Competition in Product Lines

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Abstract

We analyze competition in product lines by multi-product oligopolists. The firms are assumed to have preferential access to a local market but can supply the rival’s market by incurring distance or product adaptation costs. We show that the width and degree of overlap in the product lines chosen by these firms is greater in large markets and when the products being offered are viewed by consumers as being more differentiated. This reflects a tension between two effects: the desire to “be where the demand is” and the desire to weaken competition and intra-firm product cannibalization. Distance costs are shown to affect not only the desire to supply the distant market but also the firms’ product line choices. Product lines are broader and more overlapped when distance costs are low and when they are high; while product lines are narrower, with less overlap, when distance costs are moderate.

JEL Codes: D43, F12, L13
1. **Introduction**

Competition between multi-product firms is the norm rather than the exception but until recently there has been limited progress in our understanding of such competition. In this paper we develop an analysis of multi-product firms that can supply a set of horizontally differentiated products. The firms have the technological capacity to supply identical product lines but we allow for the possibility that each firm has preferential access to a “local” market while being able to adapt its product line to the consumer characteristics of its rival’s local market.\(^1\)

We might expect that multi-product firms would prefer to avoid overlapping their product lines, specializing in particular market niches as a means of softening competition. The problem with this type of mutual forbearance, as we shall show, is that it is unsustainable as a non-cooperative equilibrium in larger markets or if the degree of product differentiation is high. There is a simple intuition for these results. Multi-product firms face competing forces in choosing the breadth of their product lines. On the one hand, an extensive product line enables a firm to “be where the demand is” in each segment of the market (Tirole, 1988, p. 286). On the other hand, an extensive product line runs the risk that the firm will cannibalize its own products while also creating a more competitive environment as a result of the greater overlap with the product lines of competing firms. An immediate implication is that, in spite of the increased competition, there is likely to be greater benefit from being where the demand is in larger markets, while there is less risk of intra-firm cannibalization when consumers view the different products on offer as being highly differentiated.

\(^1\) Dobson and Waterson (1996) adopt a similar approach in that they allow consumers to have preferences with respect to the seller they choose.
Our analysis is consistent with some common characteristics of the behavior of multi-product firms. Large markets containing consumers with diverse tastes tend to be populated by firms that are involved in “head-to-head” competition, offering broad product lines with extensive overlap. “Probably the most common pattern is that each firm produces a wide range of varieties within a product group, and a number of firms produce very similar, and sometimes virtually identical, products. Ford and General Motors produce closely competing product lines, as do Nikon, Canon and Minolta in the camera industry.” (Brander and Eaton, 1984, p. 323) By contrast, smaller markets with less diverse consumer tastes tend to be supplied by firms with more specialized, narrower product lines. Consider, for example, the much narrower, less overlapped product lines offered by BMW, Jaguar and Mercedes.

The majority of the current literature on product differentiation is based on one or the other, or occasionally a combination, of two paradigms, both of which focus on single-product firms. First, there is the “address” literature building on the seminal analysis of Hotelling (1929), seeking to explain location and pricing behavior and oscillating between models that predict Hotelling’s “excessive sameness” and d’Aspremont et al.’s (1979) maximum differentiation. Second, there is the literature based on Cournot’s non-spatial duopoly analysis, a model that has become a workhorse in industrial organization for the study of imperfectly competitive markets.

The literature on multi-product firms is less extensive: Manez and Waterson (2001) provide a valuable recent review. Gal-Or (1983) and Champsaur and Rochet (1989) developed some of the earliest studies of multi-product firms offering products of different qualities – vertical product differentiation. De Fraja (1996) and Johnson and
Myatt (2006) extend this analysis, in both cases assuming Cournot competition and identifying equilibria in which firms compete head-to-head.

Much of the multi-product literature assuming *horizontal* product differentiation employs address models building on the original Salop (1979) analysis. Eaton and Lipsey (1979), Schmalensee (1978) and Judd (1985) focused on product proliferation as an entry-deterring device. Eaton and Schmitt (1994) and Norman and Thisse (1999) consider firms offering multiple products by employing flexible manufacturing techniques, focusing on the width of each firm’s product range but by assumption ruling out head-to-head competition. Klemperer (1992) develops a model in which duopolists choose whether to interlace their products or engage in head-to-head competition, given that consumers face differential shopping costs when buying from the two firms, thus preventing head-to-head competition from resulting in marginal-cost pricing. Shaked and Sutton (1990) and Dobson and Waterson (1996) develop non-address models. Since the former treats all goods symmetrically the analysis rules out the possibility that competing firms will offer overlapped product lines. By contrast, Dobson and Waterson assume that consumers care about both the product and the seller from whom they buy, leading to the possibility that the firms will offer broad product lines.

