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A Comment on The Role of Prices for Excludable Public Goods

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### Abstract

Blomquist and Christensen (2005) argue that welfare is initially decreasing in the price of an excludable public good and that the case for a positive public good price is weak. We argue that this result follows from their particular characterization of the public good and that a more reasonable characterization overturns their result. Hence the policy case for a positive price on the public good is stronger than Blomquist and Christiansen suggest. We also provide a more flexible characterization of public goods that nests a wide variety of public goods models.

### KEYWORDS

public goods  
optimal second-best taxation

## **I. Introduction**

In a recent article Blomquist and Christiansen (2005) argue that a necessary condition to charge a price for an excludable public good in the presence of a non-linear income tax is that the marginal valuation of the public good be increasing in leisure. Furthermore, they argue that given this condition, welfare is initially non-increasing in the price of the public good (in fact decreasing before possibly increasing). Thus if it is uncertain what the optimal second-best price of the public good is, they argue that it may be better to set a zero price than to set a low price in hopes of avoiding overshooting the optimal price.

We argue that this result follows from their particular characterization of the public good and that with another more reasonable assumption, the condition that the marginal valuation of the public good is increasing in leisure is both necessary and sufficient for a positive price for the excludable public good. Hence the policy case for a positive price on the public good is stronger than Blomquist and Christiansen suggest.

In the next section, we discuss different ways to characterize excludable public good access and provide an interpretation for the different results between our model and that of Blomquist and Christiansen. In the following section we set up and solve the model. Section 4 provides an analysis of the Samuelson Condition in our model. We conclude in Section 5 with some summary observations on the flexibility of our model to handle the various types of public goods that have been studied in the literature.

## **II. Modeling Excludable Public Goods**

Blomquist and Christiansen (henceforth B&C) model the government as providing  $G$  units of an excludable public good to which individuals can obtain access by

paying a per unit price  $q$ . Upon payment of  $qg$  they may consume  $g$  units of the public good with the constraint that  $g \leq G$ . They motivate this characterization of excludable public goods with such examples as weather forecasts with  $g$  measuring the amount of information the consumer purchases up to the maximal amount available; an art gallery with  $g$  measuring the number of rooms visited and  $G$  the total number of rooms available; and TV broadcasting where  $g$  measures the number of channels purchased and  $G$  the total number available.

Implicit in B&C's model is the restriction that consumers can purchase the public good only once. Alternatively we characterize excludable public goods as goods that can be consumed repeatedly thereby allowing the government to charge a fee per use of the public good. An on-line web service could charge a fee each time an individual wished to access up-to-date weather information; an art gallery could charge an entrance fee per visit; and a TV network could offer on-demand movies. This strikes us as a reasonable characterization of most excludable public goods (public parks, uncongested highways, museums, for example). Our modeling framework also allows for variation in the consumption of the public good across consumers based on differences in preferences.

In addition, the capacity constraint plays a key role in their model in a way that may mischaracterize most public goods. Excludable public goods can be charged on a per use basis or with all-or-nothing pricing (access pricing). B&C's model appears to be an example of per-unit pricing since different consumers may pay different amounts for the public good (depending on demand). But in fact, their model is better understood as a model of multiple public goods with access pricing once the capacity constraint is binding.

To understand the importance of our distinctions, consider the following examples. There are two drivers using a road. One uses the entire road once per day; the other uses it twice each day. In B&C's model, which (if either) of these drivers is rationed (presuming they'd drive further on a longer road)? If they're both rationed, are they consuming the same amount of the public good? Obviously not.

As another example, consider an art gallery with five sections. I choose to view three of the five sections. In B&C's model, I am not rationed. But now the gallery owner spends money to improve all five sections. The amount of the public good has gone up as has my utility since the sections I do visit have been improved. But in B&C's model, an expansion of  $G$  only raises utility if consumers are rationed. For this example, an increase in  $G$  in B&C's model is the addition of a sixth room on the art gallery. They argue that consumers get no benefit from this sixth room if they only wish to visit three rooms. Their pricing assumption really amounts to an offer of multiple public goods (where each room is a public good and accessible at price  $q$ ).<sup>1</sup>

In fact, all the examples that B&C offer to motivate their model can be viewed as bundles of multiple public goods. Cable television service with ten channels can be viewed as a bundle of ten public goods. The cable company could offer service on a per-channel basis.<sup>2</sup> A weather forecast can be a bundle of multiple public goods. One public good could be a basis service; a second with more detail.

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<sup>1</sup> Note too the strong assumption that consumers receive no benefit from the expansion of  $G$  that might arise from the option value arising their potential desire to visit the sixth gallery room in the future. We do not pursue this idea further in this paper.

<sup>2</sup> In fact, the chairman of the Federal Communications Commission recently proposed just this pricing scheme for satellite and cable television companies. See Labaton (2005) for details.

It may well be that B&C's modeling of public goods is appropriate in certain cases. It is important to determine, however, if B&C's conclusion that there is at best a weak case to be made for charging for excludable public goods is robust to the modeling of the public good. We turn to that question now.

### III. Optimal Pricing of Excludable Public Goods

Following B&C, we assume that there are two types of consumers (high ability and low ability) and that the government utilizes a non-linear income tax to finance the public good. Type  $i$  consumers obtain utility over a consumption good,  $c_i$ , and the public good,  $g_i$ , and leisure,  $L_i$ . They face a time endowment  $Z = H_i + L_i$  where  $H_i$  is the number of hours worked. Their income  $Y_i = w_i H_i$  where  $w_i$  is their wage rate ( $w_2 > w_1$ ). The government observes  $Y_i$  but not hours worked or the wage rate separately. Also as in B&C, the government is decentralized in that the tax agency cannot share income information with the agency providing the public good.

