Macia (2015) who emphasizes misallocation between tangible and intangible capital.
decline in these expenditures during and after the 2001-2002 recession, which raises the possibility that the productivity slowdown prior to the Great Recession was also in part a response to cyclical factors. Overall, this evidence, together with descriptive evidence on the cyclical behavior of technology diffusion that we present is consistent with endogenous growth factors contributing to the productivity slowdown.

Motivated by the evidence in Section 2, we develop and estimate a macroeconomic model modified to allow for endogenous technology via R&D and adoption. We then use the model to address the following three issues: (i) how much of the recent productivity decline reflects an endogenous response to the Great Recession; (ii) whether the mechanism can also account for the productivity slowdown prior to the Great Recession; and (iii) more generally the extent to which endogenous productivity can help account for business cycle persistence.

The endogenous productivity mechanism we develop is based on Comin and Gertler (2006), which uses the approach to connect business cycles to growth. The Comin/Gertler framework, in turn, is a variant of Romer (1990)'s expanding variety model of technological change, modified to include an endogenous pace of technology adoption. We include adoption to allow for a realistic period of diffusion of new technologies, and we allow for endogenous adoption intensity to capture cyclical movements in productivity that may be the product of cyclical adoption rates. To justify our formulation, in Section 2 we present panel data evidence showing that the speed of technology diffusion strongly varies over the cycle.

In addition to the literature cited above, there are several other papers related to our analysis. Queralto (2015), Guerron-Quintana and Jinnai (2014) and Garcia-Macia (2013) have appealed to endogenous growth considerations to explain the persistence of financial crises. The paper most closely related to ours is Bianchi and Kung (2014), who first estimated a macroeconomic model with endogenous growth using R&D data. In addition to focusing on somewhat different issues, our analysis differs in the following ways: first, we develop an explicit model of R&D and adoption with realistic lags which aids in both the empirical identification and the interpretation of the mechanism; second, we use data on business R&D expenditures produced by the NSF as opposed to the NIPA measure. The former measure corresponds more closely to the model counterpart of R&D since unlike the latter it excludes public expenditures on R&D and includes expenditures on software development. As a consequence, it exhibits cyclical properties more in keeping with the predictions of the theory. Third, we investigate quantitatively the ability of the model to explain not only the post- but also the pre-crisis slowdown in U.S. productivity. Finally, our solution method allows us to take account of a binding zero lower bound on monetary
policy, which turns out to be an important factor propagating the endogenous decline in productivity in the wake of the Great Recession.

The rest of the paper is organized as follows. Section 2 presents some evidence on the cyclical behavior R&D and technology adoption. In addition, we present some new panel data evidence on the speed of technology diffusion that suggests strong cyclical effects, consistent with the approach we take in our model. Section 3 presents the model. Section 4 describes the econometric implementation and present the estimates. Section 5 analyzes the extent to which the endogenous growth mechanism can account for the evolution of productivity both before and after the Great Recession.

2 Evidence on R&D and the speed of technology diffusion

In this section, we present evidence on the cyclical behavior of R&D and various measures of technology adoption. We are interested in exploring not only general cyclical patterns, but also behavior during the last fifteen years, which is the main focus of our study. While reasonably broad measures are available for R&D, the same is not true for adoption. Accordingly, for the latter we consider several different specific measures.

We begin with Figure 2 which plots expenses on R&D conducted by US corporations. During the Great Recession, there was a sharp decline in R&D potentially consistent with endogenous growth factors contributing to the productivity slowdown.\(^3\) There was also a large decline in R&D expenditures following the 2001-2002 recession, which raises the possibility that the productivity slowdown prior to the Great Recession was also in part a response to cyclical factors.

The evidence on R&D is consistent with the general procyclical behavior found in other studies (see e.g. Comin and Gertler (2006)). Less however is known about the behavior of adoption. We therefore resort to datasets that contain traditional measures of diffusion from the productivity literature to explore the cyclicity of the speed of diffusion of new technologies.\(^4\) The data we have available is a sample of 26 production technologies that diffused at various times over the period 1947-2003 in the US (5) and the UK (21).\(^5\) We

\(^3\)There is a long literature documenting the cyclicality of R&D expenditures (see Barlevy (2007), for a summary). Barlevy (2007) presents evidence based on firm-level data on the importance of both sectoral demand as well as firms’ financial conditions for the pro-cyclicality of R&D expenditures.

\(^4\)See for example, Griliches (1957) and Mansfield (1961).

\(^5\)The data on UK technologies comes from Davies (1979) and covers special presses, foils, wet suction boxes, gibberellic acid, automatic size boxes, accelerated drying hoods, basic oxygen process, vacuum degassing, vacuum melting, continuous casting, tunnel kilns, process control by computer, tufted carpets, computer typesetting, photo-electrically controlled cutting, shuttleless looms, numerical control printing presses, numerical control turning machines and numerical control turbines. The data for the five tech-
Figure 2: R&D Expenditures by US Corporations, 1983-2013

Log-linearly detrended data. Source: R&D Expenditure by US corporations (National Science Foundation). Data are deflated by the GDP deflator and divided by the civilian population older than 16 (see Appendix A.1 for data sources).
use the data to estimate the effect of the business cycle on the speed of diffusion, after controlling for the normal diffusion process.

Specifically, we denote by $m_{it}$ the fraction of potential adopters that have adopted a specific technology $i$ in $t$. The ratio of adopters to non-adopters $r_{it}$ is

$$r_{it} = m_{it} / (1 - m_{it}).$$  

(1)

The speed of diffusion is then the percentage change in $r_{it}$:

$$\text{Speed}_{it} = \Delta \ln(r_{it})$$  

(2)

As shown by Mansfield (1961), if the diffusion process follows a logistic curve, the speed of diffusion (2) is equal to a constant $\alpha_i$. In reality, however, the speed of diffusion is not constant, it tends to be faster in the early stages. Therefore, $r_{it}$ declines with the age of the technology. Additionally, we want to explore whether the speed of technology diffusion varies over the cycle. To this end, we consider the following specification

$$\text{Speed}_{it} = \alpha_i + G(\text{lag}_{it}) + \beta * \hat{y}_t + \epsilon_{it},$$  

(3)

where $G(.)$ is a polynomial in the years since the technology was first introduced, and $\hat{y}_t$ is a cyclical indicator, specifically detrended GDP per person over 16 years old.

Table 1 presents the estimates of equation (3). The main finding is that the estimates of the elasticity of the speed of diffusion with respect to the cycle, $\beta$, are robustly positive and significant. In particular, the point estimate is between 3.6 and 4.1 depending on the specification. The effect of years since the technology started diffusing is negative and convex (i.e. it vanishes over time). The results are robust to specifying the function $G$ as a second order polynomial or in logarithms. Finally, we do not observe any significant differential effect of the cycle on US versus UK technologies.

To further illustrate the cyclicity of the speed of technology diffusion, Figure 3 plots the speed of diffusion for the balanced panel of four US technologies for which we have data from 1981 to 2003. Specifically, for each of the technologies we remove the acyclical component of the diffusion rate ($\alpha_i + G(\text{lag}_{it})$). We then average the residual ($\beta * \hat{y}_t + \epsilon_{it}$) over the four technologies. The dashed line is a plot of this average, while the solid line is a three year centered moving average. The Figure reveals a positive correlation between the technologies in the US comes from Trajtenberg (1990), and Bartel et al. (2009) and covers the diffusion of CT scanners, Computerized numerical controlled machines, automated inspection sensors, 3-D CAD, and flexible manufacturing systems.
Table 1: Cyclicality of the Speed of Technology Diffusion

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
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</thead>
<tbody>
<tr>
<td>$\hat{y}_t$</td>
<td>3.73</td>
<td>3.7</td>
<td>3.64</td>
<td>4.12</td>
</tr>
<tr>
<td></td>
<td>(3.59)</td>
<td>(2.81)</td>
<td>(3.94)</td>
<td>(3.17)</td>
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<tr>
<td>$\hat{y}_t \times$ US</td>
<td>0.07</td>
<td>-0.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.53)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>lag$_{it}$</td>
<td>-0.057</td>
<td>-0.057</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(5.22)</td>
<td>(4.76)</td>
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<tr>
<td>lag$_{it}^2$</td>
<td>0.001</td>
<td>0.001</td>
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<tr>
<td></td>
<td>(2.52)</td>
<td>(2.12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(lag$_{it}$)</td>
<td></td>
<td>-0.29</td>
<td>-0.29</td>
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<tr>
<td></td>
<td></td>
<td>(6.68)</td>
<td>(6.65)</td>
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<tr>
<td>R2 (within)</td>
<td>0.11</td>
<td>0.11</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>N technologies</td>
<td>26</td>
<td>26</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td>N observations</td>
<td>327</td>
<td>327</td>
<td>327</td>
<td>327</td>
</tr>
</tbody>
</table>

Notes: (1) dependent variable is the speed of diffusion of 26 technologies, (2) all regressions include technology specific fixed effects, (3) t-statistics in parenthesis, (4) $\hat{y}_t$ denotes the cycle of GDP per capita in the country and represents the high and medium term components of output fluctuations, (5) $\hat{y}_t \times$ US is the medium term cycle of GDP per capita times a US dummy, (6) lag represents the years since the technology first started to diffuse.