Our analysis extends the literature on multi-product firms in horizontally differentiated markets in a number of ways, relating to the role that market characteristics play in determining firms’ endogenous choices of product lines. We show that product lines are likely to be broader and to involve more head-to-head competition in large markets or when consumers consider the products on offer to be highly differentiated.
We extend this analysis by introducing the possibility that each firm has preferential access to a local market but can, by incurring additional costs, supply a rival’s market. In an explicitly spatial context, these may be transport and other distance costs in shipping a product from a firm’s local market to that of its rival. In a more general characteristics space these could be costs of customizing the firm’s products to the precise characteristics that are preferred by consumers in the “distant” market. Our analysis shows that these distance costs affect more than just a firms’ choice of the quantity of a product that should be shipped to the distant market. They also affect the choice of product line and, if a broad product line is adopted, which of the products should actually be shipped to the distant market. We further show that the relationship between the breadth of a firm’s product line and distance or customization costs is non-monotonic: broader product lines are likely to be adopted when distance costs, or customization costs, are low and when they are high.

This part of our analysis introduces an additional twist to the literature on intra-industry trade based on the seminal analysis of Brander (1981) and Brander and Spencer (1985). Our analysis shows that increased barriers to trade may harm consumers by reducing the quantity of goods exported but may benefit consumers by broadening the product line choices of exporting firms.

The remainder of the paper is organized as follows. Section 2 presents the basic model. Section 3 develops and discusses the subgame perfect equilibria of the model. Some welfare implications of our analysis are outlined in section 4 and our main conclusions are presented in section 5.
2. The Model

We assume that the market consists of two consumer locations \( a \) and \( b \), each of which contains \( s \) identical consumers. These may be locations in geographic space or in some more general characteristics space. The locations are supplied by Cournot duopolists \( A \) and \( B \), each of which can choose to produce one or both of horizontally differentiated products 1 and 2. \( A \) (\( B \)) has its production facility in location \( a \) (\( b \)) and each firm can choose to export its product(s) to, or customize its product(s) for, the other location at a cost of \( t \) per unit.

Following Singh and Vives (1984) we assume that individual consumer utility in both locations is quadratic and given by

\[
 u(q_1, q_2) = V(q_1 + q_2) - \left[ (\beta (q_1^2 + q_2^2) + 2\gamma q_1 q_2) / 2 \right] + y
\]

where \( y \) is an outside good produced competitively and \( q_i \geq 0 \) is individual consumption of differentiated product \( i \). \( V \) is a positive parameter and \( \gamma \) is an inverse measure of the degree of product differentiation, with \( 0 \leq \gamma < \beta \). Thus consumers have a taste for product variety. Since the utility function is quasi-linear, consumers’ decisions with respect to consumption of the differentiated goods are independent of their consumption decisions with respect to the outside good.

Standard analysis gives the individual indirect demands

\[
p_1 = V - \beta q_1 - \gamma q_2; \quad p_2 = V - \gamma q_1 - \beta q_2
\]

Inverting to obtain the direct demands, aggregating over the \( s \) consumers and reinverting gives aggregate inverse demand for the differentiated goods at each location:
We follow Eaton and Schmitt (1994) and assume that each firm incurs three sets of production costs. Suppose that firm $K$ chooses to produce $n = 1, 2$ of the differentiated goods, aggregate output of product $i$ being $Q_{Ki}$. Then firm $K$’s total production costs are

$$C_K = F + (n_K - 1)c + v(Q_{K1} + Q_{K2}) \quad (K = A, B)$$

In (2) $F$ is a fixed (set-up) cost, $v$ is constant marginal cost, and $c$ can be thought of as the additional set-up cost that a firm incurs to install a multi-product technology plus a switching cost of re-tooling as firm $K$ switches production from one differentiated product to another. Since we ignore the possibility of entry, we can normalize $F$ and $v$ to zero without loss of generality.

