Product Line Rivalry: How Box Office Revenue Cycles Influence Movie Exhibition Variety**

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Abstract

We analyze how product line rivalry by multi-product oligopolists is affected by market size and product substitutability. We show that the width and degree of overlap in competing product lines is determined by the tension between two effects: the drive to “be where the demand is” and the desire to weaken competition and intra-firm product cannibalization. Product lines are shown to be wider and more overlapped in large markets and when product substitutability is weak. We provide econometric support for our main hypotheses using data on weekly film programming choices by first-run movie theatres in a large US metropolitan market.

Keywords: Product line rivalry; market size; product substitutability; movie exhibition

JEL Codes: D21, D43; L13; L82

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1. **Introduction**

We analyze the impact of consumer attitudes, market size and strategic interaction on firms’ choices of product lines. The seminal analysis by Hotelling (1929) and its subsequent critique by d’Aspremont *et al.* (1979) have sparked considerable debate on the extent to which we should expect firms to differentiate themselves from their rivals. The competing forces at work have been aptly characterized by Tirole (1988, p. 286). On the one hand, firms seek to imitate each other to “be where the demand is”; on the other hand, they seek to differentiate themselves in order to soften competition. The implication is that we are more likely to see Hotelling’s (1929) “excessive sameness” in markets where price competition is muted.\(^1\)

When we extend the analysis to multi-product firms additional forces affect the product line/output choices. On the one hand, extending a firm’s product line potentially constrains product line choices by the firm’s rivals. On the other hand, extending a firm’s product line potentially cannibalizes sales from the firm’s own products\(^2\) and, if rivals are not constrained in their product line choices, leads to a tougher competitive environment.

Our analysis addresses two further issues that are largely ignored in the literature. We should expect the choice of product line and the degree of product line overlap to be affected by whether the products on offer are viewed as being close substitutes or not.\(^3\) We should also expect to find that these choices are affected by the size of the market, with firms choosing broader product lines and more product line overlap in larger as opposed to smaller markets because there is greater benefit from “being where the demand is” in larger markets.

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1. A classic example is the Anderson/ Neven (1991) result that single-product Cournot competitors supplying a line market will perfectly agglomerate.
2. This underpins the argument by Spence (1976) that close substitutes will be produced by different firms.
3. Brander and Eaton (1984) consider a case with four products, with products 1 and 2 (3 and 4) being closer substitutes for each other than for products 3 and 4 (1 and 2). Our analysis assumes that products are symmetrically differentiated and explicitly considers the impact on product line choice of variation in the degree of substitutability.
Casual empiricism supports our suggestion that the choice between broad and narrow product lines is affected by market size and consumer attitudes. In large markets with diverse consumer tastes we find broad product lines, with firms offering multiple products that are largely indistinguishable from each other: mass automobile production (Ford, GM, Toyota); low-end cameras (Nikon, Minolta, Pentax); low-end computers (Dell, HP, Compaq) are obvious examples. In smaller markets and/or markets in which consumer tastes are rather less diverse, by contrast, we find narrower product lines with firms seeking to supply particular niches: in automobile production BMW and Jaguar or Mercedes-Benz and Lexus are obvious examples.

We test these hypotheses directly using data from the motion-pictures exhibition market in a major metropolitan area. A first-run movie theatre can be thought of as a multi-product firm, with the individual products being the films showing at the theatre on a particular week. Comparing the weekly film programming choices of competing first-run theatres over a period of a year allows us to investigate the impact on programming similarity (product line overlap) of market size and of the degree of substitutability between theatres (proxied as the distance between the competing theatres).

While there is now an extensive literature on spatial competition, the literature on product line rivalry is not extensive: for a recent review, see Manez and Waterson (2001). Eaton and Lipsey (1979) and Schmalensee (1978) were among the first to consider multi-product production, but primarily as an entry-deterring device. Klemperer (1992) develops an address model in which duopolists choose whether to interlace their products or engage in head-to-head competition, on the assumption that consumers incur shopping costs and so cannot freely switch from one seller to the other. Shaked and Sutton (1990), Dobson and Waterson (1996) and Lal and Matutes (1989) develop non-address models of horizontal product differentiation with

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4 Philips (1983) provides an early description of this type of product proliferation.
endogenous product-range decisions, but in each case firms are confined to producing no more than two products. Draganska and Jain (2005) provide an interesting recent analysis of how firms might use product line length strategically. Their analysis focuses on “firms’ decisions about how many products to offer but not which products to offer.” (p. 4) By contrast, we consider both decisions. Anderson and De Palma (2006) provide one of the most general treatments, in which they endogenize both the entry and product line choices, using a nested logit demand formulation. Their focus is on comparing equilibrium with socially optimal outcomes. In doing so, they confine attention to symmetric equilibria, not surprisingly given the technical challenges posed by their approach. We show below that we cannot, in general, assume that equilibria in product line choices will be symmetric.

Brander and Eaton (1984) provide a non-address analysis that is closest to ours. Their formal analysis considers the case in which two firms each produce two products, with two of the four products being closer substitutes than the other two. In their concluding remarks they briefly consider the possibility that there might be equilibria in which both firms produce all four products but this possibility is not explored. Moreover, while the role of market size in determining the width of the product line is hinted at, it is not formally analyzed.

In summary, our theoretical analysis extends the existing literature in a number of important directions. First, we endogenize the product line decision. Second, we investigate the impact of market size and the degree of product substitutability on the product line choice. Third, we show that all but one of the equilibria exhibit prisoner’s dilemma characteristics, with firms choosing product lines that are “too broad”.