The speed of diffusion and the cycle. Diffusion speed was lowest in the deep 1981-82 recession; it recovered during the 80s and declined again after the 1990 recession. It increased notably during the expansion in the second half of the 90s and declined again with the 2001 recession.

Next, we turn our attention to study the evolution of the speed of technology diffusion during the Great Recession. Due to its recent nature, the evidence we have is more anecdotal. Eurostat provides information on the diffusion of three relevant internet-related technologies in the UK. Figure 4 plots their average diffusion from 2004 until 2013 with the business cycle downturns in the UK. The figure confirms the pro-cyclicality of the speed of diffusion of these technologies. In particular, during the downturn corresponding to the Great Recession (2008-2009), the average speed of diffusion of our three technologies sharply declined by 75%. After the Great Recession, the speed of diffusion recovered and converged.

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6 The measures we consider are the fraction of firms that (i) have access to broadband internet, that (ii) actively purchase online products and services and that (iii) actively sell online products and services (actively is defined as constituting at least 1% of sales/purchases). For each of these three measures we construct the speed of technology diffusion using expression (2), and then filter the effect of the lag since the introduction of the technology using expression (3) and the estimates from column 3 of Table 1. The resulting series are demeaned so that they can be interpreted as percent deviations from the average speed of technology diffusion.
to approximately 8% below average. Beyond its cyclicality, the second observation we want to stress from the Figure is that fluctuations in the speed of diffusion are very wide, ranging from 86% above average in 2004 to 74% below the average diffusion speed in 2009.

Finally, Andrews et al. (2015) have recently provided complementary evidence that technology diffusion in OECD countries may have slowed during the Great Recession. In their study, they show that the gap in productivity between the most productive firms in a sector (leaders) and the rest (followers) has increased significantly during the Great Recession.\(^7\) Andrews et al. (2015) show that the most productive firms have much greater stocks of patents which suggests that they engage in more R&D activity. They interpret the increase in the productivity gap as evidence that followers have slowed down the rate at which they incorporate frontier technologies developed by the leaders.

These co-movement patterns between the business cycle and measures of investments in technology development as well as measures of the rate of technology adoption is, in our view, sufficiently suggestive evidence to motivate the quantitative exploration we conduct through the lens of our model.

\(^7\)In manufacturing the productivity gap increased by 12% from 2007 and 2009, and in services by approximately 20%.
Figure 4: Diffusion Speed for 3 Internet Technologies in the UK, 2004-2013

Source: Eurostat; see footnote 6 for details of calculations. Shaded areas are UK recession dates as dated by UK ONS.


3 Model

Our starting point is a New Keynesian DSGE model similar to Christiano et al. (2005) and Smets and Wouters (2007). We include the standard features useful for capturing the data, including: habit formation in consumption, flow investment adjustment costs, variable capital utilization and “Calvo” price and wage rigidities. In addition, monetary policy obeys a Taylor rule with a binding zero lower bound constraint.

The key non-standard feature is that total factor productivity depends on two endogenous variables: the creation of new technologies via R&D and the speed of adoption of these new technologies. Skilled labor is used as an input for the R&D and adoption processes. We do not model financial frictions explicitly; however, we allow for a shock that transmits through the economy like a financial shock, as we discuss below.

We begin with the non-standard features of the model before briefly describing the standard ones:

3.1 Production Sector and Endogenous TFP: Preliminaries

In this section we describe the production sector and sketch how endogenous productivity enters the model. In a subsequent section we present the firm optimization problems.

There are two types of firms: (i) final goods producers and (ii) intermediate goods producers. There are a continuum, measure unity, of monopolistically competitive final goods producers. Each final goods firm \(i\) produces a differentiated output \(Y^i_t\). A final good composite is then the following CES aggregate of the differentiated final goods:

\[
Y_t = \left( \int_0^1 (Y^i_t)^{\frac{1}{\mu_t}} \, di \right)^{\mu_t}
\]

where \(\mu_t > 1\) is given exogenously.

Each final good firm \(i\) uses \(Y^i_{mt}\) units of intermediate goods composite as input to produce output, according to the following simple linear technology

\[
Y^i_t = Y^i_{mt}
\]

We assume each firm sets its nominal price \(P^i_t\) on a staggered basis, as we describe later.

There exists a continuum of measure \(A_t\) of monopolistically competitive intermediate goods firms that each make a differentiated product. The endogenous predetermined variable \(A_t\) is the stock of types of intermediate goods adopted in production, i.e., the stock of adopted technologies. Intermediate goods firm \(j\) produces output \(Y^j_{mt}\). The intermediate
goods composite is the following CES aggregate of individual intermediate goods:

\[ Y_{mt} = \left( \int_0^A \left( Y_{mt}^j \right)^{\frac{1}{\vartheta}} dj \right)^{\vartheta} \]  

(6)

with \( \vartheta > 1 \).

Let \( K_{jt} \) be the stock of capital firm \( j \) employs, \( U_{jt} \) be how intensely this capital is used, and \( L_{jt} \) the stock of labor employed. Then firm \( j \) uses capital services \( U_{jt} K_{jt} \) and unskilled labor \( L_{jt} \) as inputs to produce output \( Y_{mt}^j \) according to the following Cobb-Douglas technology:

\[ Y_{mt}^j = \theta_t \left( U_{jt} K_{jt} \right)^{\alpha} \left( L_{jt} \right)^{1-\alpha} \]  

(7)

where \( \theta_t \) is an exogenous random disturbance. As we will make clear shortly, \( \theta_t \) is the exogenous component of total factor productivity. Finally, we suppose that intermediate goods firms set prices each period. That is, intermediate goods prices are perfectly flexible, in contrast to final good prices.

Let \( \bar{Y}_t \) be average output across final goods producers. Then the production function (4) implies the following expression for the final good composite \( Y_t \)

\[ Y_t = \Omega_t \cdot \bar{Y}_t \]  

(8)

where \( \Omega_t \) is the following measure of output dispersion

\[ \Omega_t = \left( \int_0^1 \left( Y_t^i / \bar{Y}_t \right)^{\frac{1}{\mu_t}} di \right)^{\mu_t} \]  

(9)

In a first order approximation, \( \Omega_t \) equals unity, implying that we can express \( Y_t \) simply as \( \bar{Y}_t \).

Next, given the total number of final goods firms is unity, given the production function for each final goods producer (5), and given that \( Y_t \) equals \( \bar{Y}_t \), it follows that to a first order

\[ Y_t = Y_{mt} \]  

(10)

Finally, given a symmetric equilibrium for intermediate goods (recall prices are flexible in this sector) it follows from equation (6) that we can express the aggregate production
function for the finally good composite $Y_t$ as

$$Y_t = \left[ A_t^{\theta-1} \theta_t \right] \cdot (U_t K_t)^\alpha (L_t)^{1-\alpha}$$  \hspace{1cm} (11)

where the term in brackets is total factor productivity, which is the product of a term that reflects endogenous variation, $A_t^{\theta-1}$, and one that reflects exogenous variation $\theta_t$. Note that equation (11) holds to a first order since we impose $\Omega_t$ equals unity.

In sum, endogenous productivity effects enter through the expansion in the variety of adopted intermediate goods, measured by $A_t$. We next describe the mechanisms through which new intermediate goods are created and adopted.

### 3.2 R&D and Adoption

The processes for creating and adopting new technologies are based on Comin and Gertler (2006). Let $Z_t$ denote the stock of technologies, while as before $A_t$ is the stock of adopted technologies (intermediate goods). In turn, the difference $Z_t - A_t$ is the stock of unadopted technologies. R&D expenditures increase $Z_t$ while adoption expenditure increase $A_t$. We distinguish between creation and adoption because we wish to allow for realistic lags in the adoption of new technologies. We first characterize the R&D process and then turn to adoption.