The firms compete in a three-stage game. In the first stage they select their product lines by choosing the number of differentiated goods they will produce. In the second stage, if a firm chooses to produce both products, it chooses which of these products to export. We assume that distance costs $t$ are low enough or, equivalently, that the consumer reservation price $V$ is high enough that if a firm produces only one product it will always choose to export that product. A sufficient condition for this to hold is that $t < V/2$. In the third stage the firms compete in quantities, choosing quantities $Q_{Kji}$ where $Q_{Kji} \geq 0$ is the quantity supplied by firm $K$ to location $j$ of the differentiated product $i$: $K = A, B; j = a, b; i = 1, 2$. The solution concept is, as usual, subgame perfection.

$$p_i = V - \frac{\beta}{s}Q_i - \frac{\gamma}{s}Q_2$$

$$p_2 = V - \frac{\gamma}{s}Q_i - \frac{\beta}{s}Q_2$$
3. **Analysis**

In subsequent analysis it proves useful to define:

\[
g = \frac{\gamma}{\beta} \in [0, 1); \quad \sigma = \frac{c\beta}{sV^2}; \quad \tau = \frac{t}{V} \in [0, 0.5)
\]

since this allows us to characterize the equilibria fully in terms of three parameters: \( g \), an inverse measure of the degree of product differentiation defined on the interval \([0, 1)\); \( \sigma \), an inverse measure of market size and a direct measure of the set-up and switching costs incurred in installing the multi-product technology and \( \tau \), a measure of distance costs defined on the interval \([0, 0.5]\). (Recall that the constraint on \( \tau \) ensures that if either firm offers only one product it will choose to export that product.)

Rather than developing a generic solution to the quantity subgame for all possible product-line configurations, exposition is considerably eased if we identify the equilibria to the export and quantity subgames separately for each possible product-line configuration and export choice. In doing so, and if at least one firm adopts the full product line, we use the notation \( \pi_{n_i n_j}^{e_i e_j} \) for firm \( i \)'s profits, in relation to firm \( j \)'s choices, where \( n_i (n_j) \) is the number of products offered by firm \( i \) (\( j \)) and \( e_i (e_j) \) is the number of products exported by firm \( i \) (\( j \)).

### 3.1 Both Firms Produce Both Products

If both firms choose the full product line there is no reason to believe *a priori* that they will each export both goods. Admittedly, exporting both increases market share in the distant market. But exporting both also introduces the possibility that the exporting

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2 The calculations in this section have been made using Mathematica®. The notebook is available from the authors on request.
firm cannibalizes sales of its own products. Suppose first that both firms do, indeed, choose to export both goods. The equilibrium quantities are:

\[
\begin{align*}
Q_{Aa1} &= Q_{Aa2} = Q_{Ba1} = Q_{Ba2} = \frac{sV(1 + \tau)}{3\beta(1 + g)} \\
Q_{Ab1} &= Q_{Ab2} = Q_{Bb1} = Q_{Bb2} = \frac{sV(1 - 2\tau)}{3\beta(1 + g)}
\end{align*}
\]

Profit of each firm is:

\[
\pi^{22}_{22} = \frac{sV^2}{\beta} \left[ \frac{2(2 - 2\tau + 5\tau^2)}{9(1 + g)} - \gamma \right]
\]

These results exhibit the familiar tension between the import protection offered by distance costs and the export cost they impose. Domestic output increases with an increase in distance costs while exports decrease. As a result, the import protection effect on profit dominates for \( \tau > 0.2 \): \( \frac{\partial \pi^{22}_{22}}{\partial \tau} > 0 \Leftrightarrow \tau > 0.2 \). In addition, and as expected, profit is increasing in market size and in the degree of product differentiation.

Now suppose that both firms choose to export only one good. We can show that the firms earn the same profit and consumers enjoy the same utility whether they export the same or different goods. Assume, therefore, that the firms choose to export different goods, firm \( A \) exporting good 2 and firm \( B \) exporting good 1. The equilibrium quantities are:

\[
\begin{align*}
Q_{Aa2} &= Q_{Bb1} = \frac{sV}{2\beta(1 + g)}; Q_{Aa1} = Q_{Bb2} = \frac{sV(2(1 + \tau) - g(1 - 2\tau))}{6\beta(1 + g)} \\
Q_{Ab2} &= Q_{Bb1} = \frac{sV(1 - 2\tau)}{3\beta}
\end{align*}
\]

Profit of each firm is:

---

3 If the firms export the same products, we simply switch the product subscripts in equation (6).
Import competition reduces domestic output \((Q_{aa1} < Q_{aa2} \text{ and } Q_{bb2} < Q_{bb1})\). Domestic output of the imported good \((Q_{aa1} \text{ and } Q_{bb2})\) is increasing in \(\tau\) as a result of the import protection effect while exports \((Q_{ab2} \text{ and } Q_{ba1})\) are decreasing in \(\tau\). Profit and domestic output of both goods are increasing in the degree of product differentiation:

\[
\frac{\partial \pi^{11}_{22}}{\partial g} = -sV^2/2(1+g)^2.
\]

As in the previous case, the import protection effect on profit dominates for \(\tau > 0.2\): \(\frac{\partial \pi^{11}_{22}}{\partial \tau} > 0 \iff \tau > 0.2\).