We assume that the public good consumption by type  $i$  consumers is a function of the number of visits to (or utilizations of) the public good,  $v_i$ , and the amount of the public good provided,  $G$ :

$$(1) \quad g_i = f(v_i, G)$$

where the function  $f$  has the following properties:

$$(2a) \quad f(0, G) = f(v, 0) = 0$$

$$(2b) \quad \frac{\partial f}{\partial v_i} > 0$$

$$(2c) \quad \frac{\partial f}{\partial G} > 0$$

$$(2d) \quad \frac{\partial^2 f}{\partial v_i \partial G} > 0.$$

The last condition means that the marginal consumption value of access to the public good increases with the amount of the public good provided. Without loss of generality, we make the further assumption that public good consumption is linear in visits (or utilization).<sup>3</sup> With that assumption, the function  $f(v_i, G)$  takes the form  $v_i h(G)$  with  $h'(v) > 0$  and  $h(0) = 0$ . One simple characterization of public good consumption is  $g_i = v_i G$ .

Thus individuals of type  $i$  maximize the utility function

$$(3) \quad U^i(c_i, g_i, H_i)$$

subject to the budget constraint

$$(4) \quad c_i + p_v \cdot v_i = w_i H_i - T_i = Y_i - T_i(Y_i) = B_i$$

where  $p_v$  is the price per usage for the public good,  $T_i(Y_i)$  is a non-linear income tax and  $B_i$  is after-tax income. We model the price per usage of the public good as comprising a private cost (normalized at 1) and a public charge ( $\tilde{p}_v$ ). The government sets and receives  $\tilde{p}_v$  per visit with  $\tilde{p}_v \geq 0$ . Thus  $p_v = \tilde{p}_v + 1$ . In the absence of a private cost of utilizing the public good, we would observe that consumers make infinite use of the public good if the price were set to zero. Since we don't observe this in public parks and other excludable public goods with zero entrance fees, this seems a reasonable assumption. It costs money and/or time to utilize a public good and we simply want to capture this opportunity cost in our model. The government can choose  $p_v$  or  $\tilde{p}_v$  as its choice variable in its optimization problem described below.

It will be convenient to define the price of the public good,  $p_g$

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<sup>3</sup> This does not imply that utility is linear in visits.

$$(5) \quad p_g = \frac{p_v v_i}{g_i} = \frac{p_v}{h(G)} = \frac{\tilde{p}_v + 1}{h(G)} \equiv \tilde{p}_g + \frac{1}{h(G)}.$$

The price of  $g$  rises with  $p_v$  and falls with  $G$ . While the government's public good instruments are  $\tilde{p}_v$  and  $G$ , we can equivalently characterize them in terms of  $\tilde{p}_g$  and  $G$ .<sup>4</sup>

Lastly, we will find it convenient to work with the utility function in terms of

$$\text{observables, } V_i(B_i, Y_i, \tilde{p}_g, G) = \max_{c_i, g_i} U\left(c_i, g_i, \frac{Y_i}{w_i}\right) + \lambda(B_i - c_i - p_g g_i).$$

The government sets a non-linear income tax for the two types of individuals (with  $N_i$  of each type) in a pareto efficient fashion to raise revenue to help pay for the public good. Specifically, we assume the planner maximizes:

$$(6) \quad \text{Max}_{B_1, Y_1, B_2, Y_2, \tilde{p}_g, G} V_1(B_1, Y_1, \tilde{p}_g, G)$$

subject to

$$(7) \quad V_2(B_2, Y_2, \tilde{p}_g, G) \geq \bar{V}_2$$

$$(8) \quad V_2(B_2, Y_2, \tilde{p}_g, G) \geq \hat{V}_2(B_1, Y_1, \tilde{p}_g, G)$$

$$(9) \quad N_2(Y_2 - B_2) + N_1(Y_1 - B_1) + \tilde{p}_g(N_1 \cdot g_1 + N_2 \cdot g_2) - mG \geq 0$$

where  $m$  is the marginal cost of producing the public good and  $\hat{V}^2$  refers to the utility of a high-ability type mimicking a low-ability type.

The Lagrangian and first order conditions for this problem are

$$(10) \quad \Lambda = V_1(B_1, Y_1, \tilde{p}_g, G) + \beta[V_2(B_2, Y_2, \tilde{p}_g, G) - \bar{V}_2] + \\ \rho[V_2(B_2, Y_2, \tilde{p}_g, G) - \hat{V}_2(B_1, Y_1, \tilde{p}_g, G)] +$$

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<sup>4</sup> Following B&C, we assume that  $\tilde{p}_v$  and  $\tilde{p}_g$  are non-negative.