#### 3.2.1 R&D: Creation of $Z_t$

There are a continuum of measure unity of innovators that use skilled labor to create new intermediate goods. Let $L_{p srt}$ be skilled labor employed in R&D by innovator $p$ and let $\phi_t$ be the number of new technologies at time $t + 1$ that each unit of skilled labor at $t$ can create. We assume $\phi_t$ is given by

$$\phi_t = \chi_t Z_t L_{srt}^{\rho_z-1}$$  \hspace{1cm} (12)

where $\chi_t$ is an exogenous disturbance to the R&D technology and $L_{srt}$ is the aggregate amount of skilled labor working on R&D, which an individual innovator takes as given. Following Romer (1990), the presence of $Z_t$, which the innovator also takes as given, reflects public learning-by-doing in the R&D process. We assume $\rho_z < 1$ which implies that increased R&D in the aggregate reduces the efficiency of R&D at the individual level. We introduce this congestion externality so that we can have constant returns to scale in the creation of new technologies at the individual innovator level, which simplifies aggregation.
but diminishing returns at the aggregate level. Our assumption of diminishing returns is consistent with the empirical evidence (see Griliches (1990)); further, with our specification the elasticity of creation of new technologies with respect to R&D becomes a parameter we can estimate, as we make clear shortly.\footnote{An added benefit from having diminishing returns to R&D spending is that, given our parameter estimates, steady state growth is relatively insensitive to tax policies that might affect incentives for R&D. Given the weak link between tax rates and long run growth, this feature is desirable.}

Let $J_t$ be the value of an unadopted technology, $\Lambda_{t,t+1}$ the representative household’s stochastic discount factor and $w_{st}$ the real wage for a unit of skilled labor. We can then express innovator $p$’s decision problem as choosing $L_{srt}^p$ to solve

$$\max_{L_{srt}^p} E_t\{\Lambda_{t,t+1}J_{t+1}\varphi_t L_{srt}^p\} - w_{st}L_{srt}^p$$  \hspace{1cm} (13)

The optimality condition for R&D is then given by

$$E_t\{\Lambda_{t,t+1}J_{t+1}\varphi_t\} - w_{st} = 0$$

which implies

$$E_t\{\Lambda_{t,t+1}J_{t+1}\chi_t Z_t L_{srt}^{\rho_z-1}\} = w_{st}$$  \hspace{1cm} (14)

The left side of equation (14) is the discounted marginal benefit from an additional unit of skilled labor, while the right side is the marginal cost.

Given that profits from intermediate goods are pro-cyclical, the value of an unadopted technology, which depends on expected future profits, will be also be pro-cyclical. This consideration, in conjunction with some stickiness in the wages of skilled labor which we introduce later, will give rise to pro-cyclical movements in $L_{srt}$.\footnote{Other approaches to motivating procyclical R&D, include introducing financial frictions Aghion et al. (2010), short term biases of innovators Barlevy (2007), or capital services in the R&D technology function Comin and Gertler (2006).}

Finally, we allow for obsolescence of technologies.\footnote{We introduce obsolescence to permit the steady state share of spending on R&D to match the data.} Let $\phi$ be the survival rate for any given technology. Then, we can express the evolution of technologies as:

$$Z_{t+1} = \varphi_t L_{srt} + \phi Z_t$$  \hspace{1cm} (15)

where the term $\varphi_t L_{srt}$ reflects the creation of new technologies. Combining equations (15)
and (12) yields the following expression for the growth of new technologies:

$$\frac{Z_{t+1}}{Z_t} = \chi_t L_{sat}^{\rho_z} + \phi$$

(16)

where $\rho_z$ is the elasticity of the growth rate of technologies with respect to R&D, a parameter that we estimate.

### 3.2.2 Adoption: From $Z_t$ to $A_t$

We next describe how newly created intermediate goods are adopted, i.e. the process of converting $Z_t$ to $A_t$. Here we capture the fact that technology adoption takes time on average, but the adoption rate can vary pro-cyclically, consistent with evidence in Comin (2009). In addition, we would like to characterize the diffusion process in a way that minimizes the complications from aggregation. In particular, we would like to avoid having to keep track, for every available technology, of the fraction of firms that have and have not adopted it.

Accordingly, we proceed as follows. We suppose there are a competitive group of “adopters” who convert unadopted technologies into ones that can be used in production. They buy the rights to the technology from the innovator, at the competitive price $J_t$, which is the value of an adopted technology. They then convert the technology into use by employing skilled labor as input. This process takes time on average, and the conversion rate may vary endogenously.

In particular, the pace of adoption depends positively on the level of adoption expenditures in the following simple way: an adopter succeeds in making a product usable in any given period with probability $\lambda_t$, which is an increasing and concave function of the amount of skilled labor employed, $L_{sat}$:

$$\lambda_t = \lambda(Z_t L_{sat})$$

(17)

with $\lambda' > 0$, $\lambda'' < 0$.\(^{11}\) We augment $L_{sat}$ by a spillover effect from the total stock of technologies $Z_t$ - think of the adoption process as becoming more efficient as the technological state of the economy improves. The practical need for this spillover is that it ensures a balanced growth path: as technologies grow, the number of new goods requiring adoption increases, but the supply of labor remains unchanged. Hence, the adoption process must become more efficient as the number of technologies expands.

\(^{11}\)In the estimation, we assume that

$$\lambda(\bullet) = \tilde{\lambda} * (\bullet)^{\rho\lambda}.$$
Our adoption process implies that technology diffusion takes time on average, consistent with the evidence. If $\lambda$ is the steady state value of $\lambda_t$, then the average time it takes for a new technology be adopted is $1/\lambda$. Away from the steady state, the pace of adoption will vary with skilled input $L_{sat}$. We turn next to how $L_{sat}$ is determined.

Once in usable form, the adopter sells the rights to the technology to a monopolistically competitive intermediate goods producer that makes the new product using the production function described by equation (11). Let $\Pi_{mt}$ be the profits that the intermediate goods firm makes from producing the good, which arise from monopolistically competitive pricing. The adopter sells the new technology at the competitive price $V_t$, which is the present discounted value of profits from producing the good, given by

$$V_t = \Pi_{mt} + \phi E_t\{\Lambda_{t,t+1}V_{t+1}\}$$

(18)

Then we may express the adopter’s maximization problem as choosing $L_{sat}$ to maximize the value $J_t$ of an unadopted technology, given by

$$J_t = \max_{L_{sat}} E_t\{-w_{st}L_{sat} + \phi \Lambda_{t,t+1}[\lambda_t V_{t+1} + (1 - \lambda_t)J_{t+1}]\}$$

(19)

subject to equation (17). The first term in the Bellman equation reflects total adoption expenditures, while the second is the discounted benefit: the probability weighted sum of the values of adopted and unadopted technologies.

The first order condition for $L_{sat}$ is

$$Z_t \lambda_t \cdot \phi E_t\{\Lambda_{t,t+1}[V_{t+1} - J_{t+1}]\} = w_{st}$$

(20)

The term on the left is the marginal gain from adoption expenditures: the increase in the adoption probability $\lambda_t$ times the discounted difference between an adopted versus unadopted technology. The right side is the marginal cost.

The term $V_t - J_t$ is pro-cyclical, given the greater influence of near term profits on the value of adopted technologies relative to unadopted ones. Given this consideration and the stickiness in $w_{st}$ which we alluded to earlier, $L_{sat}$ varies pro-cyclically. The net implication is that the pace of adoption, given by $\lambda_t$, will also vary pro-cyclically.

Given that $\lambda_t$ does not depend on adopter-specific characteristics, we can sum across adopters to obtain the following relation for the evolution of adopted technologies

$$A_{t+1} = \lambda_t \phi [Z_t - A_t] + \phi A_t$$

(21)
where \( Z_t - A_t \) is the stock of unadopted technologies.

### 3.3 Households

The representative household consumes and saves in the form of capital and riskless bonds which are in zero net supply. It rents capital to intermediate goods firms. As in the standard DSGE model, there is habit formation in consumption. Also as is standard in DSGE models with wage rigidity, the household is a monopolistically competitive supplier of differentiated types of labor.

The household’s problem differs from the standard setup in two ways. First it supplies two types of labor: unskilled labor \( L^h_t \) which is used in the production of intermediate goods and skilled labor which is used either for R&D or adoption, \( L^h_{st} \).

Second, we suppose that the household has a preference for the safe asset, which we motivate loosely as a preference for liquidity and capture by incorporating bonds in the utility function, following Krishnamurthy and Vissing-Jorgensen (2012). Further, following Fisher (2015), we introduce a shock to liquidity demand \( \varrho_t > 0 \). As we show, the liquidity demand shock transmits through the economy like a financial shock. It is mainly for this reason that we make use of it, as opposed to a shock to the discount factor.\(^{12}\)

Let \( C_t \) be consumption, \( B_t \) holdings of the riskless bond, \( \Pi_t \) profits from ownership of monopolistically competitive firms, \( K_t \) capital, \( Q_t \) the price of capital, \( R_{kt} \) the rate of return, and \( D_t \) the rental rate of capital. Then the households’ decision problem is given by

\[
\max_{C_t, B_{t+1}, L^h_t, L^h_{st}, K_{t+1}} E_t \sum_{\tau=0}^{\infty} \beta^\tau \left\{ \log(C_{t+\tau} - bC_{t+\tau-1}) + \varrho_t B_{t+1} - \left[ \frac{\nu(L^h_t)^{1+\varphi} + \nu_s(L^h_{st})^{1+\varphi}}{1 + \varphi} \right] \right\}
\]

subject to

\[
C_t = w^h_t L^h_t + w^h_{st} L^h_{st} + \Pi_t + R_{kt} Q_{t-1} K_t - Q_t K_{t+1} + R_t B_t - B_{t+1}
\]

with

\[
R_{kt} = \frac{D_t + Q_t}{Q_{t-1}}
\]