Finally, suppose that one firm exports both goods while the other exports only one. To fix ideas, assume that firm \(A\) exports only good 1 while firm \(B\) exports both goods. The equilibrium quantities are:

\[
\begin{align*}
Q_{aa1} &= Q_{aa2} = \frac{sV(1 + \tau)}{3\beta(1 + g)}; Q_{ab1} = \frac{sV(1 - 2\tau)}{3\beta} \\
Q_{bb1} &= \frac{sV(2(1 + \tau) - g(1 - 2\tau))}{6\beta(1 + g)}; Q_{bb2} = \frac{sV}{2\beta(1 + g)}; Q_{ba1} = Q_{ba2} = \frac{sV(1 - 2\tau)}{3\beta(1 + g)}
\end{align*}
\]

Profits of the two firms are:

\[
\begin{align*}
\text{Two-product firm: } \pi^{21}_{22} &= \frac{sV^2}{\beta} \left( \frac{3(7 - 8\tau + 12\tau^2) - g(1 - 2\tau)(5 + 2\tau)}{36(1 + g)} \right) - \sigma \\
\text{One-product firm: } \pi^{12}_{22} &= \frac{sV^2}{\beta} \left( \frac{3 + 6\tau^2 + g(1 - 2\tau)^2}{9(1 + g)} \right) - \sigma
\end{align*}
\]

Comparative static properties of the equilibrium outputs are as discussed above.

The additional market share gained by exporting both products more than offsets the potential cannibalization that arises in the export market when exporting both goods. As a result, profit of the firm exporting both products is greater than profit of the firm exporting only one product. Profit of the firm exporting only one product is increasing in
$\tau$ for $\tau > g/(2g-3)$ while profit of the firm exporting both products is increasing in $\tau$ for $\tau > (3-g)/(9+g)$. Clearly, $g/(3+2g) < (3-g)/(9+g)$. The single-product-export firm gains more from the import protection effect and suffers less from the export cost effect than the two-product-export firm.

The final question to which we turn in this section is the determination of the number of products each firm actually chooses to export. Table 1 gives the pay-off matrix, drawing on equations (5), (7) and (9), and allows us to conclude:

**Result 1:** When both firms choose a full product line they choose to export both goods.

**Proof:** (5), (7) and (9) give

$$\pi_{22}^{22} - \pi_{22}^{12} = \pi_{22}^{21} - \pi_{22}^{11} = \frac{s \nu^2}{\beta} \left( \frac{(1-g)(1-2\tau)^2}{9(1+g)} \right) > 0.$$  

While Result 1 might, on first sight seem obvious, this conceals the fact that $\pi_{22}^{22} < \pi_{22}^{11}$. In other words, this subgame is a prisoner’s dilemma game. Exporting both products is a dominant strategy for each firm although both would be better off if they could at least partially forebear from invading each other’s market.4

**Table 1: Pay-Off Matrix for the Export Subgame – full product line**

<table>
<thead>
<tr>
<th></th>
<th>Firm B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Export One Good</td>
</tr>
<tr>
<td>Firm A</td>
<td>Export One Good</td>
</tr>
<tr>
<td></td>
<td>Export Both Goods</td>
</tr>
</tbody>
</table>

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4 Indeed, exporting neither good would be best for both firms in that this confers a local monopoly in both goods.
3.2 Both Firms Produce One Product

When both firms choose to produce and export only one product from the two-product line it is tempting to believe that they will choose to offer different products in order to soften competition between them. However, this ignores the import protection effect offered by distance costs. This effect is stronger when the firms offer the same good as a result of which, as we shall see, the firms choose to offer the same products when distance costs are “high” and the degree of product differentiation is “low”.