$$\mu[N_2(Y_2-B_2) + N_1(Y_1-B_1) + \tilde{p}_g(N_1 \cdot g_1 + N_2 \cdot g_2) - mG]$$

$$(11) \quad \frac{\partial \Lambda}{\partial B_1} = \frac{\partial V_1}{\partial B} - \rho \cdot \frac{\partial \hat{V}_2}{\partial B} - \mu \cdot N_1 + \mu \cdot \tilde{p}_g \cdot N_1 \cdot \frac{\partial g_1}{\partial B} = 0$$

$$(12) \quad \frac{\partial \Lambda}{\partial Y_1} = \frac{\partial V_1}{\partial Y} - \rho \cdot \frac{\partial \hat{V}_2}{\partial Y} + \mu \cdot N_1 + \mu \cdot \tilde{p}_g \cdot N_1 \cdot \frac{\partial g_1}{\partial Y} = 0$$

$$(13) \quad \frac{\partial \Lambda}{\partial B_2} = \beta \cdot \frac{\partial V_2}{\partial B} + \rho \cdot \frac{\partial V_2}{\partial B} - \mu \cdot N_2 + \mu \cdot \tilde{p}_g \cdot N_2 \cdot \frac{\partial g_2}{\partial B} = 0$$

$$(14) \quad \frac{\partial \Lambda}{\partial Y_2} = \beta \cdot \frac{\partial V_2}{\partial Y} + \rho \cdot \frac{\partial V_2}{\partial Y} + \mu \cdot N_2 + \mu \cdot \tilde{p}_g \cdot N_2 \cdot \frac{\partial g_2}{\partial Y} = 0$$

$$(15) \quad \frac{\partial \Lambda}{\partial \tilde{p}_g} = \frac{\partial V_1}{\partial \tilde{p}_g} + \beta \cdot \frac{\partial V_2}{\partial \tilde{p}_g} + \rho \cdot \frac{\partial V_2}{\partial \tilde{p}_g} - \rho \cdot \frac{\partial \hat{V}_2}{\partial \tilde{p}_g} +$$

$$\mu[N_1 \cdot g_1 + N_2 \cdot g_2] + \mu \cdot \tilde{p}_g [N_1 \cdot \frac{\partial g_1}{\partial \tilde{p}_g} + N_2 \cdot \frac{\partial g_2}{\partial \tilde{p}_g}] \leq 0, \quad \tilde{p}_g \left( \frac{\partial \Lambda}{\partial \tilde{p}_g} \right) = 0.$$

$$(16) \quad \frac{\partial \Lambda}{\partial G} = \frac{\partial V_1}{\partial G} + \beta \cdot \frac{\partial V_2}{\partial G} + \rho \cdot \frac{\partial V_2}{\partial G} - \rho \cdot \frac{\partial \hat{V}_2}{\partial G} +$$

$$\mu \frac{\partial \tilde{p}_g}{\partial G} [N_1 \cdot g_1 + N_2 \cdot g_2] + \mu \cdot \tilde{p}_g [N_1 \cdot \frac{\partial g_1}{\partial G} + N_2 \cdot \frac{\partial g_2}{\partial G}] - \mu \cdot m = 0$$

Using Roy's Identity and the Slutsky decomposition, equation (15) can be written as<sup>5</sup>

$$(15') \quad \frac{\partial \Lambda}{\partial \tilde{p}_g} = \rho \frac{\partial \hat{V}_2}{\partial B} [\hat{g}_2 - g_1] + \mu \tilde{p}_g \sum S^i_{gg} \leq 0,$$

where  $S^i_{gg} < 0$  is the  $i^{\text{th}}$  individual's own-price Slutsky term. As B&C note, the first term captures the social benefit from relaxing (or the social cost from tightening) the self-

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<sup>5</sup> Details are in Appendix.

selection constraint while the second term describes the distortionary effect due to the wedge between the price of the public good ( $p_g$ ) and the marginal cost of access (0).

Note that, from equation (15'), if the optimal price is positive, it is given by the formula

$$(17) \quad \tilde{p}_g^* = \frac{\rho \frac{\partial \hat{V}_2}{\partial B} [g_1 - \hat{g}_2]}{\mu \sum S_{gg}}.$$

The analysis done so far is analogous to that of Blomquist and Christiansen. Let us now consider the demand for the public good for the type 1 consumer relative to the mimicking type 2 consumer. Blomquist and Christiansen's Proposition 2 rules out the possibility of a positive price in the case that  $\hat{g}_2 \leq g_1$ . This also follows directly from inspection of equation (16').

Their result that  $\hat{g}_2 > g_1$  is a necessary but not sufficient condition for a positive price on the public good depends crucially on their modeling assumption for the public good. Once we replace rationing with our assumptions embodied in equations (1) and (2), we can show

**Proposition 1.** The condition that  $\hat{g}_2 > g_1$  is a sufficient condition for a positive price on the excludable public good to be pareto improving from an initial position with  $\tilde{p}_g$  equal to zero.

*Proof.* At  $\tilde{p}_g = 0$ , the first order condition for the access price is

$$\left. \frac{\partial \Lambda}{\partial \tilde{p}_g} \right|_{\tilde{p}_g=0} = \rho \frac{\partial \hat{V}_2}{\partial B} [\hat{g}_2 - g_1] > 0. \quad \text{QED}$$

The intuition for why  $\hat{g}_2 > g_1$  is a necessary and sufficient condition for a positive price on the excludable public good to be pareto improving is straightforward. At a positive price for the public good

$$(18) \quad \rho \frac{\partial \hat{V}_2}{\partial B} [\hat{g}_2 - g_1] + \mu \tilde{p}_g \sum S^i_{gg} = 0$$

Consider a small increase in the price for the public good,  $dp > 0$  with a revenue neutral decrease in the income tax  $dT_i = -g_i dp$ . Utility for both consumer types will be unchanged since  $dV_i = -V_{iB}(dT_i + g_i dp) = 0$  by Roy's Identity. The utility for the mimicking consumer is  $d\hat{V}_2 = -V_{2B}(dT_1 + \hat{g}_2 dp) = -V_{2B}(\hat{g}_2 - g_1) dp < 0$ . This reform makes mimicking less attractive and allows a relaxation of the self-selection constraint thereby allowing a reduction of the distortion on type 1 consumers required to ensure a separating equilibrium.<sup>6</sup>