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5 There are also analyses of product line competition with vertical product differentiation. See, for example, De Fraja (1996) and Manez and Waterson (2001)
The final point leads us to investigate another possibility. There is an extensive literature on multimarket contact as a strategy to sustain tacitly collusive agreements between competing firms, building on the seminal work of Bernheim and Whinston (1990): recent contributions include Busse (2000), Gupta (2001), and Feinberg (2003). Our analysis implies an alternative, rather different, strategy that can sustain tacit cooperation; what we term mutual forbearance. Firms can increase their profits by agreeing not to invade each other’s markets: in our analysis by reducing the degree of product line overlap. The strategy underpinning such an agreement is simple enough to articulate. Each firm adopts a restricted product line with the trigger strategy that if either firm switches to a more extended product line the non-deviating firm will revert to the Nash equilibrium product line, leading to more product line rivalry and tougher competition.

Mutual forbearance offers several advantages compared to tacit coordination based on multimarket contact. First, on a purely technical note, this strategy has bite even when the markets are symmetric. Second, mutual forbearance is much easier to monitor than are agreements on prices or outputs: I might not know precisely how much my rival is producing of a particular product but I certainly know whether she is producing that product. Third, mutual forbearance is difficult to detect by the antitrust authorities: the fact that firms are not competing head-to-head may be obvious but the firms can argue that this is a simple commercial decision.

The remainder of the paper is organized as follows. We outline the basic model in section 2. Sections 3 and 4 present and discuss the nature and welfare properties of the subgame perfect equilibria of the model. In section 5 we investigate the conditions under which firms might wish to adopt mutual forbearance in their choice of product lines. Section 6 tests our main

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6 In the Bernheim and Whinston analysis, slack enforcement power in one market is used to sustain cooperation in another market where cooperation would not otherwise be sustainable. This is effective only if the two markets are asymmetric.
hypotheses using data on the weekly programming choices of first-run movie theatres in the Greater Boston metropolitan area. Brief concluding remarks are given in section 7.

2. The Model

We use a variant of a consumer demand model first formulated by Shubik and Levitan (1980). The market is supplied by Cournot duopolists $A$ and $B$ who choose their product lines from $M$ differentiated goods. The output by firm $k$ of differentiated product $m$ is $q_{km}$ ($m = 1, \ldots, M; k = A, B$). Denote by $N \subseteq M$ the set of differentiated goods for which there is strictly positive aggregate supply: $N = \{i \in M: q_{Ai} + q_{Bi} > 0\}$. Suppose that $N$ contains $n$ of the $M$ differentiated goods. Then individual consumer utility is assumed to be

$$U = V \sum_{i \in N} q_i - \frac{n}{2(1+\gamma)} \left[ \sum_{i \in N} q_i^2 + \frac{\gamma}{n} \left( \sum_{i \in N} q_i \right)^2 \right] + y$$

In (1) $y$ is an outside good produced competitively, $q_i > 0$ is individual consumption of the $i$th differentiated good in $N$ supplied to the market, $V$ is a positive parameter and $\gamma \in [0, \infty]$ is a direct measure of the degree of substitutability between the differentiated goods. Note that since the utility function is quasi-linear, consumers’ decisions with respect to consumption of the differentiated goods are independent of their consumption decisions with respect to the outside good.

Maximizing (1) subject to the standard budget constraint gives the inverse demand function for the $i$th good:

$$p_i = V - \frac{1}{1+\gamma} \left( nq_i + \gamma \sum_{j \in N} q_j \right) \quad (i \in N)$$

Inverting this system and aggregating over $s$ consumers (where $s$ is an index of market size) gives the aggregate direct demand functions (Motta (2004)):
(3) \[ Q_i = s q_i = \frac{s}{n} \left( V - p_i (1 + \gamma) + \sum_{j \in N_i} \gamma p_j \right) \quad (i \in N) \]

This demand system has two particularly attractive features for our purposes. First, aggregate demand “does not depend upon the degree of substitution” between products. Second, “in the case of symmetry … aggregate demand does not change with the number of products \( n \)” (Motta op. cit., p. 253). In other words, the utility function (1) does not exhibit a “love of variety”, by contrast, for example, with the Dixit/Stiglitz (1977) utility function. This feature is particularly important for our analysis. It implies that a decision by a firm to extend its product line does not change aggregate demand but rather results in a finer segmentation of the existing market. There is little reason to believe, for example, that adding (deleting) an additional variant of the Ford Focus, or Head and Shoulders shampoo, or Dell Latitude laptop will increase (decrease) aggregate demand for small cars, shampoo or laptops.\(^7\)

We follow Eaton and Schmitt (1994) and assume that each firm incurs three sets of costs. Suppose that firm \( k \) chooses to produce a subset \( N_k \) of the differentiated goods, with \( N_k \) containing \( n_k \) goods. Then firm \( k \)’s total costs are

(4) \[ C_k = F + (n_k - 1) c + v \sum_{i \in N_k} q_{ik} \quad (k = A, B) \]

In (4) \( F \) is a set-up cost, \( v \) is constant marginal cost, and \( c \) is a switching cost; the cost of re-tooling as firm \( k \) switches production from one product to another. As can be seen, production is assumed to exhibit both economies of scale and economies of scope.

Since we do not model the entry decision we can normalize \( F \) and \( v \) to zero without loss of generality. It follows that the profit of firm \( k \), given that it chooses product line \( N_k \) is:

\(^7\) These properties also characterize the Hotelling and logit (discrete choice) models that have become the standard workhorses in the analysis of product differentiation. Note further that while aggregate demand may be unaffected by the width of the product line, the individual firm’s market share will be affected by its choice of product line.
\( \Pi_k = \sum_{i \in N_k} p_i q_{ki} - (n_k - 1)c \) \hspace{1cm} (k = A, B) 

where \( p_i \) is given by (2), \( N = N_A \cup N_B \) and \( n \) is the number of distinct products in \( N \). The firms compete in a two-stage game. In the first stage they choose their product lines \( N_k \) and in the second stage they compete in quantities. The solution concept is, as usual, subgame perfection.