\( \Lambda_{t,t+1} \), the household’s stochastic discount factor, is given by

\[
\Lambda_{t,t+1} = \beta u'(C_{t+1})/u'(C_t)
\]

\(^{12}\)Another consideration is that the liquidity demand shock induces positive co-movement between consumption and investment, while that is not always the case for a discount factor shock.
where \( u'(C_t) = 1/(C_t - bC_{t-1}) - b/(C_{t+1} - bC_t) \). In addition, let \( \zeta_t \) be the liquidity preference shock in units of the consumption good:

\[
\zeta_t = \varrho_t / u'(C_t)
\]

Then we can express the first order necessary conditions for capital and the riskless bond as, respectively:

\[
1 = E_t\{\Lambda_{t,t+1}R_{kt+1}\}
\]

\[
1 = E_t\{\Lambda_{t,t+1}R_{t+1}\} + \zeta_t
\]

As equation (28) indicates, the liquidity demand shock distorts the first order condition for the riskless bond. A rise in \( \zeta_t \) acts like an increase in risk: given the riskless rate \( R_{t+1} \) the increase in \( \zeta_t \) induces a precautionary saving effect, as households reduce current consumption in order to satisfy the first order condition (which requires a drop in \( \Lambda_{t,t+1} \)). It also leads to a drop in investment demand, as the decline in \( \Lambda_{t,t+1} \) raises the required return on capital, as equation (27) implies. The decline in the discount factor also induces a drop in R&D and investment.

Overall, the shock to \( \zeta_t \) generates positive co-movement between consumption and investment similar to that arising from a monetary shock. To see, combine equations (27) and (28) to obtain

\[
E_t\{\Lambda_{t,t+1}(R_{kt+1} - R_{t+1})\} = \zeta_t
\]

To a first order an increase in \( \zeta_t \) has an effect on both \( R_{kt+1} \) and \( \Lambda_{t,t+1} \) that is qualitatively similar to that arising from an increase in \( R_{t+1} \). In addition, note that an increase in \( \zeta_t \) raises the spread \( R_{kt+1} - R_{t+1} \). In this respect it transmits through the economy like a financial shock. Indeed, we show later that our identified liquidity demand shock is highly correlated with credit spreads.

Since it is fairly conventional, we defer until later a description of the household’s wage-setting and labor supply behavior.

### 3.4 Firms

#### 3.4.1 Intermediate goods firms: factor demands

Given the CES function for the intermediate good composite (6), in the symmetric equilibrium each of the monopolistically competitive intermediate goods firms charges the markup
\( \vartheta \). Let \( p_{mt} \) be the relative price of the intermediate goods composite. Then from (6) and the production function (7), cost minimization by each intermediate goods producer yields the following standard first order conditions for capital, capital utilization, and unskilled labor:

\[
\alpha \frac{p_{mt} Y_{mt}}{K_t} = \vartheta [D_t + \delta(U_t)Q_t] \quad (30)
\]

\[
\alpha \frac{p_{mt} Y_{mt}}{U_t} = \vartheta \delta'(U)Q_t K_t \quad (31)
\]

\[
(1 - \alpha) \frac{p_{mt} Y_{mt}}{L_t} = \vartheta w_t \quad (32)
\]

### 3.4.2 Final goods producers: price setting

Let \( P^i_t \) be the nominal price of final good \( i \) and \( P_t \) the nominal price level. Given the CES relation for the final good composite, equation (4), the demand curve facing each final good producer is:

\[
Y^i_t = \left( \frac{P^i_t}{P_t} \right)^{-\mu_t/(\mu_t - 1)} Y_t \quad (33)
\]

where the price index is given by:

\[
P_t = \left( \int_0^1 (P^i_t)^{-1/(\mu_t - 1)} \, di \right)^{-(\mu_t - 1)}, \quad (34)
\]

Following Smets and Wouters (2007), we assume Calvo pricing with flexible indexing. Let \( 1 - \xi_p \) be the i.i.d probability that a firm is able to re-optimize its price and let \( \pi_t = P_t / P_{t-1} \) be the inflation rate. Firms that are unable to re-optimize during the period adjust their price according to the following indexing rule:

\[
P^i_t = P^i_{t-1} \pi_{t-1}^{1-\xi_p} \pi^{1-\xi_p} \quad (35)
\]

where \( \pi \) is the steady state inflation rate and \( t_p \) reflects the degree of indexing to lagged inflation.

For firms able to re-optimize, the optimization problem is to choose a new reset price \( P^*_t \) to maximize expected discounted profits until the next re-optimization, given by

\[
E_t \sum_{\tau=0}^{\infty} \xi^{\tau}_p \Lambda_{t, t+\tau} \left( \frac{P^*_t \Gamma_{t, t+\tau}}{P_{t+\tau}} - p_{mt+\tau} \right) Y^i_{t+\tau} \quad (36)
\]
subject to the demand function (33) and where

$$\Gamma_{t,t+\tau} \equiv \prod_{k=1}^{\tau} \pi_{t+k-1}^{i_p} \pi_{t-1}^{1-i_p}$$  \hspace{1cm} (37)$$

The first order condition for \(P_t^*\) and the price index that relates \(P_t\) to \(P_t^*, P_{t-1}\) and \(\pi_{t-1}\) are then respectively:

$$0 = E_t \sum_{\tau=0}^{\infty} \xi_p \Lambda_{t,t+\tau} \left[ \frac{P_t^* \Gamma_{t,t+\tau}}{P_{t+\tau}} - \mu_{t+\tau} p_{mt+\tau} \right] Y_{t+\tau}$$ \hspace{1cm} (38)

$$P_t = \left[ (1 - \xi_p) (P_t^*)^{-1/\mu_t-1} + \xi_p (\pi_{t-1}^{i_p} \pi_{t-1}^{1-i_p} P_{t-1})^{-1/\mu_t-1} \right]^{-(\mu_t-1)}$$ \hspace{1cm} (39)

Equations (38) and (39) jointly determine inflation. In the loglinear equilibrium, current inflation is a function of current real marginal cost \(p_{mt}\), expected future inflation, and lagged inflation.

3.4.3 Capital producers: investment

Competitive capital producers use final output to make new capital goods, which they sell to households, who in turn rent the capital to firms. Let \(I_t\) be new capital produced and \(p_{kt}\) the relative price of converting a unit of investment expenditures into new capital (the replacement price of capital), and \(\gamma_y\) the steady state growth in \(I_t\). In addition, following Christiano et al. (2005), we assume flow adjustment costs of investment. The capital producers’ decision problem is to choose \(I_t\) to solve

$$\max_{I_t} E_t \sum_{\tau=0}^{\infty} \Lambda_{t,t+\tau} \left\{ Q_{t+\tau} I_{t+\tau} - p_{kt+\tau} \left[ 1 + f \left( \frac{I_{t+\tau}}{(1+\gamma_y)I_{t+\tau-1}} \right) \right] I_{t+\tau} \right\}$$ \hspace{1cm} (40)

where the adjustment cost function is increasing and concave, with \(f(1) = f'(1) = 0\) and \(f''(1) > 0\). We assume that \(p_{kt}\) follows an exogenous stochastic process.

The first order condition for \(I_t\) relates the ratio of the market value of capital to the replacement price (i.e. “Tobin’s Q”) to investment, as follows:

$$\frac{Q_t}{p_{kt}} = 1 + f \left( \frac{I_t}{(1+\gamma_y)I_{t-1}} \right) + \frac{I_t}{(1+\gamma_y)I_{t-1}} f' \left( \frac{I_t}{(1+\gamma_y)I_{t-1}} \right) - E_t \Lambda_{t,t+1} \left( \frac{I_{t+1}}{(1+\gamma_y)I_t} \right)^2 f' \left( \frac{I_{t+1}}{(1+\gamma_y)I_t} \right)$$ \hspace{1cm} (41)
3.4.4 Employment agencies and wage adjustment

As we noted earlier, the household is a monopolistically competitive supplier of labor. Think of the household as supplying its labor to form a labor composite. Firms then hire the labor composite. The only difference from the standard DSGE model with wage rigidity, is that households now supply two types of labor, skilled and unskilled. It also sets wages for each type.

Let \( X_t = \{L_t, L_{st}\} \) denote a labor composite. As is standard, we assume that \( X_t \) is the following CES aggregate of the differentiated types of labor that households provide:

\[
X_t = \left[ \int_0^1 X_t^h \frac{1}{\mu_{wt}} \, dh \right]^{\mu_{wt}}. \tag{42}
\]

where \( \mu_{wt} > 1 \) obeys an exogenous stochastic process.\(^{13}\)

Let \( W_{xt} \) denote the wage of the labor composite and let \( W_{xt}^h \) be the nominal wage for labor supplied of type \( x \) by household \( h \). Then profit maximization by competitive employment agencies yields the following demand for type \( x \) labor:

\[
X_t^h = \left( \frac{W_{xt}^h}{W_{xt}} \right)^{-(\mu_{wt} - 1)/(\mu_{wt} - 1)} X_t, \tag{43}
\]

with

\[
W_{xt} = \left[ \int_0^1 W_{xt}^h \frac{1}{\mu_{wt} - \tau} \, dh \right]^{-(\mu_{wt} - 1)}. \tag{44}
\]

As with price setting by final goods firms, we assume that households engage in Calvo wage setting with indexation. Each period a fraction \( 1 - \xi_w \) of households re-optimize their wage for each type. Households who are not able to re-optimize adjust the wage for each labor type according to the following indexing rule:

\[
W_{xt}^h = W_{xt}^h \pi_{t-1} \pi^{1-\epsilon_w} \gamma. \tag{45}
\]

where \( \gamma \) is the steady state growth rate of labor productivity.