Summing up, the condition  $\hat{g}_2 > g_1$  is a necessary and sufficient condition for a positive price on the public good. B&C's conclusion that "[t]he policy case for a price may thus appear rather weak" (p. 61) depends on utility initially being unchanged over some interval of  $q$  from  $\{0, \bar{q}\}$ , then falling starting at price  $\bar{q}$ , and then eventually rising to the socially optimal  $q$ . With our characterization of excludable public goods, we find that utility unambiguously increases as the price is increased starting at zero. We thus find a stronger policy case to be made for pricing excludable public goods.

When will  $\hat{g}_2 = g_1$ ? If utility is weakly separable between the consumption good and the public good on the one hand and leisure on the other hand, public good

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<sup>6</sup> This is analogous to the intuition for commodity taxation and self-selection provided in Edwards et al. (1994). That paper investigates the interplay between commodity taxation, non-linear income taxation and the provision of a non-excludable public good.

consumption of type 1 consumers and mimicking type 2 consumers are identical. This follows since differences in the consumption of type 1 consumers and mimicking type 2 consumers result only from difference in labor supply, which does not affect the desired consumption of the public good with this form of weakly separable utility. In that case, the optimal price for the public good is zero.<sup>7</sup> This emphasizes the importance of labor distortions in the optimal pricing of the public good.

We next turn to the appropriate characterization of the Samuelson Condition in this second-best framework. As has been noted in other models of public goods in second-best frameworks (Atkinson and Stern (1974), King (1986), Boadway and Keen (1993), and Edwards, et al. (1994) among others) the rule must be modified to the extent that leisure and the public good are related. We make this relationship more precise in the next section.

#### IV. The Samuelson Condition Revisited

The analysis in the previous section shows that introducing  $p_g$  as the price of the public good allows us to draw on the literature on optimal commodity taxation with a non-linear income tax (e.g. Christiansen (1984), Edwards, et al. (1994)). Analogous to Edwards, et al.'s Proposition 3 is

**Proposition 2.** Pareto efficiency in the provision of the public good,  $G$ , in the presence of self-selection constraints requires

$$(19) \quad \sum \text{MRS}_i = m + \frac{\rho}{\mu} \cdot \frac{\partial \hat{V}_2}{\partial B} \cdot [\hat{\text{MRS}}_2 - \text{MRS}_1] - \tilde{p}_v \sum \frac{\partial v_i^c}{\partial G}$$

*proof:* see appendix.

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<sup>7</sup> Edwards et al. (1994) find an analogous result for non-excludable public goods.

The Samuelson condition that the sum of the marginal rates of substitution should equal the marginal rate of transformation is altered in two ways. First, there is a term that depends on the desire to deter mimicking by high-ability types. And second, there is a revenue term. Focusing on this latter term first, the marginal cost of the public good is lowered to the extent that increased provision of the public good increases utilization and therefore entry fee revenue. The modification of the Samuelson condition for revenue effects of increased public good provision is well known (see Atkinson and Stern (1974) for example).

The term including the difference in the *MRS* between the mimicking and low-ability consumer reflects the benefits arising from the public good's ability to help distinguish between high and low-ability consumers. To see why this is so, let's assume that visits ( $v$ ) are unaffected by changes in the amount of the public good ( $G$ ) so we can ignore the third term in (19). Further assume that  $\hat{MRS}_2 - MRS_1 > 0$ . Then  $\sum MRS_i > m$  at the second-best optimum. If  $\sum MRS_i$  was monotonically decreasing in  $G$ , then applying the Samuelson condition would lead to an overprovision of the public good.<sup>8</sup> Consider an initial provision of  $G$  where the Samuelson condition holds and decrease  $G$  marginally. At the same time, lower the high ability type's taxes by  $MRS_2$  and the low-ability type's taxes by  $MRS_1$ . Utility for both types will be unchanged and the government's budget remains balanced. But the mimicker has a loss of utility since the value of the loss of the public good ( $\hat{MRS}_2$ ) exceeds the gain from lower taxes ( $MRS_1$ ). This makes mimicking less attractive and allows for a relaxation of the self-selection

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<sup>8</sup> Atkinson and Stern (1974) note that there may not be a monotone relationship between the aggregate *MRS* and  $G$ . We will assume it here to develop intuition.

constraint which in turn allows for a increase in utility for the low-ability types without adversely affecting the high-ability types.<sup>9</sup>

As Boadway and Keen (1993) point out in the context of non-excludable public goods, the term  $\frac{\rho}{\mu} \cdot \frac{\partial \hat{V}_2}{\partial B} \cdot [\hat{MRS}_2 - MRS_1]$  in equation (19) equals zero if utility is weakly separable between the consumption good and the public good on the one hand and leisure on the other hand. Since the only difference between a mimicking high-ability consumer and a low-ability consumer is their labor supply, the *MRS* for both must be the same since weak separability of this kind implies that the *MRS* between the private and the public good is independent of labor supply. And as discussed in the last section, weak separability also implies the planner should not charge a positive price for public good usage so that both the second term and the third term vanish with this weak separability. We formalize this result as

**Proposition 3.** If utility for both types of consumers is of the form  $U^i(c_i, g_i, H_i)$ , then the Samuelson condition holds:

$$(20) \quad \sum MRS_i = m.$$

In closing we note that an appealing feature of our model is its general applicability to non-excludable as well as excludable public goods. Public goods differ in two key aspects: first, they may be excludable or non-excludable and second, it may be possible to consume varying amounts of the public good. Thus there are four possible types of public goods.