3. Analysis

In the interests of tractability, we follow Brander and Eaton and assume in the remainder of the analysis that \( M = 4 \); that is, each firm can produce up to four products. This implies that there are 225 potential strategy combinations in the second-stage quantity game. However, many of these are strategically equivalent: for example, \( \{1, 2, 3\}, \{3, 4\} \) is strategically equivalent to any strategy combination in which firm \( A \) (\( B \)) produces three (two) of the four products, with the firms having one product in common, such as \( \{1, 2, 4\}, \{1, 3\} \). We adopt the numbering convention that when the product lines are not perfectly overlapped firm \( A \) produces the lower indexed products and firm \( B \) the higher indexed products.

Solution of the two-stage game is considerably eased by the following important result that characterizes the pay-off matrix for the first-stage product line game:

**Lemma 1:** Suppose that inverse demand is given by (1) and that \( n_k < 4 \) \( (k = A, B) \). Then a strategy for firm \( k \) \( (k = A, B) \) that exhibits less product line overlap strictly dominates a strategy with more product line overlap.

For example, suppose that firm \( A \) chooses \( \{1, 2, 3\} \). Then for firm \( B \) \( \{1, 2, 3\} \) is strictly dominated by any three-product strategy that includes product 4, \( \{1, 2\} \) is strictly dominated by any two-product strategy that includes product 4 and \( \{1\}, \{2\} \) and \( \{3\} \) are strictly dominated by

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8 Recall from our discussion of equation (1) that \( n \) does not necessarily equal \( n_A + n_B \).
9 Note, however, that we assume these four products to be symmetrically differentiated.
10 This is analogous to the assumption in the spatial model that one firm always locates “to the left” of the other.
11 In the interest of brevity, the lengthy proof is omitted. Details can be obtained from the authors on request.
The intuition underlying Lemma 1 is simple enough. When given the choice, firms prefer indirect competition – introducing a product that the rival is not offering – to direct competition – introducing a product that goes head-to-head with a rival’s product.\textsuperscript{12} Our numbering convention and Lemma 1 imply that we can restrict the product line strategy space to \{\{1, 2, 3, 4\}, \{1, 2, 3\}, \{1, 2\}, \{1\}\} for firm A and \{\{1, 2, 3, 4\}, \{2, 3, 4\}, \{3, 4\}, \{4\}\} for firm B. The equilibrium quantities and prices for the second-stage quantity subgame are presented in Table 1.

(Table 1 near here)

With one exception, there is no problem with existence of equilibrium in this subgame. The exception arises with any product line choices in which one firm produces all four products and the other firm produces only one. Output by the four-product firm of the one product in which there is head-to-head competition is positive only if the degree of substitutability, $\gamma$, is less than 8. For any higher degree of substitutability, the four-product firm prefers not to offer the overlapped product.\textsuperscript{13} This is an extreme version of a pattern that characterizes all of the equilibria in Table 1. Suppose that firm B narrows its product line by dropping a product that is in head-to-head competition with one of firm A’s products. This obviously benefits firm A, leading firm A to increase its output of that product. But firm B also benefits from reduced intra-firm product competition, allowing firm B to increase output of its remaining products. The result is that firm A reduces the output of those products that are still in head-to-head competition with firm B’s product line while firm B increases its output of these same products.

Equilibrium prices behave as we might have expected. Prices are lower when product lines are more extensive and are lowest, at $V/3$, when products are in head-to-head competition.\textsuperscript{14}

\textsuperscript{12} This result contrasts sharply with spatial Cournot models of single-product firms, where agglomeration is the typical equilibrium – Anderson and Neven (1991).
\textsuperscript{13} We take this into account in solving the product line first-stage game; it has no practical effect on the equilibria.
\textsuperscript{14} Given Lemma 1, this arises only in strategy combinations such that all four products are offered.
Now consider the first-stage product line game. Our model is characterized by four parameters, $s$, $V$, $c$ and $\gamma$. However, given the results of the second-stage quantity subgame in Table 1, profit to firm $k$ in any strategy combination in which $k$ produces $n_k$ products is of the form $sV^2f(\gamma) - (n_k - 1)c$. Define:

$$\alpha = \frac{sV^2}{c}$$

Then profit to firm $k$ in any strategy combination is of the form $sV^2[f(\gamma) - (n_k - 1)/\alpha]$. In other words, equilibrium for the first-stage product line game is fully determined by the two parameters $\alpha$ and $\gamma$. We then have the following:

**Proposition 1:** Equilibrium to the first-stage product line game is:

- $\{1, 2, 3, 4\}, \{1, 2, 3, 4\}$ for $\alpha > \alpha_{44}(\gamma)$
- $\{1, 2, 3\}, \{2, 3, 4\}$ for $\alpha \in (\alpha_{33}(\gamma), \alpha_{33}\uparrow(\gamma))$
- $\{1, 2\}, \{3, 4\}$ for $\alpha \in (\alpha_{22}(\gamma), \alpha_{22}\downarrow(\gamma))$
- $\{1, 2\}, \{4\}$ for $\alpha \in (\alpha_{11}(\gamma), \alpha_{22}\downarrow(\gamma))$
- $\{1\}, \{4\}$ for $\alpha < \alpha_{11}(\gamma)$.

