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COMPLEXITY AND ORGANIZATIONAL ARCHITECTURE

by

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Abstract

This paper revisits the literature on modelling organizations by means of networks of agents. Individual agents are engaged in screening projects, and architectural features of organizations, that is how each agent's decision combines with those of others, affect the organization's screening performance. It emphasizes how an organization of several agents may improve upon individual performance by a suitable arrangement of the flow of decisions. The paper is motivated, in part, by a theorem due to Von Neumann (also exploited by Moore and Shannon) on how to build reliable networks using unreliable components. It also extends previous contributions by Sah and Stiglitz by recasting their original model in more standard firm-theoretic terms and by endogenizing its features.

For an organization to perform better than an individual in terms of screening, its screening function must be *sigmoid* as a function of individual screening performance, as measured by the probability that a good (bad) project be accepted (rejected). This property is indeed satisfied by organizations with *mixed* Sah-stiglitz architectures, such as hierarchies made up of components that are polyarchies, and polyarchies made up of components that are hierarchies. This property is in turn critical for determining the optimal number of levels of a hierarchy, and for endogenizing individual screening performance. The models are extended to allow for individuals' own screening to be influenced by the opinions of superiors and subordinates. The paper examines the implications of such interactions for the limits to organizational performance.

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1 Introduction

This paper revisits the literature on modelling organizations by means of networks of agents. Individual agents are engaged in screening projects, and architectural features of organizations, that is the specifics of how an agent's decision combines with those of others, affect an organization's screening performance. It emphasizes how an organization of several agents may improve upon individual screening performance by a suitable arrangement of the flow of decisions, that is, organizational architecture. The possibility that is pursued in this paper is motivated, in part, by a theorem due to von Neumann (1956) and to Moore and Shannon (1956). The paper extends previous contributions by Sah and Stiglitz (1985; 1986; 1988) by recasting their original model in more standard firm-theoretic terms.

For an organization's screening performance to improve over that of an individual, its screening function must be *sigmoid* in individual performance, as measured by the probability that a good (bad) project be accepted (rejected). This is, indeed, the case for organizations with *mixed* Sah–Stiglitz architectures, such as hierarchies made up of components that are polyarchies and polyarchies made up of components that are hierarchies, but not for pure Sah–Stiglitz architectures. This property is in turn critical for endogenizing individual screening performance. The models are extended to allow for individuals' own screening outcomes to combine with influence from others, subordinates and superiors. The paper examines the implications of such interactions for the limits to organizational performance of complex organizational architectures.

It appears to be particularly timely to revisit the literature on organizational architecture. The revival of interest in economic consequences of social interactions has been attracting a lot of attention, and so have applications of economic tools to understanding cognition and other aspects of actual human behavior. Noteworthy contributions to this literature emphasize that individuals attack complex problems by means of intelligent shortcuts, which may be studied as approximations of fully optimal decisions [Gabaix and Laibson (2001); MacLeod (1997)]. A separate strand of research has been motivated by the increasing importance of information and communication technologies in modern organizations. These technologies may be changing the organization of the place of work, exactly as past epochs of rapid technological innovations have been associated with organizational change [Ahuja and Carley (1999)]. Such experiences behoove us to reconsider whether the basic economic tools for studying organizations within the canonical model of the firm are good models for contemporary organizations [*c.f.* Allen (2000)]. This paper aims at contributing to such an effort by viewing organizations as being made up of sets of interacting agents.

Some of the current research on organizations invokes the notion of organizational *complexity.* Complexity is a well-defined concept in computer science: *computational* complexity pertains to how hard it is to perform computations needed to solve computationally well-defined problems [Papadimitriou (1994)]. Yet, it is an often-used but not so well-defined problem in the economics of organizations. For example, many authors appeal to Simon (1962), who emphasizes the hierarchical nature of complex systems in nature. The notion of complexity invoked here is similar to the one developed by Mount and Reiter (1998; 2002). Rivkin (2001), and especially Visser (2001a), define as *complex* organizational structures that are made up of a "large number of divisions or hierarchical layers or if they contain many interdependent parts the individual functioning of which is of importance to the overall performance of the organization" [Visser (2001a), p. 1.] One of the objectives of this paper is to help assess the notion of complexity in the context of the theory of organizations.[2] Fioretti and Visser (2002) focuses away from defining organizational complexity directly in terms of organizational architecture and emphasize instead the importance of linking with the human cognition of "a structure or behavior."