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<sup>9</sup> Boadway and Keen (1993) use this same intuition in the case of a non-excludable public good with no entry fee.

We next give specific examples for each category and consider how our model nests this category. Recall that in our model individual maximization problem<sup>10</sup> is

$$\max_{c_i, v_i, G, \frac{Y_i}{w_i}} U\left(c_i, v_i, G, \frac{Y_i}{w_i}\right) + \lambda(B_i - c_i - p_v v_i) \text{ and the optimal provision rule is given by equation}$$

(19) repeated here for convenience:

$$(19) \quad \sum MRS_i = m + \frac{\rho}{\mu} \cdot \frac{\partial \hat{V}_2}{\partial B} \cdot [MRS_2 - MRS_1] - \tilde{p}_v \sum \frac{\partial v_i^c}{\partial G}.$$

We shall show how this equation is modified for the different types of public goods.

In the first case, the public good is non-excludable and consumption levels fixed. This is the classic public good first studied by Samuleson (1954) and analyzed by Boadway and Keen (1993) in the presence of a non-linear income tax; national defense is an oft-cited example of such a good. In the context of our model, we normalize the usage level ( $v_i$ ) for every citizen at 1 so that public good consumption for every consumer is  $v_i G = G$ . Because the good is non-excludable, the provider cannot charge a per-use price for consuming the public good so  $\tilde{p}_v = 0$ . Thus the policy instruments available to the social planner are  $B_1, B_2, Y_1, Y_2$  and  $G$  just as in Boadway and Keen and the optimal provision rule becomes

$$(19') \quad \sum MRS_i = m + \frac{\rho}{\mu} \cdot \frac{\partial \hat{V}_2}{\partial B} \cdot [MRS_2 - MRS_1]$$

Note that this modified rule is the same as equation (9) in Boadway and Keen (1993).

Next is the case of an excludable public good with uniform consumption levels across consumers (conditional on consuming the public good). Fraser (1996) is a good example of such a model. While our model is not precisely analogous to that of Fraser, it

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<sup>10</sup> Without loss of generality, we assume  $h(G) = G$ .

is easily modified by allowing the low-ability types (for example) to opt out of consumption of the public good and only tax the high-ability types to finance the public good. The term  $\hat{MRS}_2 - MRS_1$  in equation (19') reflecting the distortion arising from the self-selection constraint is analogous to the term in Fraser's equation (5) measuring the change in the marginal consumer of the public good due to a change in the tax level used to finance its production. As argued above, we feel that the Blomquist and Christiansen model falls in this category in the case where their capacity constraint binds.

The third case allows for varying levels of consumption. As we've argued in this paper, many public goods can be consumed in differing amounts based on the amount of access. In this case, we allow for endogenous consumption of a non-excludable public good. An example of such a good may be my enjoyment of a neighbor's beautiful home as I walk by it on my way to work.<sup>11</sup> In our model, every consumer is assumed to have the same access to the public good and to consume the same amount of the public good. We can model non-excludability by simply constraining  $p_v$  to zero and each consumer type consumes a fixed amount of the public good.<sup>12</sup>

Finally, the last case is an excludable public good with endogenous consumption, the case we focus most attention on in this paper. We are not aware of other papers that analyze this case. We do note an historic example of such a public good from Coase (1974) – that of British lighthouses. According to Coase, ships using the lighthouse paid fees that were "so much per net ton payable per voyage for all vessels arriving at, or

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<sup>11</sup> Excludability is clearly a technological phenomenon. My neighbor could presumably build a sufficiently tall fence so as to exclude me from viewing the house. This good is non-excludable if the cost of doing so is prohibitive.

<sup>12</sup> Equation (19) is again the same as Boadway and Keen's equation (9) with one minor difference: the consumption of the ability types may differ in our model. But this is observationally equivalent to the situation where both types consume the same amount of  $G$  but their marginal utility of public good consumption differs.

departing from, ports in Britain" (p. 361). Because only ships would come near the British lighthouses if they planned to enter or leave British ports, it was possible to levy a usage charge. Presumably if ships evaded the payment, they would be barred from future entry into (or exit from) these ports, thereby denying them the use of the public good.<sup>13</sup>

Summing up, we feel that our characterization of the public good benefits in utility deriving from the amount of the public good provided ( $G$ ) as well as usage ( $v$ ) provides a flexible framework for thinking about a wide range of public good models.

## **V. Conclusion**

We have argued that Blomquist and Christiansen's result that utility decreases in entry price for an excludable public good before (possibly) increasing depends importantly on their characterization of the public good. With a more natural characterization of excludable public goods, we find that utility rises if an incremental entry fee is applied to a public good (starting from zero) so long as utility is not weakly separable between private and public good consumption on the one hand and leisure on the other. We find, therefore, that their conclusion that there is at best a weak case to be made for charging for excludable public goods is not robust to the modeling of the public good. Hence the policy case for a positive price on the public good is stronger than Blomquist and Christiansen suggest.

We also characterize the modified Samuelson condition for the provision of an excludable public good with entry fee and find that the original Samuelson condition must be modified for an entry fee revenue impact as well as a distortion term arising from the desire to distinguish high-ability from low-ability consumers.