Where:

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15 We do not explicitly report the pay-off matrix. This can be obtained from the authors on request.
\( \alpha_{44}(\gamma) \equiv 9(4 + 3\gamma) \)
\( \alpha_{33}(\gamma) \equiv \frac{36(4 + 3\gamma)(8 + 7\gamma)^2}{256 + 432\gamma + 185\gamma^2} \)
\( \alpha_{33}(\gamma) \equiv \frac{9(8 + 7\gamma)^2(32 + 40\gamma + 11\gamma^2)^2}{(1 + \gamma)(2048 + 6016\gamma + 6552\gamma^2 + 3110\gamma^3 + 535\gamma^4)} \)
\( \alpha_{22}(\gamma) \equiv \frac{9(4 + 3\gamma)^2(32 + 40\gamma + 11\gamma^2)^2}{(1 + \gamma)(512 + 1344\gamma + 1332\gamma^2 + 584\gamma^3 + 93\gamma^4)} \)
\( \alpha_{22}(\gamma) \equiv \frac{9(4 + 3\gamma)^2(6 + 6\gamma + \gamma^2)^2}{(1 + \gamma)(216 + 540\gamma + 477\gamma^2 + 173\gamma^3 + 21\gamma^4)} \)
\( \alpha_{11}(\gamma) \equiv \frac{18(4 + 3\gamma)^2(6 + 6\gamma + \gamma^2)^2}{(1 + \gamma)(432 + 1080\gamma + 977\gamma^2 + 356\gamma^3 + 39\gamma^4)} \)

Each of these functions is increasing in \( \gamma \). Proposition 1 is illustrated in Figure 1.

(Figure 1 near here)

The intuition underlying Proposition 1 is relatively straightforward. Consider the strategy combination ({1}, {4}). The binding constraint for this to be an equilibrium configuration is that \( \alpha < \alpha_{11}(\gamma) \), in which case firm A (B) prefers to offer only one product given that firm B (A) is also offering only one product. By contrast, for \( \alpha \in (\alpha_{11}(\gamma),\alpha_{22}(\gamma)) \), firm A (B) prefers to offer two products given that firm B (A) is offering one, and the ({1}, {4}) strategy combination breaks down. Similar properties apply to each of the other equilibria: the critical values \( \alpha_{33u}, \alpha_{22u} \) and \( \alpha_{11} \) of equation (7) are those above which a firm prefers to offer \( n + 1 \) products if its rival is offering \( n \) products, with \( n = 3, 2 \) and 1 respectively.

Product lines are broader and product line overlap more extensive when \( \alpha \) is high or \( \gamma \) is low. The product line choice balances two forces: competition with the rival and cannibalization of the firm’s own products. A low value of \( \gamma \) is equivalent to the degree of substitutability between products being low, implying that cannibalization and competition
effects are both weak, encouraging broader product lines and more product line overlap. The opposite is the case for high values of $\gamma$.

We know from equation (6) that a high value of $\alpha$ can result from any combination of three factors: large market size ($s$), higher consumer reservation price ($V$) and/or low product switching costs ($c$). For any given degree of substitutability there is a critical value of $\alpha$ ($\alpha_{44}(\gamma)$) above which both firms choose the broadest product line with perfect product line overlap and a critical value of $\alpha$ ($\alpha_{11}(\gamma)$) below which the firms choose the narrowest product lines with no overlap.16 These critical values are higher the greater is the degree of substitutability between the products, reflecting a tension between, for example, large market size that encourages broad product lines and high product substitutability that encourages narrow product lines. Given our specific interest in the relationship between market size and product line rivalry we can state:

**Corollary 1:** No matter the degree of substitutability, as market size increases equilibrium moves from narrow product lines with no product line overlap to broad product lines with perfect product overlap.

### 4. Welfare Comparisons

In this type of differentiated product model total surplus is aggregate utility minus total cost. A complicating factor in evaluating total surplus in the different strategy combinations, however, is that these comparisons are not functions solely of $\alpha$ and $\gamma$. Define:

\[
\alpha_{s44}(s, \gamma) = \frac{9s(8 + 7\gamma)^2}{2(5 + 4\gamma)}
\]

\[
(8)
\]

\[
\alpha_{s33}(s, \gamma) = \frac{9s(32 + 52\gamma + 21\gamma^2)^2}{160 + 384\gamma + 299\gamma^2 + 75\gamma^3}
\]

Then we have:

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16 They are, of course, still in competition with each other since the products are substitutes.
**Lemma 2:** (a) Total surplus is less with strategy combination \((\{1, 2, 3, 4\}, \{1, 2, 3, 4\})\) than \((\{1, 2, 3\}, \{2, 3, 4\})\) if \(\alpha < \alpha_{\text{ts}44}(s, \gamma)\);

(b) Total surplus is less with strategy combination \((\{1, 2, 3\}, \{2, 3, 4\})\) than \((\{1, 2\}, \{3, 4\})\) if \(\alpha < \alpha_{\text{ts}33}(s, \gamma)\)

(c) Total surplus is always less with strategy combination \((\{1, 2\}, \{3, 4\})\) than \((\{1\}, \{4\})\).

Comparison with the critical values of \(\alpha\) in equation (7) implies that, provided market size is not “very large”, total surplus will typically be reduced by more extensive product lines. The potentially ambiguous impact of broader product lines on total surplus derives from the very different effects that broader product lines have on producers and consumers. If we confine our attention solely to the equilibrium outcomes of Proposition 1, it is easy to show the following:

**Lemma 3:** Consumers gain and producers lose from broader product lines.

Broader product lines with more product line overlap lead to more intense inter- and intra-firm competition, lowering prices and benefiting consumers. Since we assume that there is no “love of variety” in our setting, there is no countervailing increase in aggregate demand. Rather, more extensive product lines lead to a finer segmentation of the overall market. The result is that profits fall and consumer surplus rises as product lines broaden.\(^17\)

5. **Mutual Forbearance**

An immediate implication of Lemma 3 is that every symmetric subgame perfect product line configuration other than \((\{1\}, \{4\})\) (and its equivalents) has prisoner’s dilemma characteristics. Both firms would prefer narrower product lines and less product line overlap but in a one-shot-game are unable to sustain a narrowing of their product lines. Suppose, however, that the game is repeated indefinitely. This raises the possibility that the firms can tacitly agree

\(^{17}\) There is one partial exception. In comparing \((\{1, 2\}, \{4\})\) with \((\{1\}, \{4\})\), when the former is an equilibrium, firm \(A\) gains from the more extensive product line and firm \(B\) loses. However, aggregate profit in the configuration \((\{1, 2\}, \{4\})\) is less than aggregate profit with \((\{1\}, \{4\})\).
to adopt narrower product lines, with the agreement supported by a trigger strategy, even if they continue to act non-cooperatively in the second-stage quantity subgame.