The remaining fraction of households choose an optimal reset wage \( W_{xt}^* \) by maximizing

\[
E_t \left\{ \sum_{\tau=0}^{\infty} \xi_{wt}^\beta \beta^\phi \left[ -\frac{X_t^{h_{t+\tau}} + \mu_{wt}^{t+\tau} X_t^{h_{t+\tau}}}{1 + \phi} + u'(C_{t+\tau}) \frac{W_{xt}^* \Gamma_{wt,t+\tau} X_t^{h_{t+\tau}}}{P_{t+\tau}} \right] \right\} \tag{46}
\]

\(^{13}\)In estimating the model we introduce wage markup shocks to the wage setting problem of unskilled labor only, so the markup for skilled labor is constant at its steady state level.
subject to the demand for type $h$ labor and where the indexing factor $\Gamma_{xt,t+\tau}$ is given by

$$\Gamma_{wt,t+\tau} \equiv \prod_{k=1}^{\tau} \pi_{t+k-1}^{1-\mu w} \gamma$$

The first order condition for the re-set wage and the equation for the composite wage index as a function of the reset wage, inflation and the lagged wage are given, respectively, by

$$E_t \left\{ \sum_{\tau=0}^{\infty} \xi^\tau \Lambda_{t,\tau} \left[ \frac{W^*_x \Gamma_{xt,t+\tau}}{P_t} - \mu_{wt} \frac{X^h_{t+\tau}}{u'(C_{t+\tau})} \right] X^h_{t+\tau} \right\} = 0$$

$$W_{xt} = \left(1 - \xi_w \right) \left( W^*_x \right)^{-1/\left(\mu_{wt}-1\right)} + \xi_p \left( \gamma \pi_{t-1}^{1-\mu w} W_{xt-1} \right)^{-1/\left(\mu_{wt}-1\right)} - \left(\mu_{wt}-1\right)^{-(\mu_{wt}-1)}$$

### 3.4.5 Fiscal and monetary policy

We assume that government consumption $G_t$ is financed by lump sum taxes $T_t$.

$$G_t = T_t$$

Further, the (log) deviation of $G_t$ from the deterministic trend of the economy follows an AR(1) process. Formally,

$$\log\left(\frac{G_t}{1 + \gamma y}\right) = (1 - \rho_g) \bar{g} + \rho_g \log\left(\frac{G_{t-1}}{1 + \gamma y}\right) + \epsilon^g_t, \quad (51)$$

Next, we suppose that monetary policy obeys a Taylor rule. Let $R_{nt+1}$ denote the gross nominal interest rate, $R_n$ the steady state nominal rate, $\pi^0$ the target rate of inflation, $L_t$ total employment and $L^{ss}$ steady state employment. The (nonlinear) Taylor rule for monetary policy that we consider is given by

$$R_{nt+1} = \left[ \frac{\pi_t}{\pi^0} \right]^{\phi\pi} \left( \frac{L_t}{L^{ss}} \right)^{\phi_y} R_n \right]^{1-\rho} \cdot R^n_{nt}$$

where the relation between the nominal and real rate is given by the Fisher relation:

$$R_{nt+1} = R_{t+1} \cdot \pi_{t+1}$$

and where $\phi\pi$ and $\phi_y$ are the feedback coefficients on the inflation gap and capacity util-
lization gap respectively. We use the employment gap to measure capacity utilization as opposed to an output gap for two reasons. First, Berger et al. (2015) show that measures of employment are the strongest predictors of changes in the Fed Funds rate. Second, along these lines, the estimates of the Taylor rule with the employment gap appear to deliver a more reasonable response of the nominal rate to real activity within this model than does one with an output gap.\(^\text{14}\)

In addition, we impose the zero lower bound constraint on the net nominal interest rate, which implies that the gross nominal rate cannot fall below unity.

\[ R_{nt+1} \geq 1 \]  \hfill (54)

### 3.5 Resource constraints and equilibrium

The resource constraint is given by

\[ Y_t = C_t + p kt \left[ 1 + f \left( \frac{I_{t+\tau}}{(1 + \gamma_y)I_{t+\tau-1}} \right) \right] I_t + G_t \]  \hfill (55)

Capital evolves according to

\[ K_{t+1} = I_t + (1 - \delta(U_t))K_t \]  \hfill (56)

The market for skilled labor must clear:

\[ L_{st} = L_{sat} + L_{srt} \]  \hfill (57)

Finally, the market for risk-free bonds must clear, which implies that in equilibrium, risk-free bonds are in zero net supply

\[ B_t = 0 \]

This completes the description of the model.

### 4 Estimation

We estimate our model using Bayesian methods (see for example An and Schorfheide (2007)). As is common practice in the literature (for example Smets and Wouters (2007))

\(^{14}\)Part of the problem may be that the behavior of the flexible price equilibrium output is quite complex in the model, particularly given the endogenous growth sector. As a robustness check on our specification of the Taylor rule, we estimate a version of the model in which we adjust the employment gap for demographic effects on the size of the labor force; our estimation results are robust to this change.
and Justiniano et al. (2010)), we calibrate a subset of the parameters of the model and estimate the remainder.

We estimate using quarterly data from 1984:I to 2008:III on eight US series: real output, consumption, investment, hours worked, real wages, inflation (as measured by the GDP deflator), the nominal risk-free interest rate and expenditures on R&D by US corporations. Unlike the other series, R&D expenditures are annual. We deal with the mixed frequency of the data in estimation using a version of the Kalman filter adapted for this purpose. Appendix A.1 describes the data in detail.

We do not use data beyond 2008:III in the estimation of the structural parameters because the zero lower bound (ZLB) on the nominal interest was binding after that period, rendering estimation using a log-linear approximation our baseline model problematic. However we do use the data from 2008:III to 2013:IV to identify shocks and other latent variables of our model, including the endogenous component of TFP. We do so by modifying the standard log-linear approximation of the model with the technique introduced by Guerrieri and Iacoviello (2015) to deal with occasionally binding constraints, as described in Appendix A.2.

We next discuss the calibrated parameters and then proceed to describe the prior and posterior estimates of the remaining parameters.

4.1 Calibrated parameters

We calibrate standard real business cycle model parameters (i.e., the rates of time preference and capital depreciation, and the capital share); the steady state share of government spending in output; the trend growth rate; steady state markups for intermediate and final goods and for wages ($\vartheta$, $\mu$ and $\mu_w$ respectively); and three of the four endogenous technological change parameters.

Of the four endogenous technological change parameters, we calibrate the expenditure elasticity of the adoption probability, $\rho_\lambda$, the obsolescence rate $(1 - \phi)$ and the steady state adoption lag $\bar{\lambda}$. The elasticity of $\lambda$ with respect to adoption expenditures, $\rho_\lambda$ is set to 0.95 to induce a ratio of private R&D to GDP consistent with the U.S. post-1970 experience (of approximately 2% of GDP). $\bar{\lambda}$ is set to produce an average adoption lag of 7 years which is consistent with the estimates in Comin and Hobijn (2010) and Cox and Alm (1996). Finally, the obsolescence rate $(1 - \phi)$ is set to 8% which falls in the middle of the broad range of estimates for the obsolescence rate in the literature (see Caballero and Jaffe (1993) and Pakes and Schankerman (1984) for the two extremes).

Table 2 presents the calibrated parameters and their values.
4.2 Parameter estimates

Tables 3 and 4 below present the prior and posterior distributions for the parameters that we estimate. For the conventional parameters we use similar priors to the literature (e.g. Justiniano et al. (2010)). For the new parameter we estimate, the elasticity of R&D parameter ($\rho_z$) we use a fairly loose beta prior centered around a mean of 0.6, which is at the lower end of estimates provided in Griliches (1990).

Most of our estimates are similar to those in the literature. The price and wage rigidity parameters, though, are on the high side, likely reflecting that inflation was low and stable over our sample. Our estimate of the elasticity of new technologies with respect to R&D, $\rho_z$, is 0.34, which is somewhat below the Griliches (1990) estimates. This discrepancy, however, may reflect the fact that (effectively) we use quarterly data while the literature uses annual data: one would expect greater diminishing returns to R&D at higher frequencies due to frictions in adjusting skilled labor input.

Finally, with respect to the shocks, we find lower estimates of the persistence of exogenous TFP than in the literature. This reflects the fact that our model produces significant endogenous persistence in TFP.