Assume first that the firms choose to offer different goods, with firm A (B) producing good 1 (2). The equilibrium quantities are:

\[
Q_{Aa1} = Q_{Bb2} = \frac{sV}{\beta} \left( \frac{2 - g(1 - \tau)}{4 - g^2} \right); Q_{Ab1} = Q_{Ba2} = \frac{sV}{\beta} \left( \frac{2(1 - \tau) - g}{4 - g^2} \right)
\]

Exports are decreasing in \(\tau\) and \(g\) and domestic output is increasing in \(\tau\). Domestic output generally decreases with \(g\) but actually increases in \(g\) for \(\tau > \frac{(2 - g)^2}{(4 + g^2)}\).

Intuitively, when \(\tau\) and \(g\) are “high” domestic production benefits both from the import protection effect and from notably weakened competition as firms avoid head-to-head competition in the face of diminished product differentiation.

Profits of the two firms are:

\[
\pi_{1d} = \frac{sV^2}{\beta} \left[ \left( \frac{2 - g(1 - \tau)}{4 - g^2} \right)^2 + \left( \frac{2(1 - \tau) - g}{4 - g^2} \right)^2 \right]
\]

where \(d\) indicates that firms offer different goods. The first term is profit from domestic sales and the second is profit from exports. Profit is decreasing in \(\tau\) if \(g < 0.536\). In other words, as noted above, when the firms offer different products the import protection
effect of increased distance costs is weakened and, unless the products are “reasonably” similar, is dominated by the export cost effect.

Assume, by contrast, that the firms offer the same good, say product 1. The equilibrium quantities are now:

\[ Q_{d,1} = Q_{b,1} = \frac{sV}{3\beta} (1 + \tau); Q_{d,1} = Q_{b,1} = \frac{sV}{3\beta} (1 - 2\tau) \]

Profits of the two firms are:

\[ \pi^s = \frac{sV^2}{9\beta} \left( (1 + \tau)^2 + (1 - 2\tau)^2 \right) \]

where \( s \) indicates that firms offer the same good. As in (11) the first term is profit from domestic sales and the second is profit from exports.

Define:

\[ \tau_1(g) = \min \left[ 0.5, \frac{3\sqrt{\left(4 - g^2\right)^2(3 - g)(5 + g) - (2 - g)^2(5 + g)}}{(1 + g)(44 - 5g^2)} \right] \]

We have \( \tau_1(g) < 0.5 \) for \( g > 0.3469 \), \( \tau_1(1) = 0.3228 \) and \( \frac{d\tau_1(g)}{dg} < 0 \). Comparison of (11) and (13) gives:

**Result 2:** When each firm chooses a single-product line they offer different (the same) products if \( \tau < (>) \tau_1(g) \).

The intuition underlying Result 2 is simply explained. When the firms offer the same products two competing forces come into play. On the one hand competition from the rival firm’s exports is direct and so stronger. On the other hand, the import protection effect is direct and so stronger. (The elasticities of domestic output and profit with

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Note that our restriction that \( \tau \in [0, .5) \) still holds, ensuring each firm exports when producing only one product.
respect to \( \tau \) are greater when the firms offer the same good than when they offer different goods.) Equations (10) and (12) indicate that exports fall and domestic production increases as distance costs \( \tau \) increase. As a result, the import protection effect dominates the export competition effect at higher distance costs.

### 3.3 One Firm has a Full Product Line and the Other a Single Product Line

Assume that firm A adopts the full product line while firm B produces only one product, say product 1. We need to check whether the two-product firm chooses to export both products or, in order to avoid intra-firm competition, chooses to export only one product.

Assume first that firm A chooses to export both products. The equilibrium quantities are:

\[
Q_{A1} = \frac{sV}{\beta} \left( \frac{2(1 + \tau) - g(1 - 2\tau)}{6(1 + g)} \right); \quad Q_{A2} = \frac{sV}{2\beta(1 + g)}; \quad Q_{B1} = \frac{sV}{3\beta(1 + \tau)}; \quad Q_{B2} = \frac{sV}{2\beta(1 + g)}; \quad Q_{B1} = \frac{sV}{3\beta(1 - 2\tau)}
\]

Note that the non-negativity constraint on output is potentially binding in this case.\(^6\)

Define:

\[
\tau_2(g) = \frac{2 - g}{4 + g}
\]

**Result 3:** Assume that one firm chooses a full product line and the other chooses a single-product line. Exports of the overlapped product by the full-product line firm are positive if and only if \( \tau < \tau_2(g) \).