There is the further possibility, of course, that the firms might coordinate both stages of the game – product lines and outputs. An obvious reason for concentrating solely on cooperation in the first-stage product line game is that cooperation in the quantity-setting subgame will typically be more difficult to monitor in any “real life” setting. By contrast, an agreement to restrict product lines is relatively easy to police: I might not know precisely how much of a product a rival is producing, but I certainly know whether she is producing that product.

In demonstrating the sustainability of a tacit agreement to restrict product lines, but not outputs, we confine our attention to the case in which the subgame perfect equilibrium strategy combination is (\{1, 2, 3, 4\}, \{1, 2, 3, 4\}). Now consider the familiar trigger strategy for firm \(k\): “I shall continue to restrict my product line in the current period so long as we have both restricted our product lines in every previous period. Otherwise I shall extend my product line to four products.” Standard analysis indicates that the agreement to restrict product lines to \(r\) products is sustainable for any discount factor greater than \(d_r\), where

\[
\begin{align*}
  d_3 &= \frac{\alpha(256 + 432\gamma + 185\gamma^2) - 36(4 + 3\gamma)(8 + 7\gamma)^2}{5\alpha(8 + 7\gamma)^2} \\
  d_2 &= \frac{\alpha(64 + 88\gamma + 33\gamma^2) - 72(2 + \gamma)(4 + 3\gamma)^2}{5\alpha(4 + 3\gamma)^2} \\
  d_1 &= \frac{\alpha(208 + 304\gamma + 123\gamma^2) - 108(4 + \gamma)(4 + 3\gamma)^2}{15\alpha(4 + 3\gamma)^2}
\end{align*}
\]

Figure 2 presents some contour plots of \(d_r\). In all three cases the critical discount factor is increasing in \(\alpha\) and decreasing in \(\gamma\). In other words, a tacit agreement to restrict product lines is more likely to be sustainable in small markets and/or when the degree of product substitutability is high. No matter the market size or degree of substitutability, it further follows from (9) that an
agreement to restrict product lines to 2 or 3 products is sustainable for any discount factor greater than 4/5 and to a single product for any discount factor greater than 13/15.\(^{18}\)

(Figure 2 near here)

6. **An Application to the Movie Exhibition Market**

The weekly movie programming choices by first-run movie theatres in a well-defined metropolitan area provides an interesting test of our theoretical analysis for several reasons. First, if we take the screening of an individual movie as a single product, the individual movie theatre is an obvious example of a multi-product firm. Second, given that competing first-run movie theatres can, and do, show many of the same films, this is an obvious case of the type of head-to-head competition that our theoretical analysis envisages. Third, given that we can identify the movie theatre programming choices of competing first-run movie theatres, we can generate a simple measure of similarity in programming choices.\(^{19}\) Fourth, there is considerable seasonality in movie theatre attendance, with movie audiences building up as we approach major holidays and declining thereafter.\(^{20}\) This provides us with a direct test of our first hypothesis, that product line overlap as measured by movie programming similarity, is greater when the market is larger. Finally, the first-run movie theatres in our sample are located at different distances from each other. It seems reasonable to suggest that we can treat distance between theatre pairs as a measure of substitutability – the more distant are two theatres from each other, the lower the degree of substitutability they exhibit. Then we have a test of our second hypothesis, that the degree of product line overlap (programming similarity) is inversely related to substitutability.

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\(^{18}\) These are equivalent respectively to per period discount rates less than 25 per cent and 15.4 per cent.

\(^{19}\) We exclude from our study art house and second-run theatres on the assumption that such theatres serve markets with consumer preferences for different attributes (i.e., film offerings and currency of film) from the first-run theatre market.

Our econometric analysis uses data drawn from the first-run motion-pictures exhibition market within the I-95 loop of the Boston greater metropolitan area, which contains 13 first-run theatres. For each theatre, for each week from June 30, 2000 through the week of June 22, 2001, we have information from Nielsen EDI on which films were playing and on the revenues generated at each theatre by each film for that week. We supplemented these data by recording screening times on the Friday of each week, for each film, for each theatre in our data set. Screening-time information was determined by reviewing *Boston Globe* movie advertisements on microfilm.

Our measure of programming similarity uses an approach suggested by Jaffe (1986): see also Chisholm, McMillan and Norman (2006) and Sweeting (2006). For each theatre pair $ij$ we construct a weekly angular similarity index $a_{ijt}$ distributed on the interval $[0, 90]$. $a_{ijt} = 0$ indicates perfect overlap in programming choices between theatres $i$ and $j$ in week $t$ and $a_{ijt} = 90$ reflects no overlap. In other words, $a_{ijt}$ is an inverse measure of programming similarity – or a direct measure of disimilarity. Details of how $a_{ijt}$ is constructed are provided in the Appendix. We then divided the total set of $ij$ pairs into three subsets - theatre pairs within five miles of each other, theatre pairs between five and ten miles apart and theatre pairs more than 25 miles apart – and calculated the weekly mean of $a_{ijt}$ for each subset, denoted $A_{5t}$, $A_{10t}$ and $A_{25t}$, respectively. These are illustrated in Figure 3, together with $HOLIDAYDIST_t$, the number of weeks that week $t$ is from the nearest major holiday.\(^{21}\)

(Figure 3 near here)

Summary statistics for the similarity measures are given in Table 2. These statistics indicate that while there is an underlying degree of similarity in film programming choice across

\(^{21}\) The holidays that we consider are Memorial Day, the Fourth of July, Thanksgiving, and Christmas. We discuss the interpretation of $HOLIDAYDIST_t$, below.
the theatre pairs in our sample, there is also considerable variability in film programming choice. The data are also supportive of our hypothesis that programming similarity will decrease (the angular measure $A_{dt}$ should increase) as substitutability increases. Applying a $t$-test to compare the time series for $A_{5t}$, $A_{10t}$ and $A_{25t}$, we cannot reject the hypothesis that $A_{5t} > A_{10t}$ and $A_{10t} > A_{25t}$ at the .01 significance level.