The present paper retains a "connectionist" approach and defines complexity in terms of mixing specific patterns of interdependence ("connections") between members of an organization. In particular, it associates complexity with organizational architectures that are made up of components that are organized in a particular way, while each of them is made up of agents organized in a different way. For example, a hierarchy typically involves a clear line of authority among its members, from higher to lower levels. We consider organizations whose components are arranged hierarchically, but within each component decisions are made, say, by consensus among individual

[2] Visser (2001a) defines *complexity* as the level of detail that is necessary to correctly assign agents to positions in the organizational structure; *performance* is defined in terms of the maximum expected profit associated with the structure; and, *robustness* is measured in terms of the maximal extent in which expected profits on implemented projects fall short of the optimal case which would obtain if agents had been correctly assigned. Visser shows that increasing organizational complexity is associated with reducing robustness, at least for the specific organizational forms which he examines in the paper. Visser (2001b) argues in favor of cognition aspects of complexity. He proposes to study organizations in terms of three possible types of information about agents' characteristics that may be used in designing organizations, that is, no information, ordinal information, and cardinal information.

agents. Such organizations are of course quite common, but they have not been studied extensively. As we see in further detail below, it is an important result of the paper that it is such "mixing" of architectures that makes it possible for an organization to perform better in terms of screening relative to the screening performance of an individual member.

The remainder of this paper is organized as follows. Section 2 defines firms as networks of interdependent decisions, elaborates on architectural features and motivates the principle of composition by drawing on the von Neumann–Moore–Shannon theorem of optimal network design. Section 3 endogenizes individual screening performance via setting the wage rate so as to maximixe profit and studies organizations that involve both hierarchical and polyarchical features. Section 4 turns to a model of organizations, which employs a cognitive component. This allows us to distinguish between uncertainties affecting an agent's knowledge that are inherent in decision making, on one hand, and with the influence from co-workers, on the other, and poses the problem of optimizing organizational architecture.

2 The Firm as a Network of Interdependent Decisions

A vibrant literature in organizational science has developed in the last few years that emphasizes the complexity of interactions over decisions.[3] Suppose that in evaluating an investment project, a firm must carry out different pairwise comparisons that may be construed in a nested setting, such as what technology to adopt, how to implement it, and so on. An example is the problem of having to decide over different materials that may be used in a particular production setting, where materials must be combined with one another, sort of like when one must decide how to dress, as in an example invoked by MacLeod (1997). Another example is a firm's deciding whether to have its own sales staff or to sell through third parties, whether to pursue its own R&D or not, etc. Thus, a firm may be modelled as a network of interdependent decisions, where each individual decision pertains to elementary choices, as above.

Let the set $\mathcal{I} = \{i : i = 1, \ldots, I\}$ represent the members of a firm (or organization). A firm that needs to make I binary decisions about how to configure its activities allocate each

[3] Some of these works also utilize the so-called NK model of Kauffman (1993), such as Marengo *et al.* (2000), Rivkin (2000), and Rivkin and Siggelkow (2001). Rivkin and Siggelkow emphasize how a firm's formal organizational structure affects its ability to cope with interdependent decisions.