\(^6\) In every other case this constraint is satisfied so long as \( \tau < 0.5 \).
Comparison of (15) and (4) indicates that when the single-product firm chooses to produce only one product (in our example product 1) it increases its output of that product as compared to its full-product line choice. Moreover, its equilibrium output is independent of the degree of product differentiation since there is no intra-firm competition. As a result, competition is much tougher for the overlapped export product of the full-product line firm to the extent that, if distance costs are high enough and the degree of product differentiation low enough, the full-product line firm will cease exports of this product.

Given that \( \tau < \tau_2(g) \) profit of the full-product line firm is:

\[
\pi_{21}^{12} = \frac{sV^2}{\beta} \left( \frac{26 - 26\tau + 29\tau^2 - g(10 - 10\tau + 11\tau^2)}{36(1 + g)} - \sigma \right)
\]

and of the single product firm is:

\[
\pi_{12}^{12} = \frac{sV^2}{\beta} \left[ \frac{1}{9}(1 + \tau)^2 + \frac{1}{9}(1 - 2\tau)^2 \right]
\]

As in other cases, profit initially falls as \( \tau \) rises but increases with \( \tau \) as \( \tau \) approaches 0.5.

Now suppose that the full-product line firm chooses to export only one product. Will it export the same (overlapped) product as its rival or the different (non-overlapped) product? We can show:

**Result 4:** Assume that one firm has a full-product line and the other a single-product line. If the full-product line firm chooses to export only one product it will export the non-overlapped product.

The contrast with Result 2 is easily explained. The full-product line firm benefits from the import protection effect whether it exports one or both of its products. As a result, the
only consideration that this firm has to take into account is competition in its export market. This is softer if the firm exports the product that its rival is not producing.

The equilibrium quantities are:

\[ Q_{Aa1} = \frac{sV}{\beta} \left( \frac{2(1 + \tau) - g(1 - 2\tau)}{6(1 + g)} \right), \quad Q_{Aa2} = \frac{sV}{2\beta(1 + g)}; \quad Q_{Ab1} = \frac{sV}{\beta} \left( \frac{2 - g(1 - \tau)}{4 - g^2} \right) \]

\[ Q_{Ab2} = \frac{sV}{\beta} \left( \frac{2(1 - \tau) - g}{4 - g^2} \right); \quad Q_{ba1} = \frac{sV}{3\beta} (1 - 2\tau) \]

Profits of the two firms are:

\[ \pi_{21}^{11} = \frac{sV^2}{\beta} \left[ \frac{(1 + \tau)(2(1 + \tau) - g(1 - 2\tau))}{18(1 + g)} + \frac{3 - g(1 - 2\tau)}{12(1 + g)} + \frac{(2 - g(1 - \tau))^2}{(4 - g^2)^2} - \alpha \right] \]

\[ \pi_{12}^{11} = \frac{sV^2}{\beta} \left[ \frac{(2 - g(1 - \tau))^2}{(4 - g^2)^2} + \frac{1}{9} (1 - 2\tau)^2 \right] \]

While we have shown that \( \tau < \tau_2(g) \) is a necessary condition for the full-product line firm to choose to export both goods this does not mean that it is also a sufficient condition.

Comparison of (20) and (17), however, gives:

**Result 5:** Assume that one firm has a full-product line and the other a single-product line. The full-product line firm chooses to export both products if and only if \( \tau < \tau_2(g) \).

**Proof:** From (20) and (17) \( \pi_{21}^{11} < \pi_{21}^{21} \) if and only if \( \tau < \tau_2(g) \). ■

The full-product line firm balances two forces in deciding whether to export both products. On the one hand, shipping both products wins market share from the rival single-product firm. On the other hand, shipping both products leads to intra-firm competition in the export market. When the degree of product differentiation is “high”
and distance costs are “low” intra-firm competition is relatively weak and the market share force, with its benefits from “being where the demand is,” dominates.

3.4 The Product Line Game

Define

(20) \[ \pi_{22} = \pi_{22}^2; \pi_{11} = \max\{\pi_{11}^d, \pi_{11}^s\}; \pi_{21} = \max\{\pi_{21}^1, \pi_{21}^2\} \]

In addition define \( \pi_{12} = \pi_{12}^1 \) if \( \tau < \tau_2(g) \) and \( \pi_{12}^2 \) otherwise. The pay-off matrix for the product line game is then given in Table 2.