In order to investigate the impact of market size on programming similarity, we construct two measures of market size. $HOLIDAYDIST_t$, as noted above, is the number of weeks that week $t$ is from the nearest major holiday. We use $HOLIDAYDIST_t$ as an inverse measure of market size, reflecting the work of Einav (2007), who finds in his analysis of seasonality in movie theatre revenues that movie audiences build up as we approach major holidays and decline thereafter. Our second market size measure is $AGGREV_t$, aggregate revenue (in millions of dollars) in week $t$ of the first-run theatres in our sample. While appealing on its face, there are at least two reasons for our having reservations in taking $AGGREV_t$ as a measure of market size. First, this is an ex post realization of revenues in week $t$ whereas we would have preferred a measure of expected revenues. Second, there is the possibility that aggregate revenue overstates the true seasonality in movie theatre attendance (Einav op. cit.) since the expected boxoffice appeal and revenue-generating potential of the movies being released is likely to be greater nearer to major holidays.

We estimate the following reduced form:

$$A_{dt} = \alpha_d + \beta_d \cdot MARKETSIZE_t + \varepsilon_{dt}$$

where $d = 5, 10, 25$ and $MARKETSIZE_t$ is one of the two proposed measures of market size in week $t$ ($t = 1 \ldots 52$).
Recall that $A_{dt}$ is an inverse measure of similarity, $HOLIDAYDIST_t$ is an inverse measure of market size and $AGGREV_t$ is a direct measure of market size. We expect, therefore, that $\beta_d$ will be positive (negative) when market size is measured by $HOLIDAYDIST_t$ ($AGGREV_t$). Second, we expect that $\alpha_5 > \alpha_{10} > \alpha_{25}$. Returning to Figure 1, we can draw the further inference that the degree of product line overlap, or programming similarity, is likely to be more sensitive to market size the lower is the degree of substitutability. In other words, we should expect to find $|\beta_{25}| > |\beta_{10}| > |\beta_5|$.

The results are summarized in Table 3. With $HOLIDAYDIST_t$ as our measure of market size, the results are consistent with all three hypotheses. With $AGGREV_t$ as our measure of market size, there is some support for the hypothesized relationship between programming similarity and either market size or substitutability, but not for the hypothesis that programming similarity is likely to be more sensitive to market size when substitutability is low. This is, perhaps, a result of the reservations we noted above regarding the use of $AGGREV_t$ as a measure of market size, but is something that should be investigated in more detail in subsequent work, for example by using Einav’s approach to identify the “true” seasonality in aggregate revenues.

(Table 3 near here)

Some further points are perhaps necessary in support of our suggested interpretation of these results. It might be suggested that the impact of distance reflects the operation of the clearance system in film programming, which constrains theatres close to each other from showing the same films. There are at least two reasons for rejecting this suggestion. First, as can be seen from Figure 3, there is considerable underlying pair-wise similarity in programming choice even for theatres that are very close together, which suggests that the clearance system is not of particular importance. Second, if (di)similarity is driven by the clearance system we
would not find the systematic effect of distance on (di)similarity that we do find and that is predicted by the model.

It might also be argued that the impact of $HOLIDAYDIST_t$ and $AGGREV_t$ on programming similarity results from the wide release of major movies at holiday weekends, to some extent forcing similarity in these weekends, rather than from the impact of market size. There are at least three reasons for rejecting this interpretation. First, “forced” similarity in having to carry “blockbusters” can be unwound by the theatre manager in his choice of the other films to be shown in any given week. Second, if there is such a “blockbuster” effect, we would not find a systematic impact of $DISTANCE$ on similarity, since all theatres would be forced to carry the same films, no matter their relative locations. Third, if programming similarity were driven by a “blockbuster” effect rather than the market size effect that we suggest we would not find the build-up in programming similarity as we approach the holiday weekend “from the left”.

7. **Concluding Remarks**

Will competing multi-product firms choose wide product lines with extensive product line overlap or narrow product lines with little overlap? We have shown in this paper that the answer to this question is critically dependent upon the interplay between market size and product substitutability. This reflects the balance that firms must strike in their choice of product lines. On the one hand, there are benefits from wider product lines to “be where the demand is”.

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22 We are grateful to an industry expert on the movie exhibition market for this argument.

23 There is the further point that if the blockbuster interpretation is to be accepted $HOLIDAYRIGHT$ (a measure of the number of weeks from the most recent past holiday) should dominate $HOLIDAYDIST$ as an explanatory variable - it does not (details can be obtained from the authors on request.) Moreover, Einav’s analysis (op. cit.) identifies a steady build-up in movie-going as we approach major holidays and a decline thereafter, again consistent with our market size interpretation of $HOLIDAYDIST$ and $AGGREV$. 
On the other hand, there are costs of wider product lines in tougher competition as a result of more extensive product line overlap and increased intra-firm product cannibalization.

We have shown that the benefits of broad product lines are more likely to outweigh the costs in large markets, which favor being where the demand is, or when product substitutability is low, since this weakens inter-firm competition and intra-firm product cannibalization. In other words, our analysis yields three testable hypotheses. We should expect to find broader product lines and more product line overlap in large markets or in markets where product substitutability is low. We should further expect to find that product line choice is more sensitive to market size in markets where product substitutability is low.