decision to a single agent of the organization. Any set of technological constraints affecting a firm's decisions component may be expressed in terms of patterns of interdependence among a set of interdependent binary decisions [MacLeod (1997)]. Let the $I-$tuple of binary variables d_i, $D = (d_1, \ldots, d_I)$, denote the outcomes. Each decision d_i contributes to the firm's objective a term depends in general on decision i and on all other decisions, d_{-i}, $C_i = C_i(d_i; d_{-i})$. The interdependence of decisions is represented by an adjacency matrix $\mathbf{E}$, an $I \times I$ matrix, where entry ij being equal to 1, $e_{ij} = 1$, suggests that the decision of agent i influences agent j, and $e_{ij} = 0$ suggests that the decisions of agent i does not influence agent j. We set $e_{ii} = 1$. If $\mathbf{E}$ is a diagonal matrix of 1s, then all decisions are independent; if it is full, then all decisions are interdependent. Block structure suggests that decisions may be decided by groups of interdependent agents, within which decisions are interdependent and across them are independent.[4]

The pattern of how an agent influences others may be represented by $\mathbf{E}$, or alternatively, by a graph with the set $\mathcal{I}$ as its nodes. Such a general graph-theoretic description of organizations may accommodate easily different notions of authority. E.g., let sets $\mathcal{I}_k,\ k = 1, \ldots, K,\ \mathcal{I} = \bigcup_k \mathcal{I}_k$ represent the sets of agents at the different hierarchy levels, with $k = 1$ being the single agent at the apex of the hierarchy; $\nu(i)$ is the set of agents who report to i with $|\nu(i)|$ being their number, or i's in-degree; and $\mu(i)$ is the agent whom i reports to, agent i's boss, that is each node has out-degree equal to 1, $|\mu(i)| = 1$. So, if $i \in \nu(j)$, then $j = \mu(i)$. And, if $i \in \mathcal{I}_k$, then $\nu(i) \in \mathcal{I}_{k+1}$. The graph representing the organization is directed, with direction representing authority, flowing from superior to subordinate.

The problem of organizational architecture is to allocate different decisions to different agents and to set jointly interdependencies among decisions, that is to determine $\mathbf{E}$, given the types of decisions to be made and associated technological constraints. Complex problems facing firms are those that are highly dimensional and "whose solution requires the coordination of interdependent components" [Marengo *et al.* (2000)]. The present paper focuses on organizations all of whose agents face the same decision, that is, whether a particular project should be adopted or rejected. Organizational design, which is the object of the present study, affects organizational performance

[4] In Kauffman's "NK" model, each of N decisions is affected by K of other N decisions. In Kauffman's biology and genetics applications, as K increases relative to N, optima become "rugged" and "multi-peaked" [Kauffman (1993), 66-67].

as a function of the screening abilities of individual workers.

2.1 Individual Screening Performance and the Objective of the Firm

Workers screen projects. Each project's quality may be either good or bad, and is unobservable. Screening is imperfect but informative. That is, a good project is more likely to be accepted than a bad project. Let α and $1-\alpha$, $0<\alpha<1$, denote the proportions of good and bad projects, respectively, within the population of projects, from which a firm draws randomly. The firm's costs are simply labor costs, $\sum_{i\in\mathcal{I}} w_i$, with agent i receiving a wage rate w_i. We assume that individuals' screening ability depends on the wage rate, which is paid before project quality is revealed. As functions of the wage rate, the probability of an agent's accepting a good project, $p_g = P_g(w)$, and of rejecting a bad one, $1-P_b(w)$, both take values in $[0,1]$, and are increasing and concave in the wage rate. Consequently, the probability of an agent's accepting a bad project, $P_b(w)$, which also takes values in $[0,1]$, is decreasing and convex in the wage rate. We assume that wages are paid before agents have made their screening decisions. Clearly, for screening to be informative, $1>p_g>p_b>0$. It is appropriate to interpret $1-p_g$, the probability that a good project is rejected, as the Type-I error, and p_b, the probability that a bad project is accepted, as the Type-II error, for the organization.