4.3 Shocks and Volatilities

In this section we establish the key sources of cyclical variation not only in output and hours but also in endogenous productivity. First, as a check of the fit of the estimated model, Table 5 presents the theoretical standard deviations of the observable variables generated by the model and compares them with the data in our sample. Roughly speaking the model is in line with the actual volatilities of the key variables.

We next ascertain the relative importance of each shock by calculating a set of variance decompositions. To do so we simulate the model a large number of periods taking into account the ZLB as described in Appendix A.2. Table 6 presents the results. There are several important takeaways. First, “demand” shocks are important for both output and employment volatility and for the cyclicality of endogenous productivity. The liquidity demand shock explains 29 percent of output growth, 40 percent of hours and 17 percent of endogenous productivity. The liquidity demand and money shocks combined account for more than half the volatility of output and hours and nearly 30 percent of the endogenous production variation. The other important shock is the exogenous component of total factor productivity, which accounts for 33 percent of output variation, 16 percent of hours variation, and 40 percent of the variation in endogenous TFP.
The second important takeaway is that the main way the key cyclical shocks affect endogenous productivity is through variation in technology adoption, as opposed to creation of new technologies. The liquidity demand shock, for example, explains 29 percent of the variation in the speed of technology diffusion, the money shock 14 percent, and the TFP shock 37 percent shock. By contrast, variation in new technologies is driven almost entirely by the shock to R&D productivity.

While the liquidity demand shock is one of the three major disturbances influencing unconditional cyclical volatility, it is by far the most important shock driving recessions. Figure 5 plots the historical evolution of per capita output growth as well as the components that are accounted for by the liquidity demand and the exogenous TFP shock, the disturbance that is second most important in driving recessions\(^{15}\). In each of the three recessions, the liquidity demand shock accounts for most of the decline in output. The TFP shock accounts for a comparable decline only in the 1990-91 recession.

Finally, we present evidence supporting our interpretation of the liquidity demand shock as a financial shock (as noted in section 3.3, a negative liquidity demand shock causes the spread between the return on capital and the riskless rate to widen). Figure 6 compares the spreads implied by our estimated liquidity shocks with the measure of the credit spread calculated by Gilchrist and Zakrajsek (2012).\(^{16}\) The figure shows that the two series are highly correlated (0.69). In particular, both series show increases in spreads around recessions with an absolute peak in the sample around early 2009. Our model series does show greater persistence in the spread after the Great Recession. This difference may reflect the fact that the GZ spread pertains only to publicly traded companies, while non-traded companies continued to face borrowing frictions. Overall, we consider the similarity between our estimated liquidity shocks and the GZ series as suggestive support for our interpretation.

\(^{15}\)The decomposition takes into account the ZLB (as described in Appendix A.2), which makes the model nonlinear for the period 2008:I-2013:IV. Because of this nonlinearity, the sum of the contribution of each shock does not equal the value of the smoothed variable being decomposed (output growth in this case) for the mentioned period. This “nonlinear residual” emerges because the interaction between shocks is relevant in nonlinear models. However, our results indicate that the only shock that moves the economy to the ZLB is the liquidity demand shock. We therefore assign the nonlinear residual to this shock. This comment also applies to Figures 6-12.

\(^{16}\)Gilchrist and Zakrajsek (2012) use Compustat to measure the excess interest rate paid on long-term corporate bonds over the 10 year government bonds. Therefore, their measure reflects the premia faced by publicly-traded companies.
5 Analysis

We begin by analyzing the mechanisms through which the various cyclical shocks in our model affect endogenous productivity. We subsequently use our estimated model to provide a decomposition of the forces driving TFP before, during and after the Great Recession.

5.1 Endogenous Productivity Mechanism

Figure 7 presents the responses of some key variables to a one standard deviation shocks to the three main driving shocks: liquidity demand, money, and exogenous TFP. To isolate the effects of our endogenous productivity mechanism, in each case we plot the responses of our model and a version where technology is purely exogenous.

The first column of the figure shows the effect of a liquidity demand shock. An increase in the demand for the liquid asset, everything else equal induces households to reduce their consumption demand and their saving in risky assets (See equations 27-29). As a result there is upward pressure on the required return to capital, $R_{kt+1}$, and downward pressure on the safe real rate $R_{t+1}$. The former leads to a fall in both physical investment demand as well as in the demand for productivity enhancing investments, including both $R&D$ and adoption expenditures. The latter lessens the drop in consumption. Given nominal rigidities, the overall drop in both investment and consumption demand leads to a decline in output. The drop in productivity enhancing investments, further, induce a decline in productivity, magnifying both the overall size and persistence of the output decline.

One additional interesting result is that the endogenous productivity mechanism mutes the decline in inflation following the contractionary demand shock. As in conventional New Keynesian models, inflation declines when aggregate demand falls. However, the endogenous decline in productivity growth lessens the decline in marginal costs, which in turn dampens the decline in inflation, making it almost negligible. This feature, accordingly, can offer at least part of the explanation for the surprising failure of inflation to decline by any significant amount during the Great Recession.

Columns 2 and 3 of Figure 7 depict the impulse responses to money and TFP shocks in our model. The money shock produces responses of the real economy and inflation that are qualitatively similar to the effect of the liquidity demand shock. Both shocks raise the cost of capital. The main difference is that the money shock does so by raising the risk free rate, while the liquidity demand shock does so by increasing the spread between the cost of capital and the risk free rate. The model responses to both the money and TFP shocks are qualitatively similar to the exogenous technology model. The main differences
are that the endogenous technology mechanisms increase the model’s amplification of the shocks and produce greater persistence.

Finally, the main part of our analysis involves analyzing productivity over a period where the ZLB is binding. Our historical decomposition, further, suggests that it is the liquidity demand shock that moves the economy into the ZLB. Accordingly it is useful to understand the implications of the ZLB for how a contractionary liquidity demand shocks influence endogenous movements in productivity. Figure 8 plots the impulse response functions with and without a binding ZLB. When the ZLB is binding, monetary policy cannot accommodate a recessionary shock. This results in higher interest rates than when the ZLB is not binding. The higher real rates amplify the drops in investment, R&D and adoption intensity. In the short term, this leads to lower aggregate demand and a larger output drop. It also leads to larger declines in the growth rate of the number of adopted technologies and to lower levels of TFP in the medium and long term.

5.2 Productivity dynamics

We now explore the model’s implications for the evolution of productivity, with particular emphasis on the periods before, during and after the Great Recession. We focus on TFP but also consider labor productivity. The latter allows us to consider the impact of the demand shortfall during the Great Recession on the supply side that operates via the conventional capital accumulation channel (as emphasized by Hall (2014) and others), as well as our endogenous productivity channel.

To begin, we use equation (11) to derive the following expression that links labor productivity with TFP and capital intensity:

\[ \frac{Y_t}{L_t} = \theta_t \cdot (A_t)^{\theta-1} \cdot \left( \frac{U_t K_t}{L_t} \right)^{\alpha}. \]

The first two terms capture total TFP, which is the product of an exogenous component \( (\theta_t) \) and an endogenous one \( ((A_t)^{\theta-1}) \). The third term measures capital intensity which includes both capital per hours worked and the capital utilization rate.

Figure 9 plots the evolution of (detrended) labor productivity together with TFP and capital intensity:¹⁸,¹⁹

¹⁷ The binding ZLB is achieved with a seven-standard deviation positive shock to the liquidity preference.
¹⁸ This expression holds to a first order approximation.
¹⁹ We focus on labor productivity for two reasons. First, our measure of capital includes residential investment. Therefore, there is a discrepancy between our measure of TFP and that from standard sources (e.g., BLS). Second, labor productivity also captures the effect of variation in capital per hour. This is another channel by which fluctuations in demand can affect the potential supply in the economy.
the endogenous component of TFP. Labor productivity corresponds exactly to the data. The other two series are identified from the model. It is worth noting, though, that the evolution of TFP and labor productivity are qualitatively similar both in the model and in the data.\footnote{One can obtain the capital intensity component of labor productivity from the figure by taking the difference between labor productivity and TFP.}

Except for the middle to late 1990s, the endogenous component of TFP accounts for much of the cyclical variation in TFP. The model attributes the rise in TFP during the late 90s mainly to its exogenous component; the labor productivity surge in this period is explained by both exogenous innovations to TFP and capital deepening.

After 2000, however, the endogenous component drives the overall behavior of TFP. Importantly, the endogenous component explains virtually all of the decline in TFP immediately before the Great Recession, as well as the decline during and after that episode. In particular, between the starting point of the recent productivity slowdown, 2005, and the end of our sample, 2013, total TFP declined by approximately 5 percentage points (relative to trend). The endogenous component accounts for 4.75 percentage points of decline. This factor also accounts for most of the drop in labor productivity, which declined 7 percentage points over the same period. A drop in capital intensity after 2009 mainly accounts extra the drop in labor productivity relative to TFP (consistent with Hall (2014)).