Table 2: Pay-Off Matrix for the Product Line Game

<table>
<thead>
<tr>
<th>Firm A</th>
<th>Single Product Line</th>
<th>Full Product Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Product Line</td>
<td>( \pi_{11}, \pi_{11} )</td>
<td>( \pi_{12}, \pi_{21} )</td>
</tr>
<tr>
<td>Full Product Line</td>
<td>( \pi_{21}, \pi_{12} )</td>
<td>( \pi_{22}, \pi_{22} )</td>
</tr>
</tbody>
</table>

As we noted in the introduction to this section, given the normalization in equation (2) equilibrium to the product line game is determined by the three parameters \( \sigma, g \) and \( \tau \). Both firms offering the full product line, denoted \{2, 2\}, is an equilibrium if \( \pi_{22} > \pi_{12} \) and both firms offering a single product line, denoted \{1, 1\}, is an equilibrium if \( \pi_{11} > \pi_{21} \). One firm offering the full product line and the other a single product line, denoted \{2, 1\}, is an equilibrium if \( \pi_{22} < \pi_{12} \) and \( \pi_{11} < \pi_{21} \). These equilibria are illustrated in Figure 1 for two values of \( \sigma \). The shaded areas in this diagram are areas in which both \{1, 1\} and \{2, 2\} are equilibrium product line configurations.
No matter the degree of product differentiation $g$, an increase in $\sigma$ resulting from an increase in switching costs $c$ or a reduction in market size $s$ induces firms to adopt narrower product lines. Product lines are also narrower when the degree of product differentiation is low. This does not mean, however, that there is less product line overlap with increased $\sigma$ or increased $g$, since we must also take into account the impact of distance costs on the equilibrium configuration. Our analysis in section 3.2 indicates that if $\tau > \tau_1(g)$ the firms will have perfectly overlapped product lines even if they adopt the narrow product line configuration $\{1, 1\}$: in Figure 1 this is the configuration $\{1, 1\}^s$.

We can put this another way. Suppose that the degree of product differentiation is low enough or market size is small enough that both firms adopt a narrow product line. It is still the case that an increase in distance costs changes the product line configuration – from the non-overlapped configuration $\{1, 1\}^d$ to the overlapped configuration $\{1, 1\}^s$ – for reasons that we have discussed above. In this parameter range, in other words, an increase in distance costs changes the equilibrium configuration from one in which consumers are offered both products to one in which they are offered only one.

Now consider the more general impact of distance or customization costs on the product line choice. We see from Figure 1 that at very low distance costs one candidate equilibrium has the firms adopting broad, fully overlapped product lines. With such low distance costs the firms’ products are competitive in the other location even allowing for the distance costs. As a result, the drive for market share dominates intra-firm cannibalization. As distance costs increase, however, the prisoners’ dilemma properties of this configuration render it unviable and the firms switch to narrow, non-overlapped product lines: configuration $\{1, 1\}^d$. A further increase in distance costs leads to the
asymmetric configuration \{2, 1\}. Note, however, that the parameter region for which \{2, 1\} is an equilibrium configuration consists of two sub-regions: \(\{2, 1\}^2\) in which the full-product line firm exports both products and \(\{2, 1\}^1\) in which this firm exports only one (the non-overlapped) product. Finally, if distance costs are high enough the import cost protection that they offer leads the single-product firm in configuration \{2, 1\} to switch back to the full product line.

In other words, the relationship between distance or customization costs and product line width is non-monotonic. Wide, fully overlapped product lines emerge as equilibrium configurations when distance costs are low and when they are high. An immediate implication, which we consider in more detail in the next section, is that firms do not necessarily benefit from increased protection from their distant rivals. Increased distance costs lead to broader, more overlapped product lines and so to potentially tougher competition.

4. Welfare Analysis

In considering the welfare properties of the equilibria that we have identified, we focus our attention on the impact of changes in firms’ costs rather than changes in consumer taste parameters: the former are potentially endogenous, the latter are not. In addition, rather than present a full analysis of profit and consumer surplus for every possible parameter combination, we consider the impact of changes in distance costs by taking two “slices” through the equilibria illustrated in Figure 1, the first in which we assume \(\sigma = 0.05\) and \(g = 0.62\) and the second in which \(\sigma = 0.05\) and \(g = 0.7\).

Given the utility function \(u(q_1, q_2)\), standard analysis (see, for example, Singh and Vives, 1984) gives individual consumer surplus as:
The relationship between distance costs and consumer surplus for our two parameter combinations is illustrated in Figures 2(a) and (b).

For any given product line configuration, consumer surplus is decreasing in distance costs. However, with endogenous product line choice, a more complicated picture emerges. The relationship between consumer surplus and distance costs is not monotonic. Simply put, if a small change in distance costs results in a broadening of the equilibrium product line then consumers actually benefit from the increased distance costs.