Let $R_g>0$ and $R_b<0$ denote the revenue to a firm (organization) from implementing a good and bad project, respectively. The probability that an organization accept a project, conditional on its quality $j=g,b$, is given by the screening function $\mathcal{P}(p_j)$. The expected profit from considering a randomly selected project is given by

$$R_g\cdot\alpha\cdot\mathcal{P}(p_g)+R_b\cdot(1-\alpha)\cdot\mathcal{P}(p_b)-\sum_{i\in\mathcal{I}} w_i. \tag{1}$$

2.2 Organizational Architecture and Composition

Sah and Stiglitz [Sah and Stiglitz (1985; 1986; 1988), and Sah (1991)] define *polyarchies* as organizational structures with agents operating "in parallel:" a polyarchy approves of a project if at *least one* of the agents in the organization approves of it. They define *hierarchies* as organization with agents operating "in series": a hierarchy approves of a project only if *all* agents approve of it. Sah and Stiglitz compare the screening effectiveness of such stylized organizations. An organization's screening function $\mathcal{P}(p)$ is the probability that the organization accept a project as a function of the probability that each member accept a project is p. The function $\mathcal{P}(p)$ summarizes the impact of organizational architecture.

Following Sah–Stiglitz, *op. cit.*, for an organization with two members whose screening abilities are equal and given by the probability of accepting a project p, the probability that a project is accepted by a *polyarchy* of two agents is equal to the probability that a project be accepted by at least one of them: $\mathcal{P}_P(p) = 1 - (1-p)^2 = 2p - p^2$. The probability that a project is accepted by a *hierarchy* with two agents is equal to the probability that a project be accepted by both of them: $\mathcal{P}_H(p) = p^2$. It follows that $\mathcal{P}_P(p) > \mathcal{P}_H(p)$, $\forall p \in [0,1]$. Therefore, polyarchies accept more good and more bad projects than hierarchies. Since $1 - \mathcal{P}_P(p) < 1 - \mathcal{P}_H(p)$, hierarchies reject more good and bad projects than polyarchies. Both types of organizational architectures are informative, that is for both of them, the probability of accepting a project is increasing in the probability that an agent accept a project. As a result, a good project is more likely to be accepted than a bad project. Sah and Stiglitz (1988) apply their framework to study of the performance of committees, where decisions are made by different ways of aggregating the individual decisions (plurality rules). E. g., a polyarchy is a committee that operates with a plurality of 1, and a hierarchy is a committee that operates with unanimity. They do discuss briefly the possibility of more complex organizations, such as if polyarchies include hierarchies as their elements and vice versa [*ibid.,* 467–468] but do not explore their properties.

Ioannides (1987)[5] draws on von Neumann (1956) and Moore and Shannon (1956) to show that it is possible, in general, to improve the screening performance of organizations of the general class studied by Sah and Stiglitz by means of *complicating* their architecture in a very precise sense. That is, by replacing each agent with a replica of the entire organization (which resembles replacing an agent with a committee) and provided that the architecture satisfies certain basic properties (to be elaborated upon shortly below), it is possible to design an organization that satisfies arbitrary specified performance criteria. That is, the organization must reject projects, whose probability of being accepted is below a specified threshold, and must accept projects, whose probability of being accepted is above a specified threshold. This process is known as *composition.*[6]

[5]See also Koh (1992), Anderson (1999) and Visser (2000).

[6]Moore and Shannon (1956) in turn credit Von Neumann (1956) for the original idea of using (unreliable) *Sheffer*

The *composition* operation accomplishes this objective but at great increase in organization size: replacing each member of an organization of size n with n members causes the size to increase from n to n^2, and the next composition increases it to n^4, and to n^{16}, and so on. Note that this is much faster than the increase in the number of agents when the number of levels of a pyramidal hierarchy, K, while the span of control s remains fixed. In the latter case, the number of agents in a hierarchy of K levels with a span of control of s, $1 + s + \ldots + s^K$ increases by s^{K+1}, when the number of levels increases from K to $K + 1$. It is appropriate to refer to the outcome of the composition process as producing an organization of increasing *complexity*, with Mount and Reiter (1998) being in agreement. Still, composition as a way of improving an organization's screening performance is not practical because of the great accompanying increase in the number of agents.