While endogenous TFP declines steadily after 2005, the main sources of the drop vary over time. Figure 10 presents a historical decomposition of endogenous productivity that isolates the effects of the two shocks that were the main causes of the decline: (i) shocks to the productivity of R&D and (ii) the liquidity demand shock. We note first that the liquidity demand shock accounts for nearly all of the decline in endogenous TFP after the start of the recession at the end of 2007. This result is consistent with our earlier findings that: (i) the liquidity demand shock was the main disturbance driving the recession (see Figure 5); and (ii) the liquidity demand shock has a significant impact on endogenous TFP, especially at the ZLB (see Figure 8).

In the period just prior to the Great Recession, 2005-2007, however, the liquidity demand shock is unimportant. Instead the decline in endogenous TFP is mainly the result of negative shocks to the productivity of R&D.\footnote{Shocks to the productivity of R&D are identified by comparing the model implications for the cyclicality of R&D with the data. See equation (14) and Figure 2.} The downward trend in R&D productivity actually begins in the mid 1990s, which is consistent with Gordon (2012)’s hypothesis of a secular decline in the contribution of technological innovations to productivity. After a brief upturn following the 2000-01 recession, shocks to R&D productivity induce a sharp downturn in
TFP from 2005 until the height of the crisis.

Intuitively, the exogenous decline in R&D productivity generated fewer technologies for a given level of R&D spending, which ultimately slowed the pace of new technology adoption. Our finding that shocks to R&D productivity mainly account for the pre-recession slowdown in TFP is consistent with Fernald (2014)’s hypothesis that exogenous medium-term factors, as opposed to cyclical factors were at work. At the same time, our endogenous productivity mechanism allows cyclical shocks as well shocks to R&D productivity to drive TFP. In this regard, our accounting suggests that once the recession began, it was cyclical shocks in the form of liquidity demand shocks that largely drove the subsequent decline in endogenous TFP.

We next explore the relative importance of the specific mechanisms that drive endogenous TFP. From equation (21), fluctuations in the stock of adopted technologies, $A_t$, (and hence endogenous TFP), are driven by the product of two factors: the adoption rate $\lambda_t$ and the total stock of unadopted technologies, $Z_t - A_t$, where the total stock of technologies, $Z_t$, is entirely driven by R&D labor and its productivity, $\chi_t$. To analyze the relevance of these two channels, Figure 11 plots (relative to trend) the evolution of $Z_t, A_t$ and $\lambda_t$ – measured on the right axis. Note that the evolution of $A_t$ mirrors the evolution of endogenous productivity $(A_t^{\vartheta-1})$.

We find that cyclicality in $\lambda_t$ is the main driver of cyclical fluctuations in endogenous productivity: $\lambda_t$ co-moves closely with $A_t$ while $Z_t$ does not. During each of the recessions, $\lambda_t$ declines along with $A_t$, implying that the slowdown in adoption drives the cyclical contraction in endogenous TFP. This finding is consistent with our earlier observation that liquidity demand shocks are important sources of recessions (see Figure 5), as these shocks induce contractions in adoption rates and endogenous productivity (see Figure 8).

Further, the magnitude of fluctuations in $\lambda_t$ implied by our model is consistent with the evidence presented in Section 2. We find that the relative volatility of $\lambda_t$ (vs. output) over our sample period is 4.45, which is a similar magnitude to our estimate of the elasticity of the speed of diffusion with respect to output (of around 4, see Table 1). Additionally, the fall in $\lambda_t$ during the Great Recession implied by the model is plausible in light of the observed fall in adoption speeds for the sample of UK technologies (Figure 4).

Fluctuations in $Z_t$ do play a role in the evolution of endogenous productivity. Following the 2000-01 recession there is a steady decline in $Z_t$, consistent with the negative shocks to R&D productivity over this period that Figure 10 illustrates.\footnote{The decline in endogenous productivity induced by the negative shocks to R&D productivity lags the decline in $Z_t$ (compare figures 10 and 11) due to the lags in the adoption process.} This drop in $Z_t$, in
turn, helps account for the pre-Great Recession drop in productivity that Fernald (2014) emphasizes, complementing the analysis of Figure 10. After the start of the Great Recession however the fall in the adoption rate becomes the main driver of the productivity decline. The failure of the adoption rate to return to normal levels, after a brief recovery in 2010, is the reason endogenous TFP continues to decline.

Interestingly, while $\lambda_t$ remains low following the Great Recession, the stock of unadopted technologies, $Z_t - A_t$, reaches a peak over the sample. The latter occurs mostly because the stock of adopted technologies, $A_t$, declines, but also because there is a modest increase in $Z_t$. This finding is consistent with the evidence presented by Andrews et al. (2015) that suggests that innovation by leading edge firms continued after the Great Recession but adoption by followers slowed. An important implication is that the economy may not be doomed to low productivity growth for the foreseeable future. Given the high stock of unadopted technologies, to the extent that increasing aggregate demand pushes up the adoption rate, productivity growth should increase. Conversely, if the economy continues to stagnate, adoption rates will remain low, keeping productivity growth low.

We conclude our analysis by exploring the effect that demand shocks have on the supply side. To this end, Figure 12 plots the contribution of our two main demand side shocks (i.e., the liquidity and money shocks) to the evolution of labor productivity. In particular, the key demand shocks were important drivers of the acceleration in labor productivity between 1995 and 2001 through their effect on capital deepening. They also contributed to the slowdown in productivity between 2001 and 2007 through both endogenous TFP and capital deepening channels. During the Great Recession, the key demand shocks contributed to the decline in productivity only through endogenous TFP. After the Great Recession, the main demand shocks more than fully account for the decline in labor productivity. Both capital deepening and endogenous technology are significant channels. In sum, we find very strong evidence on the impact that demand shocks has had on the dynamics of productivity over our sample period and very especially over the last fifteen years.

6 Conclusions

We have estimated a monetary DSGE model with endogenous productivity via R&D and adoption. We then used the model to assess the behavior of productivity, with particular emphasis on the slowdown following the onset of the Great Recession. Our key result is that this slowdown mainly reflected an endogenous decline in the speed at which new technologies are incorporated in production. The endogenous decline in adoption, further,
was a product of the recession. We also find that our endogenous productivity mechanism can help account for the productivity slowdown that preceded the Great Recession. Though here, shocks to the productivity of the R&D process play an important role, consistent with Fernald (2014)’s view that acyclical factors were important over this period. Finally, we find a very limited role for an exogenous decline in TFP in the slowdown of productivity. Overall, the results suggest that the productivity slowdown following the start of the Great Recession was not simply bad luck, but rather another unfortunate by-product of the downturn.

Our analysis sheds light on two open debates. First, it provides a time series for the productivity of R&D activities that can be used to explore the hypothesis advanced by Gordon (2012) that the U.S. economy is experiencing a secular deterioration in its innovation capacity. Consistent with Gordon’s hypothesis we find low levels of productivity of R&D activities between 2002 and 2007 that contributed to the decline in TFP between 2005 and 2009. However, this episode is short-lived and the estimates suggest that the slowdown in productivity reflects medium term cyclical factors rather than secular ones. The second relevant debate concerns the stability of inflation during the Great Recession in spite of the very significant decline in economic activity. Our model and estimates suggests that the endogenous decline in TFP has increased production costs (relative to trend) counteracting the traditional Phillips- curve effect of economic contractions on inflation.

Overall, our results emphasize the importance of the effects that demand shocks have on the supply side over the medium term. This is an important take away that can be used to explain productivity dynamics more generally.
A Appendix

A.1 Data

The data used for estimation are available from the FRED (https://research.stlouisfed.org/fred2/) and NSF (http://www.nsf.gov/statistics/) websites. Descriptions of the data and their correspondence to model observables follow (the standard macro series used are as in Del Negro et al. (2015))\(^{23}\). To estimate the model we use data from 1984:I to 2008:III.

Real GDP (GDPC), the GDP deflator (GDPDEF), nominal personal consumption expenditures (PCEC), and nominal fixed private investment (FPI) data are produced by the BEA at quarterly frequency. Average weekly hours of production and nonsupervisory employees for total private industries (AWHNONAG), civilian employment 16 and over (CE16OV) and civilian noninstitutional population 16 and over (CNP16OVA) are released at monthly frequency by the Bureau of Labor Statistics (BLS) (we take quarterly averages of monthly data). Nonfarm business sector compensation (COMPNFB) is produced by the BLS every quarter. For the effective federal funds rate (DFF) we take quarterly averages of the annualized daily data (and divide by four to make the rates quarterly).

Letting $\Delta$ denote the temporal difference operator, the correspondence between the standard macro data described above and our model observables is as follows:

- Output growth = \(100 \times \Delta \ln((GDPC)/LNSINDEX)\)
- Consumption growth = \(100 \times \Delta \ln((PCEC/GDPDEF)/LNSINDEX)\)
- Investment growth = \(100 \times \Delta \ln((FPI/GDPDEF)/LNSINDEX)\)
- Real Wage growth = \(100 \times \Delta \ln(COMPNFB/GDPDEF)\)
- Hours worked = \(100 \times \ln((AWHNONAG \times CE16OV/100)/LNSINDEX)\)
- Inflation = \(100 \times \Delta \ln(GDPDEF)\)
- FFR = \((1/4) \times \text{FEDERAL FUNDS RATE}\)

The R&D data used in estimating the model is produced by the NSF and measures R&D expenditure by US corporations. The data is annual, so in estimating the model and extracting model-implied latent variables (see Appendix A.2) we use a version of the Kalman filter adapted for use with mixed frequency data.