We have tested these hypotheses and give them broad support, using evidence from film programming choice in the Boston first-run movie exhibition market.

Our analysis suggests several avenues for further research. The current model imposes an exogenous maximum degree of product-line expansion on firms, allowing endogenous choice of product line only within this limit. In addition, we have paid no attention to how a firm introduces an additional product. This is not unreasonable in the first-run movie exhibition market where the number of screens at a theatre, which is expensive to change, limits the number of films that can be shown and where the films themselves are supplied to the theatres by movie distributors rather than produced “in house”. In other contexts, however, fully endogenizing the product line decision, and the innovation process by which additional products are brought to market, would be useful extensions to the product-line rivalry literature.

Second, while our formal model is static, there clearly are interesting dynamic questions regarding when new products should be introduced and existing products dropped. Some progress has been made on these questions for the first-run movie exhibition market (Chisholm
and Norman, 2006) and for the computer mainframe market (Greenstein and Wade, 1998), but more extensive analysis, both analytical and empirical, is needed.
Figure 3: Programming Similarity and Distance
## Table 1: Equilibrium Quantities and Prices

<table>
<thead>
<tr>
<th>Product Line</th>
<th>Equilibrium Quantities</th>
<th>Equilibrium Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1,2,3,4), {1,2,3,4})</td>
<td>(q_{A1} = \frac{SV}{12}); (q_{A2} = q_{A3} = q_{A4} = \frac{SV(8 + 5\gamma)}{24(4 + 3\gamma)})</td>
<td>(p_1 = \frac{V}{3}); (p_2 = p_3 = p_4 = \frac{V}{3})</td>
</tr>
<tr>
<td>((1,2,3,4), {2,3,4})</td>
<td>(q_{A1} = \frac{SV}{8}; q_{A2} = \frac{SV(4 + \gamma)}{24(2 + \gamma)}); (q_{B2} = q_{B3} = q_{B4} = \frac{SV(1 + \gamma)}{6(2 + \lambda)})</td>
<td>(p_1 = p_2 = \frac{V(3 + \gamma)}{3(2 + \gamma)}); (p_3 = p_4 = \frac{V}{3})</td>
</tr>
<tr>
<td>((1,2,3,4), {3,4})</td>
<td>(q_{A1} = q_{A2} = q_{A3} = q_{A4} = \frac{SV(8 - \gamma)}{24(4 + \gamma)}); (q_{B2} = q_{B3} = \frac{2SV(1 + \gamma)}{3(8 + 7\gamma)})</td>
<td>(p_1 = p_2 = p_3 = \frac{V(6 + \gamma)}{3(4 + \gamma)}); (p_4 = \frac{V}{3})</td>
</tr>
<tr>
<td>((1,2,3,4), {4})</td>
<td>(q_{A1} = \frac{SV(1 + \gamma)}{8 + 7\gamma}); (q_{A2} = q_{A3} = q_{B2} = q_{B3} = \frac{2SV(1 + \gamma)}{3(8 + 7\gamma)})</td>
<td>(p_1 = p_4 = \frac{V(12 + 7\gamma)}{3(8 + 7\gamma)}); (p_2 = p_3 = \frac{V}{3})</td>
</tr>
<tr>
<td>((1,2,3), {2,3,4})</td>
<td>(q_{A1} = q_{B4} = \frac{SV(1 + \gamma)(8 + 3\gamma)}{2(32 + 40\gamma + 11\gamma^2)}); (q_{A2} = q_{A3} = q_{B2} = q_{B3} = \frac{2SV(1 + \gamma)(4 + \gamma)}{3(32 + 40\gamma + 11\gamma^2)}); (q_{B4} = \frac{2SV(1 + \gamma)(2 + \gamma)}{32 + 40\gamma + 11\gamma^2})</td>
<td>(p_1 = \frac{V}{3}); (p_2 = \frac{V(48 + 50\gamma + 11\gamma^2)}{3(32 + 40\gamma + 11\gamma^2)}); (p_3 = \frac{V}{3}); (p_4 = \frac{V(48 + 44\gamma + 11\gamma^2)}{3(32 + 40\gamma + 11\gamma^2)})</td>
</tr>
<tr>
<td>((1,2,3), {3,4})</td>
<td>(q_{A1} = q_{A2} = q_{A3} = \frac{SV(1 + \gamma)(4 + 3\gamma)}{64 + 64\gamma + 9\gamma^2}); (q_{B4} = \frac{SV(1 + \gamma)(8 + 3\gamma)}{64 + 64\gamma + 9\gamma^2})</td>
<td>(p_1 = p_2 = p_3 = \frac{V(8 + \gamma)(4 + 3\gamma)}{64 + 64\gamma + 9\gamma^2}); (p_4 = \frac{V(4 + \gamma)(8 + 3\gamma)}{64 + 64\gamma + 9\gamma^2})</td>
</tr>
<tr>
<td>((1,2,3), {4})</td>
<td>(q_{A1} = q_{A2} = q_{A3} = \frac{SV(1 + \gamma)(6 + \gamma)}{6(6 + 6\gamma + \gamma^2)})</td>
<td>(p_1 = p_2 = \frac{V(6 + \gamma)(3 + 2\gamma)}{6(6 + 6\gamma + \gamma^2)}); (p_3 = p_4 = \frac{V(3 + \gamma)^2}{3(6 + 6\gamma + \gamma^2)})</td>
</tr>
<tr>
<td>((1,2), {3,4})</td>
<td>(q_{A1} = q_{A2} = q_{A3} = \frac{SV(1 + \gamma)(6 + \gamma)}{3(6 + 6\gamma + \gamma^2)})</td>
<td>(p_1 = \frac{V(2 + \gamma)}{4 + 3\gamma}); (p_2 = \frac{V(6 + \gamma)(3 + 2\gamma)}{6(6 + 6\gamma + \gamma^2)}); (p_3 = p_4 = \frac{V(3 + \gamma)^2}{3(6 + 6\gamma + \gamma^2)})</td>
</tr>
<tr>
<td>((1,2), {4})</td>
<td>(q_{A1} = q_{A2} = \frac{SV(1 + \gamma)}{4 + 3\gamma})</td>
<td>(p_1 = p_4 = \frac{V(2 + \gamma)}{4 + 3\gamma})</td>
</tr>
<tr>
<td>((1), {4})</td>
<td>(q_{A1} = q_{A2} = \frac{SV(1 + \gamma)}{4 + 3\gamma})</td>
<td>(p_1 = p_4 = \frac{V(2 + \gamma)}{4 + 3\gamma})</td>
</tr>
</tbody>
</table>
Table 2: Summary Statistics for Average Angular Similarity Index by Distance