For an organizational architecture to lend itself to improvement by composition it must possess a screening function that satisfies the following property [Ioannides (1987)]: the screening function must possess a single fixed point that is strictly greater than 0 and strictly less than 1, and must cut the 45^o line from below at that point. In other words, since the 45^o line represents the response of an organization with a single agent, there must exist a value of the screening probability between 0 and 1 that makes an individual agent equivalent to the entire organization. Composition reduces the probability of a project's being accepted by the organization, if each individual agent's acceptance probability is below this characteristic value, and increases it, if it is above. See Figure 1. In Figure 1, starting with an organization with screening function $\mathcal{P}(p)$, composition associates with individual screening performance p organizational performance $\mathcal{P}\left(\mathcal{P}(p)\right)$.

The mathematical intuition of this result should be straightforward to economists familiar with dynamical systems. Just as in dynamical systems, the higher-order iterates of a time map share with the original map the same fixed point, composition is the "spatial" counterpart of iterating the time map of a dynamical system. The fixed point of the screening function reflects fundamental aspects of organizational architecture and is not disturbed by "spatially" iterating the screening function. Of course, it is the rapid increase of the number of agents that make the von Neumann–Moore–

stroke organs as components of reliable organisms. A Sheffer stroke is the Boolean operation "(`.not.` A) `.and.` (`.not.` B) on Boolean variables A and B. It has the property, which Von Neumann exploits, that all logical operations can be generated from it. A Sheffer stroke organ is the logical operation with two binary inputs A and B which performs this logical operation, possibly unreliably. The von Neumann approach is noteworthy because it deals explicitly with the stochastics of error-prone components.

Shannon theorem work. The von Neumann–Moore–Shannon theorem motivates us to explore how complicating organizational architecture by means of *cross-composition* may improve organizational screening performance.[7] As Ioannides (1987) points out, pure Sah–Stiglitz architectures are *not* improvable by means of composition. However, as we see shortly, cross-composing a hierarchy with a polyarchy (and vice versa) just *once,* that is, by replacing each agent in a pure hierarchy by a component which itself is a polyarchy, makes the organization's screening function sigmoid. Therefore, it is, in principle, possible to make an organization more effective both in screening good and bad projects. It is to developing this concept that we turn next.

Mount and Reiter (1998; 2002) and Reiter (1996) develop a model of the firm, in which organizational structure emerges as a solution of an optimization problem. The problem involves how to carry out computations and trades off the "complexity" of economic computations in a given class of economic environments, and the constraints express limitation on the abilities of agents to compute and communicate. Computations are decomposed in terms "auxiliary" computations that can be expressed as a *superposition* (composition, in our terminology) of the primitive functions performed in terms of "lower level" computations (or agents). Their measure of complexity is then simply the depth of superposition, which in our case corresponds to the number of hierarchies. The model of the present paper is conceptually similar to the Mount–Reiter model of the firm. Our agents's evaluations are akin to computations, and organizational architecture is akin to the execution of computations in the Mount and Reiter model.

3 Sah–Stiglitz Architectures as Building Blocks of Complex Organizations

We consider first pure Sah–Stiglitz organizations made up of identical agents. We study their properties as the number of agents increases. For a hierarchy of I_H members, the screening function $\mathcal{P}_H(p) = p^{I_H}$ is convex increasing in p and convex decreasing in I_H. For good projects, it is a

[7] We note that, however tempting, a direct comparison of the von Neumann–Moore–Shannon theorem to results on neural networks as universal approximators would be simplistic. The von Neumann–Moore–Shannon theorem exploits the particular nonlinearity of the probability that an organization will approve of a project as a function of the individual performance so as to satisfy desired performance standards by means of spatially iterating the function. The universal approximator theorem for neural networks [Haykin (1999), 208–209, 229] exploits the particular nonlinearity of the sigmoid function that makes it perform well as a functional form for the components of a power series.

concave (convex) increasing function of the wage rate, provided that $I_H - 1 < -\frac{P_g''/P_g'}{P_g'/P_g}$ (>) 0; for bad projects, it is decreasing convex function of the wage rate. If the probability of accepting good projects is, roughly speaking, sufficiently insensitive to the wage rate, maximization of expected profit defines an optimal wage.