\(^{23}\)Del Negro et al. (2015) include consumer durables in consumption as opposed to investment. Our results are robust to including them in investment. Neither approach, of course, is ideal.
A.2 Extracting Model-Implied Latent Variables during ZLB period

The piece-wise linear solution from the OccBin method developed by Guerrieri and Iacoviello (2015) can be represented in state space form as

\[ S_t = C(N_t, \theta) + A(N_t, \theta)S_{t-1} + B(N_t, \theta)\epsilon_t \]
\[ Y_t = H(N_t, \theta)S_t \]

Where \( \theta \) is a vector of structural parameters, \( S_t \) denotes the endogenous variables at time \( t \), \( Y_t \) are observables, and \( \epsilon_t \) are normally and independently distributed shocks. \( N_t \) is a vector that identifies whether the occasionally binding constraint binds at time \( t \) and whether it is expected to do so in the future. In particular, \( N_t \) is a vector of zeros and ones indicating when the constraint is or will be binding. For example, the vector \( N_t = (0, 1, 1, 1, 0, 0, 0...) \) is a situation in which the constraint does not bind at time \( t \) (denoted by the first zero in the vector), but is expected to bind in \( t + 1 \), \( t + 2 \) and \( t + 3 \). Note that the matrices \( A, B \) and \( C \), which in a standard linear approximation depend only on parameters are here also functions of \( N_t \).\(^{24}\)

OccBin provides a way of computing the sequence of endogenous variables \( \{S_t\}_{t=1}^{T} \) and regimes \( \{N_t\}_{t=1}^{T} \) for a given initial condition \( S_0 \) and sequence of shocks \( \{\epsilon_t\}_{t=1}^{T} \). The vector \( N_t \) is computed by a shooting algorithm and its resulting value will depend on the initial state and shocks at time \( t \). We refer the reader to Guerrieri and Iacoviello (2015) for a detailed description of the method.

We construct the Kalman filter and smoother from the nonlinear state space representation presented above by taking advantage of the fact that a given sequence of regimes, say \( \{\hat{N}_t\}_{t=1}^{T} \), uniquely defines a sequence of matrices \( \{\hat{C}_t, \hat{A}_t, \hat{B}_t, \hat{H}_t\}_{t=1}^{T} \). It follows that for that specific set of regimes the state space representation becomes linear:

\[ S_t = \hat{C}_t + \hat{A}_tS_{t-1} + \hat{B}_t\epsilon_t \]
\[ Y_t = \hat{H}_tS_t \]

For this linear state space representation it is straightforward to compute the Kalman filter and smoother. We use this fact in our algorithm by running two blocks: (i) one in

\(^{24}\)The matrix \( H \) might also be a function of \( N_t \) because some observables might become redundant when the occasionally binding constraint binds. This is the case for the Taylor rule interest rate when the ZLB binds.
which we compute the Kalman filter and smoother for a given set of regimes \( \{N_t\}_{t=1}^T \); and (ii) another where we use OccBin to compute the regimes given a sequence of shocks \( \{\epsilon_t\}_{t=1}^T \).

The algorithm steps are the following.

1. Guess a sequence of regimes \( \{N_t^{(0)}\}_{t=1}^T \);

2. Use the guess from the previous step and define the sequence of matrices \( \{C_t, A_t, B_t, H_t\}_{t=1}^T \) using OccBin;

3. With the matrices from the previous step, compute the Kalman Filter and Smoother using the observables \( \{Y_t\}_{t=1}^T \), and get the Smoothed shocks \( \{\hat{\epsilon}_t\}_{t=1}^T \) and initial conditions of endogenous variables;

4. Given the smoothed shocks and initial conditions from the previous step, use OccBin to compute a new set of regimes \( \{N_t^{(1)}\}_{t=1}^T \);

5. If \( \{N_t^{(0)}\}_{t=1}^T \) and \( \{N_t^{(1)}\}_{t=1}^T \) are the same, stop. If not, update \( \{N_t^{(0)}\}_{t=1}^T \) and go to step 2.

Once it converges, this algorithm yields a sequence of smoothed variables and shocks, consistent with the observables, and taking into account the occasionally binding constraint.
### Table 2: Calibrated Parameters

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### Table 3: Prior and Posterior Distributions of Estimated Parameters

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### Table 4: Prior and Posterior Distributions of Shock Processes

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### Table 5: Comparison of Standard Deviations

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<th>Variable</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption Growth</td>
<td>0.54</td>
<td>0.68</td>
</tr>
<tr>
<td>Output Growth</td>
<td>0.53</td>
<td>0.60</td>
</tr>
<tr>
<td>Hours Growth</td>
<td>0.47</td>
<td>0.74</td>
</tr>
<tr>
<td>Investment Growth</td>
<td>1.51</td>
<td>1.77</td>
</tr>
<tr>
<td>Nominal R</td>
<td>0.30</td>
<td>0.27</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.18</td>
<td>0.21</td>
</tr>
</tbody>
</table>
Table 6: Variance Decomposition (%)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Liquidity Demand</th>
<th>Money</th>
<th>Govt Exp</th>
<th>Price of Capital</th>
<th>TFP</th>
<th>R&amp;D</th>
<th>Mark up</th>
<th>Wage mark up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Growth</td>
<td>28.7</td>
<td>24.1</td>
<td>7.2</td>
<td>5.7</td>
<td>32.8</td>
<td>0.3</td>
<td>0.7</td>
<td>0.6</td>
</tr>
<tr>
<td>Consumption Growth</td>
<td>19.2</td>
<td>14.7</td>
<td>37.3</td>
<td>2.2</td>
<td>26.0</td>
<td>0.0</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Investment Growth</td>
<td>17.8</td>
<td>17.1</td>
<td>2.6</td>
<td>39.4</td>
<td>16.1</td>
<td>1.1</td>
<td>2.6</td>
<td>3.3</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.0</td>
<td>0.0</td>
<td>0.2</td>
<td>0.0</td>
<td>2.5</td>
<td>0.0</td>
<td>87.3</td>
<td>9.8</td>
</tr>
<tr>
<td>Nominal R</td>
<td>66.0</td>
<td>5.3</td>
<td>1.7</td>
<td>4.9</td>
<td>9.7</td>
<td>0.1</td>
<td>7.9</td>
<td>4.3</td>
</tr>
<tr>
<td>Hours</td>
<td>40.2</td>
<td>30.3</td>
<td>6.1</td>
<td>6.8</td>
<td>15.9</td>
<td>0.1</td>
<td>0.6</td>
<td>0.0</td>
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<tr>
<td>R&amp;D Growth</td>
<td>7.4</td>
<td>8.4</td>
<td>3.3</td>
<td>4.0</td>
<td>21.4</td>
<td>51.9</td>
<td>3.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Endogenous TFP</td>
<td>17.1</td>
<td>11.7</td>
<td>2.7</td>
<td>5.1</td>
<td>40.2</td>
<td>16.3</td>
<td>4.8</td>
<td>2.3</td>
</tr>
<tr>
<td>Int. Goods Varieties</td>
<td>0.4</td>
<td>0.7</td>
<td>0.3</td>
<td>0.9</td>
<td>1.9</td>
<td>94.3</td>
<td>1.2</td>
<td>0.3</td>
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<tr>
<td>Speed of Diffusion</td>
<td>28.7</td>
<td>13.7</td>
<td>3.5</td>
<td>2.7</td>
<td>37.1</td>
<td>11.3</td>
<td>2.1</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Variance decomposition with ZLB (10,000 simulations, HP filtered series, \( \lambda = 1600 \)).
Data sources are described in Appendix A.1. Smoothed shocks from model estimated using data as described in Section 4.2 and Appendix A.1.
Figure 7: Impulse Response to 1 std. dev. Shock

- Liq Demand
- Money
- TFP
- Output
- Consumption
- Invest
- Endog TFP
- Inflation
- Nominal R

Legend:
- Red: Endogenous TFP
- Blue: Exogenous TFP
Figure 8: Liquidity Demand Shock and the ZLB
Figure 9: Endogenous TFP, TFP and Labor Productivity

Labor productivity is GDP divided by hours worked (see Appendix A.1 for data sources). Smoothed shocks from model estimated using data as described in Section 4.2 and Appendix A.1.
Smoothed variables from model estimated using data as described in Section 4.2 and Appendix A.1.
Figure 11: Sources of Endogenous Technology
Figure 12: Productivity and Liquidity Demand + Money Shocks

Labor productivity is GDP divided by hours worked (see Appendix A.1 for data sources). Smoothed shocks from model estimated using data as described in Section 4.2 and Appendix A.1.
References