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<sup>13</sup> There is the slight complication in that the fees were capped at a given level of voyages (depending on whether the ship was a "Home Trade" or "Foreign-going" ship). But this complication should not distract from the point that a non-excludable public good with endogenous access exists.

Another contribution of this note is a more flexible characterization of public good consumption by allowing for varying use by different consumers, a particularly important issue in the context of excludable public goods where an entry fee is charged. Our model is general enough to subsume the non-excludable public good case by simply setting the access fee to zero. Thus our model allows us to bring together a number of results from different papers in the literature on public good provision all under the umbrella of a single model.

## Appendix

### 1. Derivation of equation (15') from equation (15)

First, we verify Roy's identity:

$$(A1) \quad \frac{\partial V_i}{\partial \tilde{p}_g} = -\frac{\partial V_i}{\partial B} g_i$$

From  $V_i(B_i, Y_i, \tilde{p}_g, G) = V_i^*(B_i, Y_i, p_g, G) = \max_{c, g} U\left(c_i, g_i, \frac{Y_i}{w_i}\right) + \lambda(B_i - c_i - p_g g_i)$ , the

envelope theorem implies

$$(A2) \quad \frac{\partial V_i^*}{\partial p_g} = -\lambda g_i = -\frac{\partial V_i^*}{\partial B} g_i.$$

Since  $V_i$  and  $V_i^*$  only differ in terms of  $\tilde{p}_g$  and  $G$ , it is obvious that  $\frac{\partial V_i}{\partial B} = \frac{\partial V_i^*}{\partial B}$ . Also

note that  $G$  is constant for this pricing problem, so it follows that  $\frac{\partial V_i}{\partial \tilde{p}_g} = \frac{\partial V_i^*}{\partial p_g}$ . Therefore,

$$(A3) \quad \frac{\partial V_i}{\partial \tilde{p}_g} = \frac{\partial V_i^*}{\partial p_g} = -\frac{\partial V_i^*}{\partial B} g_i = -\frac{\partial V_i}{\partial B} g_i.$$

By the same logic, when  $G$  is being fixed,

$$(A4) \quad \frac{\partial g_i}{\partial \tilde{p}_g} = \frac{\partial g_i}{\partial p_g}.$$

Using the Slutsky equation for  $g$ , that is,

$$(A5) \quad S_{gg}^i = \frac{\partial g_i}{\partial p_g} + g_i \frac{\partial g_i}{\partial B} = \frac{\partial g_i}{\partial \tilde{p}_g} + g_i \frac{\partial g_i}{\partial B},$$

equation (15) becomes:

$$\begin{aligned}
(A6) \quad \frac{\partial \Lambda}{\partial \tilde{p}_g} &= \frac{\partial V_1}{\partial \tilde{p}_g} + \beta \cdot \frac{\partial V_2}{\partial \tilde{p}_g} + \rho \cdot \frac{\partial V_2}{\partial \tilde{p}_g} - \rho \cdot \frac{\partial \hat{V}_2}{\partial \tilde{p}_g} + \mu [N_1 \cdot g_1 + N_2 \cdot g_2] + \\
&\quad \mu \cdot \tilde{p}_g [N_1 \cdot \frac{\partial g_1}{\partial \tilde{p}_g} + N_2 \cdot \frac{\partial g_2}{\partial \tilde{p}_g}] \\
&= -\frac{\partial V_1}{\partial B} g_1 - \beta \cdot \frac{\partial V_2}{\partial B} g_2 - \rho \cdot \frac{\partial V_2}{\partial B} g_2 + \rho \cdot \frac{\partial \hat{V}_2}{\partial B} \hat{g}_2 + \mu [N_1 \cdot g_1 + N_2 \cdot g_2] + \\
&\quad \mu \cdot \tilde{p}_g [N_1 \cdot (S^l_{gg} - g_1 \frac{\partial g_1}{\partial B}) + N_2 \cdot (S^2_{gg} - g_2 \frac{\partial g_2}{\partial B})].
\end{aligned}$$

Adding and subtracting  $\rho \cdot g_1 \frac{\partial \hat{V}_2}{\partial B}$ , (A6) becomes:

$$\begin{aligned}
(A6') \quad \frac{\partial \Lambda}{\partial \tilde{p}_z} &= -\frac{\partial V_1}{\partial B} g_1 - \beta \cdot \frac{\partial V_2}{\partial B} g_2 - \rho \cdot \frac{\partial V_2}{\partial B} g_2 + \rho \cdot \frac{\partial \hat{V}_2}{\partial B} \hat{g}_2 + \rho \cdot g_1 \frac{\partial \hat{V}_2}{\partial B} - \rho \cdot g_1 \frac{\partial \hat{V}_2}{\partial B} + \\
&\quad \mu [N_1 \cdot g_1 + N_2 \cdot g_2] + \mu \cdot \tilde{p}_g [N_1 \cdot (S^l_{gg} - g_1 \frac{\partial g_1}{\partial B}) + N_2 \cdot (S^2_{gg} - g_2 \frac{\partial g_2}{\partial B})] \\
&= \rho \cdot \frac{\partial \hat{V}_2}{\partial B} \hat{g}_2 - \rho \cdot g_1 \frac{\partial \hat{V}_2}{\partial B} + \mu \cdot \tilde{p}_g [N_1 \cdot (S^l_{gg}) + N_2 \cdot (S^2_{gg})].
\end{aligned}$$

The last equality follows directly from applying FOC conditions (11) and (13) to the equation.