<table>
<thead>
<tr>
<th>Similarity Measure</th>
<th>$A_t$</th>
<th>$A_{5t}$</th>
<th>$A_{10t}$</th>
<th>$A_{25t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>39.74</td>
<td>50.06</td>
<td>44.94</td>
<td>33.23</td>
</tr>
<tr>
<td>Variance</td>
<td>21.64</td>
<td>19.56</td>
<td>18.39</td>
<td>31.00</td>
</tr>
<tr>
<td>Minimum</td>
<td>30.61</td>
<td>39.72</td>
<td>37.30</td>
<td>22.00</td>
</tr>
<tr>
<td>Maximum</td>
<td>50.72</td>
<td>60.85</td>
<td>56.45</td>
<td>43.71</td>
</tr>
</tbody>
</table>

$A_t$ is the average weekly angular measure of similarity for theatre pairs by distance between theatres over 52 weeks: $A_{5t}$ for less than five miles; $A_{10t}$ between 5 and 10 miles; $A_{25t}$ over 25 miles; $A_t$ across all theatre pairs.

Table 3: Estimation of Average Angular Similarity Index by Distance

<table>
<thead>
<tr>
<th>Variable</th>
<th>$A_t$</th>
<th>$A_{5t}$</th>
<th>$A_{10t}$</th>
<th>$A_{25t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$HOLIDAYDIST_t$</td>
<td>1.005</td>
<td>0.722</td>
<td>0.864</td>
<td>1.182</td>
</tr>
<tr>
<td></td>
<td>(0.149)**</td>
<td>(0.180)**</td>
<td>(0.155)**</td>
<td>(0.163)**</td>
</tr>
<tr>
<td>$CONSTANT$</td>
<td>35.39</td>
<td>46.94</td>
<td>41.20</td>
<td>28.12</td>
</tr>
<tr>
<td></td>
<td>(0.808)**</td>
<td>(0.948)**</td>
<td>(0.677)**</td>
<td>(0.971)**</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.454</td>
<td>0.259</td>
<td>0.394</td>
<td>0.438</td>
</tr>
<tr>
<td>$F$ Value</td>
<td>45.21</td>
<td>16.14</td>
<td>31.16</td>
<td>52.43</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>$A_t$</th>
<th>$A_{5t}$</th>
<th>$A_{10t}$</th>
<th>$A_{25t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AGGREV_t$</td>
<td>-4.464</td>
<td>-0.914</td>
<td>-5.539</td>
<td>-4.782</td>
</tr>
<tr>
<td></td>
<td>(1.308)**</td>
<td>(1.433)</td>
<td>(1.184)**</td>
<td>(1.481)**</td>
</tr>
<tr>
<td>$CONSTANT$</td>
<td>45.44</td>
<td>51.23</td>
<td>52.02</td>
<td>39.35</td>
</tr>
<tr>
<td></td>
<td>(1.902)**</td>
<td>(2.161)**</td>
<td>(1.760)**</td>
<td>(2.120)**</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.170</td>
<td>0.008</td>
<td>0.308</td>
<td>0.136</td>
</tr>
<tr>
<td>$F$ Value</td>
<td>11.64</td>
<td>0.526</td>
<td>21.89</td>
<td>10.42</td>
</tr>
</tbody>
</table>

Estimation by OLS with robust standard errors.
Dependent variable is $A_t$, the average weekly angular measure of similarity for theatre pairs by distance between theatres: $A_{5t}$ for less than five miles; $A_{10t}$ between 5 and 10 miles; $A_{25t}$ over 25 miles; $A_t$ across all theatre pairs. Sample size is 52.
Significance levels *.10, **.05, ***.01; standard errors reported in parentheses.
References


**Appendix: Angular Similarity Index**

Following Jaffe (1986), Chisholm, Norman and McMillan (2006), and Sweeting (2006), suppose that there are $M_t$ different movies being shown in week $t$ in our population of first-run movie theatres. Order these movies alphabetically and define $m_t$ as the $m$th movie being shown (in at least one of the theatres) in week $t$. Then we can create the $M_t \times 1$ vector $p_i = (p_{1,i}, p_{2,i}, \ldots, p_{m,i}, \ldots, p_{M,i})$ for theatre $i$, where $p_{m,i}$ is the percentage of show-times at theatre $i$ devoted to film $m_t$ in week $t$.\(^{24}\)

Given the vectors $p_{ij}$ and $p_{jt}$ for theatres $i$ and $j$ we define the angular similarity index $a_{ijt}$ for this theatre pair as the angle between these two vectors:

$$a_{ijt} = \arccos \left( \frac{p_{ij} \cdot p_{jt}}{\|p_{ij}\| \|p_{jt}\|} \right).$$

An important characteristic of this angular similarity index is that the “measure of proximity is purely directional i.e. it is not directly affected by the length of the ($p$) vectors.” (Jaffe, 1986, p. 986, fn 5)

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\(^{24}\) In the empirical application we measure show-times on the Friday of each week.