For a polyarchy, the screening function $\mathcal{P}_P(p) = 1 - (1-p)^{I_P}$ is concave increasing in p and concave increasing in I_P. For good projects, it is a concave increasing function of the wage rate; for bad projects, it is convex (concave) decreasing, provided that $\frac{P_b''/P_b'}{P_b'/1-P_b} > I_P - 1$ (<) 0. If the probability of rejecting bad projects is, roughly speaking, sufficiently insensitive to the wage rate, maximization of expected profit defines an optimal wage. An optimal size exists for the polyarchy, but for the hierarchy, expected revenue is decreasing convex in size and so is expected profit.

3.1 Complex Architectures with Sah–Stiglitz Components

Since pure Sah–Stiglitz architectures make different kinds of errors, e.g., polyarchies accept more good and bad projects and hierarchies reject more good and bad projects, suitably complicating organizational architecture could offset the undesirable, and strengthen the desirable, properties of each of the two stylized architectures. We see, in fact, that *cross-composition* makes the screening performance of such organizations a sigmoid function of individual screening performance, that satisfies the condition stated above, namely that it possesses a unique fixed point in $(0,1)$ where it intersects the 45^o line from below. Therefore, there exists a specific value of individual screening probability, below (above) which, the probability of accepting a project by the organization is less (greater) than the probability that an individual agent approves a project. Therefore, as long as screening is informative, organizational design improves on individual screening effectiveness.

We establish first some general properties of two alternative organizational architectures whose agents have identical screening performance. Let the total number of agents I and the number of hierarchies K be sufficiently so that $\frac{I}{K}$ may be treated as an integer. One alternative architecture is a hierarchy with components that are polyarchies, and the other is a polyarchy with components that are hierarchies, hierarchies of polyarchies and polyarchies of hierarchies, for short. The screening function of a pure Sah–Stiglitz hierarchy, $\mathcal{P}_H(p)$, and of a pure Sah–Stiglitz polyarchy, $\mathcal{P}_P(p)$, defined earlier, are monotone increasing in $(0,1)$ with $\mathcal{P}_j(0) = 0,\ \mathcal{P}_j(1) = 1, j = H, P$. It follows that the screening function of a hierarchy of polyarchies , with a total number of I members who are arranged in K equal-sized hierarchies, is given by

$$\mathcal{P}_{HP} \equiv \mathcal{P}_H\left[\mathcal{P}_P(p)\right] = \left(1 - (1-p)^{\frac{I}{K}}\right)^K . \tag{2}$$

Similarly, the screening function of a polyarchy of $\frac{I}{K}$ hierarchies, each of which has K members, is given by

$$\mathcal{P}_{PH}(p) \equiv \mathcal{P}_P\left[\mathcal{P}_H(p)\right] = 1 - \left(1 - p^K\right)^{\frac{I}{K}} , \tag{3}$$

The first order conditions with respect to screening performance involve the first derivatives of the screening functions, $\mathcal{P}'_H\left[\mathcal{P}_P(p)\right]\mathcal{P}'_P(p)$ and $\mathcal{P}'_P\left[\mathcal{P}_H(p)\right]\mathcal{P}_H\prime(p)$, respectively. We show next that both these derivatives are equal to 0 for $p = 0, 1$. Since these functions are monotone increasing over $(0, 1)$ both possess at least one fixed point in $(0, 1)$. Their second derivatives are $\mathcal{P}''_H\left[\mathcal{P}_P\right]\left(\mathcal{P}'_P\right)^2 + \mathcal{P}'_H\left[\mathcal{P}_P\right]\mathcal{P}''_P$, and $\mathcal{P}''_P\left[\mathcal{P}_H\right]\left(\mathcal{P}'_H\right)^2 + \mathcal{P}'_P\left[\mathcal{P}_H\right]\mathcal{P}''_H$. The zero of the second derivative of these functions, indicates the respective inflection point, $\hat{p}$, which we obtain in closed form for hierarchies of polyarchies, and for polyarchies of hierarchies. Next we turn to exploring in detail these properties.