Therefore,  $\frac{\partial \Lambda}{\partial \tilde{p}_z} = \rho \cdot \frac{\partial \hat{V}_2}{\partial B} [\hat{g}_2 - g_1] + \mu \tilde{p}_g \sum S^i_{gg}$  QED.

## 2. Proof of Proposition 2

First, let us define indirect utility alternatively as  $V_i(B_i, Y_i, \tilde{p}_v, G)^{14}$  and MRS as:

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<sup>14</sup> Unlike in section 3 where we used the effective public good price ( $\tilde{p}_g$ ), we use  $\tilde{p}_v$  to prove proposition 2.

$$(A7) \quad \text{MRS} = \frac{\partial V_i}{\partial G} / \frac{\partial V_i}{\partial B}.$$

Second, let us verify the Slutsky equation for  $v$  (visits) with change in  $G$ . Consider the identity

$$(A8) \quad v_i^c(\bar{u}_i, p_v, G) = v_i(e_i(p_v, G, w_i, \bar{u}_i), p_v, G).$$

Taking derivatives with respect to  $G$ , we get

$$(A9) \quad \frac{\partial v_i^c}{\partial G} = \frac{\partial v_i}{\partial G} + \frac{\partial v_i}{\partial B} \cdot \frac{\partial e_i}{\partial G}.$$

Since  $\frac{\partial e_i}{\partial G}$  describes changes in how much *after* tax income is needed to keep utility remaining same when  $G$  gets higher, that is,

$$(A10) \quad \frac{\partial e_i}{\partial G} = -\frac{\partial V_i}{\partial G} / \frac{\partial V_i}{\partial B},$$

the equation (A9) becomes:

$$(A9') \quad \frac{\partial v_i^c}{\partial G} = \frac{\partial v_i}{\partial G} - \frac{\partial v_i}{\partial B} \cdot \frac{\partial V_i}{\partial G} / \frac{\partial V_i}{\partial B} = \frac{\partial v_i}{\partial G} - \frac{\partial v_i}{\partial B} \cdot \text{MRS}_i.$$

Now, social welfare maximization problem with this indirect utility function is:

$$(A11) \quad \Lambda = V_1(B_1, Y_1, \tilde{p}_v, G) + \beta[V_2(B_2, Y_2, \tilde{p}_v, G) - \bar{V}_2] + \rho[V_2(B_2, Y_2, \tilde{p}_v, G) - \hat{V}_2(B_1, Y_1, \tilde{p}_v, G)] + \mu[N_2(Y_2 - B_2) + N_1(Y_1 - B_1) + \tilde{p}_v(N_1 \cdot v_1 + N_2 \cdot v_2) - mG]$$

and the first order conditions are

$$(A12) \quad \frac{\partial \Lambda}{\partial B_1} = \frac{\partial V_1}{\partial B} - \rho \cdot \frac{\partial \hat{V}_2}{\partial B} - \mu \cdot N_1 + \mu \cdot \tilde{p}_v \cdot N_1 \cdot \frac{\partial v_1}{\partial B} = 0$$

$$(A13) \quad \frac{\partial \Lambda}{\partial Y_1} = \frac{\partial V_1}{\partial Y} - \rho \cdot \frac{\partial \hat{V}_2}{\partial Y} + \mu \cdot N_1 + \mu \cdot \tilde{p}_v \cdot N_1 \cdot \frac{\partial v_1}{\partial Y} = 0$$

$$(A14) \quad \frac{\partial \Lambda}{\partial B_2} = \beta \cdot \frac{\partial V_2}{\partial B} + \rho \cdot \frac{\partial V_2}{\partial B} - \mu \cdot N_2 + \mu \cdot \tilde{p}_v \cdot N_2 \cdot \frac{\partial v_2}{\partial B} = 0$$

$$(A15) \quad \frac{\partial \Lambda}{\partial Y_2} = \beta \cdot \frac{\partial V_2}{\partial Y} + \rho \cdot \frac{\partial V_2}{\partial Y} + \mu \cdot N_2 + \mu \cdot \tilde{p}_v \cdot N_2 \cdot \frac{\partial v_2}{\partial Y} = 0$$

$$(A16) \quad \frac{\partial \Lambda}{\partial G} = \frac{\partial V_1}{\partial G} + \beta \frac{\partial V_2}{\partial G} + \rho \frac{\partial V_2}{\partial G} - \rho \frac{\partial \hat{V}_2}{\partial G} - \mu \cdot m + \mu \cdot \tilde{p}_v \cdot [N_1 \cdot \frac{\partial v_1}{\partial G} + N_2 \cdot \frac{\partial v_2}{\partial G}] = 0$$