For firms matters are very different, as can be seen from Figures 3(a) and (b). With any given product line configuration, profit decreases with distance costs \( \tau \) so long as distance costs are “low” – typically, less than 0.2 – but increases with distance costs at higher levels of \( \tau \). The intuition has been discussed in the previous section. When distance costs are low (high) the export cost effect dominates (is dominated by) the import protection effect. In other words, our analysis challenges the conventional wisdom that firms always benefit from higher barriers to trade. This will be the case only when exports are small as a proportion of total output.

Matters become even more complicated once we allow for the impact of distance costs on the choice of product lines. An increase in distance costs that leads both firms to adopt narrower product lines benefits both firms (Figure 3a for \( \tau \) “very low”). By contrast, suppose that the increase in distance costs leads a firm’s rival to broaden its product line. The firm whose product line is unchanged certainly loses while the firm whose product line broadens may gain. In Figure 3, an increase in distance costs that

\[
(21) \quad cs(q_1, q_2) = \frac{\beta}{2} (q_1^2 + q_2^2) + \gamma q_1 q_2
\]

The relationship between distance costs and consumer surplus for our two parameter combinations is illustrated in Figures 2(a) and (b).
leads to a switch from configuration (1, 1) to (2, 1) harms the single-product firm but may benefit the firm that has broadened its product line. An increase in distance costs that leads to a switch from (2, 1) to (2,2) benefits the firm that changes to the broader product line while harming its rival. In other words, firms need to be aware of the unintended consequences of pushing for protection. This can backfire if the protection induces the rival to broaden its product line.\(^7\)

5. **Conclusions**

This paper has analyzed product line rivalry in a market environment in which non-cooperative firms have preferential access to a local market but, by incurring “distance” costs, can also supply a rival’s market. We show, in common with non-spatial analysis of product line markets, that product lines are broader and more overlapped in large markets and in markets in which consumers view the products on offer as being highly differentiated.

The result is that consumers unambiguously gain from increased market size or from investment by firms in product innovation that increases perceived differentiation.\(^8\) By contrast, firms need not benefit from such changes. Consider a marginal change in market size that changes the perfect equilibrium from a narrow, non-overlapped product line to a broad, fully overlapped product line. The latter equilibrium typically has prisoners’ dilemma characteristics. The firms would prefer to maintain narrow product

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\(^7\) This is reminiscent of the same kind of result in the analysis of foreign direct investment. Increased trade barriers may actually harm a domestic firm if the protective barriers induce its rivals to switch from exporting to direct production.

\(^8\) We do not model such process innovation explicitly but our analysis can be extended to incorporate process innovation using the approach, for example, of Lin and Saggi (2002). We leave this extension to subsequent research.
lines in the now larger market but cannot commit to doing so in the absence of some form of tacit or explicit cooperation.

Introducing “distance” costs enriches the analysis in a number of important ways. First, our analysis adds to the literature on intra-industry trade by highlighting the link between trade costs and firms’ choices, not just of how much to export but also of which products to produce and which of these to export.

Second, if market parameters are such that both firms adopt broad product lines we identify yet another prisoners’ dilemma. Both firms choose to export both products whereas they would both be better off exporting only one – or none.

Third, we identify a further reason for firms to seek greater protection from imports. Not only do such trade barriers make imports more expensive, they may lead to an equilibrium in which product lines are narrower and not overlapped, further reducing competitive pressures and intra-firm cannibalization. We show, however, that care should be exercised in suggesting that increased distance costs are always beneficial for the local firm. The relationship between distance costs and product line breadth is actually non-monotonic. Beyond some limit (in our formal analysis, for $\tau > 0.2$) an increase in distance costs will actually broaden product line choice. This is potentially beneficial for the firm whose product line is extended but is unambiguously harmful for a firm whose rival’s product line is extended.

Consumers typically benefit from a broad reduction in distance costs as we might expect. An important exception arises, however, when the reduction induces firms to adopt narrower product lines. This can sufficiently weaken competition such that consumer surplus actually falls.
6. References


Figure 1: Equilibrium Product Line Configurations
Figure 2 (a): Consumer Surplus – $g = 0.62$
Figure 2 (b): Consumer Surplus – $g = 0.7$
Figure 3 (a): Profit – $g = 0.62$
Figure 3 (b): Profit – $g = 0.7$