Adding and subtracting  $\rho \cdot \frac{\partial \hat{V}_2}{\partial B} \cdot \frac{\frac{\partial V_1}{\partial G}}{\frac{\partial V_1}{\partial B}}$ , we get from equation (A16):

$$(A16') \quad \begin{aligned} \frac{\partial \Lambda}{\partial G} &= \frac{\partial V_1}{\partial G} + \beta \frac{\partial V_2}{\partial G} + \rho \frac{\partial V_2}{\partial G} - \rho \frac{\partial \hat{V}_2}{\partial G} - \mu \cdot m + \mu \cdot \tilde{p}_v \cdot [N_1 \cdot \frac{\partial v_1}{\partial G} + N_2 \cdot \frac{\partial v_2}{\partial G}] \\ &= \left[ \frac{\partial V_1}{\partial B} - \rho \cdot \frac{\partial \hat{V}_2}{\partial B} \right] \cdot \frac{\frac{\partial V_1}{\partial G}}{\frac{\partial V_1}{\partial B}} + (\beta + \rho) \cdot \frac{\partial V_2}{\partial B} \cdot \frac{\frac{\partial V_2}{\partial G}}{\frac{\partial V_2}{\partial B}} + \rho \cdot \frac{\partial \hat{V}_2}{\partial B} \cdot \left[ \frac{\frac{\partial V_1}{\partial G}}{\frac{\partial V_1}{\partial B}} - \frac{\frac{\partial \hat{V}_2}{\partial G}}{\frac{\partial \hat{V}_2}{\partial B}} \right] \\ &\quad - \mu \cdot m + \mu \cdot \tilde{p}_v \cdot [N_1 \cdot \frac{\partial v_1}{\partial G} + N_2 \cdot \frac{\partial v_2}{\partial G}] \\ &= \left[ \frac{\partial V_1}{\partial B} - \rho \cdot \frac{\partial \hat{V}_2}{\partial B} \right] \cdot MRS_1 + (\beta + \rho) \cdot \frac{\partial V_2}{\partial B} \cdot MRS_2 + \rho \cdot \frac{\partial \hat{V}_2}{\partial B} \cdot [MRS_1 - \hat{MRS}_2] \\ &\quad - \mu \cdot m + \mu \cdot \tilde{p}_v \cdot [N_1 \cdot \frac{\partial v_1}{\partial G} + N_2 \cdot \frac{\partial v_2}{\partial G}] \\ &= [\mu \cdot N_1 - \mu \cdot \tilde{p}_v \cdot N_1 \cdot \frac{\partial v_1}{\partial B}] \cdot MRS_1 + [\mu \cdot N_2 - \mu \cdot \tilde{p}_v \cdot N_2 \cdot \frac{\partial v_2}{\partial B}] \cdot MRS_2 + \\ &\quad \rho \cdot \frac{\partial \hat{V}_2}{\partial B} \cdot [MRS_1 - \hat{MRS}_2] - \mu \cdot m + \mu \cdot \tilde{p}_v \cdot [N_1 \cdot \frac{\partial v_1}{\partial G} + N_2 \cdot \frac{\partial v_2}{\partial G}] \\ &= 0. \end{aligned}$$

The last equality follows directly from applying FOC conditions (11) and (13).

By dividing equation (A16') by  $\mu$  and rearranging, we get:

$$\begin{aligned}
 \text{(A16'')} \quad & N_1 \cdot \text{MRS}_1 + N_2 \cdot \text{MRS}_2 + \frac{\rho}{\mu} \cdot \frac{\partial \hat{V}_2}{\partial B} \cdot [\text{MRS}_1 - \hat{\text{MRS}}_2] \\
 & = m - \tilde{p}_v [N_1 \cdot \{ \frac{\partial v_1}{\partial G} - \frac{\partial v_1}{\partial B} \cdot \text{MRS}_1 \} + N_2 \cdot \{ \frac{\partial v_2}{\partial G} - \frac{\partial v_2}{\partial B} \cdot \text{MRS}_2 \}] \\
 & = m - \tilde{p}_v [N_1 \frac{\partial v_1^c}{\partial G} + N_2 \frac{\partial v_2^c}{\partial G}]
 \end{aligned}$$

The last equality follows from applying Slutsky equation for  $v$ .

$$\text{Therefore, } \sum \text{MRS}_i = m + \frac{\rho}{\mu} \cdot \frac{\partial \hat{V}_2}{\partial B} \cdot [\hat{\text{MRS}}_2 - \text{MRS}_1] - \tilde{p}_v \sum \frac{\partial v_i^c}{\partial G} \quad \text{QED.}$$

## References

**Atkinson, Anthony and Stern, Nicholas.** "Pigou, Taxation, and Public Goods." *Review of Economic Studies*, 1974, 41, pp. 119-28.

**Blomquist, Soren and Christiansen, Vidar.** "The Role of Prices for Excludable Public Goods." *International Tax and Public Finance*, 2005, 12, pp. 61-79.

**Boadway, Robin and Keen, Michael.** "Public Goods, Self-Selection and Optimal Income Taxation." *International Economic Review*, 1993, 34(3), pp. 463-78.

**Christiansen, Vidar.** "Which Commodity Taxes Should Supplement the Income Tax?" *Journal of Public Economics*, 1984, 24, pp. 195-220.

**Coase, Ronald H.** "The Lighthouse in Economics." *Journal of Law and Economics*, 1974, 17(2), pp. 357-76.

**Edwards, Jeremy; Keen, Michael and Tuomala, Matti.** "Income Tax, Commodity Taxes and Public Good Provision: A Brief Guide." *FinanzArchiv*, 1994, 51, pp. 472-87.

**Fraser, C. D.** "On the Provision of Excludable Public Goods." *Journal of Public Economics*, 1996, 60(1), pp. 111-30.

**King, Mervyn A.** "A Pigovian Rule for the Optimum Provision of Public Goods." *Journal of Public Economics*, 1986, 30, pp. 273-91.

**Labaton, Stephen.** "New Package Deals Urged for Cable and Satellite TV," *New York Times*, Nov. 30, 2005.

**Samuleson, Paul.** "The Pure Theory of Public Expenditure." *Review of Economics and Statistics*, 1954, 36, pp. 387-89.

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