3.1.1 Hierarchies of Polyarchies

We set first the optimal number of hierarchies, given a total number of $I = |\mathcal{I}|$ agents with identical screening performance. Let the number of agents to allocated to hierarchy levels $1, 2, \ldots, K$ be given by $(I_1, I_2, \ldots, I_K)$, where $\sum_{k=1}^{K} I_k = I$, and let I_k members at level k be organized as a polyarchy. Then the probability of accepting a project by such an organization is given by $\mathcal{P}_{HP} = [1-(1-p)^{I_1}] \times \ldots \times [1-(1-p)^{I_K}]$, where we follow the mnemonic rule that HP stands for *h*ierarchies of *p*olyarchies. The problem of maximizing the screening performance of such an organization when the total number of members I and a number of levels of the hierarchy K are given is straightforward. We assume, for simplicity, that I is much larger than K and that both are continuous quantities rather than integer ones. Since the screening effectiveness of the organization is an increasing concave function of each of the I_k's, its maximum over the convex set defined by the given total number of agents occurs when the sizes of all polyarchies are equalized, given the number of hierarchy levels. Therefore, under the assumption that I is divisible by K, there are

$\frac{I}{K}$ agents in each polyarchy and the screening function of a hierarchy of K polyarchies is given by (**??**) above. This function has the following properties. First, it is an increasing concave function of the total number of agents, I. That is, adding agents while holding constant the number of hierarchies has the effect of increasing the likelihood of acceptance of *all* projects. Second, the screening function is a decreasing convex function of the number of levels of the hierarchy K, provided that each hierarchy has at least one agent. It follows that the determination of the optimal number of hierarchy levels must rest on other considerations. Third, it is a monotonically increasing but sigmoid function of p. This follows from the fact that the first derivative of the screening function with respect to p is $\frac{\partial}{\partial p}\mathcal{P}_{HP} = I(1-p)^{\frac{I}{K}-1}\left(1-(1-p)^{\frac{I}{K}}\right)^{K-1}$, and second derivative is $\frac{I}{K}(1-p)^{\frac{I}{K}-2}\left(1-(1-p)^{\frac{I}{K}}\right)^{K-2}\left[K(I-1)(1-p)^{\frac{I}{K}}-(I-K)\right]$. The inflection point occurs at $\tilde{p} = 1-\left(\frac{I-K}{K(I-1)}\right)^{\frac{K}{I}}$, which is in $(0,1)$, provided that $I > K > 1$. The screening function $\mathcal{P}_{HP}$ intersects the 45^o line from below at a single point, the unique fixed point $\hat{p}_{HP}$, of $\mathcal{P}_{HP}$ in $(0,1)$.

As more hierarchy levels are added, the inflection point of the screening function increases making screening more demanding; as more agents are added while holding the number of hierarchy levels constant, the fixed point of the screening function decreases. It is therefore not possible both to improve the organization's performance in rejecting poor projects and in approving good projects in this fashion. A tradeoff emerges between overall probability of acceptance and effectiveness of screening performance. Increasing the number of hierarchy levels reduces the probability that an organization accept any given project. But if the probability of acceptance is high, such an increase makes it more likely that better projects would be accepted.

3.1.2 Polyarchies of Hierarchies

It is straightforward to prove that an optimal polyarchy of hierarchies should have equal number of components. The screening function of a polyarchy of $\frac{I}{K}$ hierarchies, each of which has K agents, given by (**??**) above, has the following properties. First, it is increasing concave in the total number of agents; second, it is convex decreasing in the number of hierarchy levels; and third, it is monotonically increasing and sigmoid in individual screening performance. This follows from the fact that the first derivative of the screening function with respect to p is $\frac{\partial}{\partial p}\mathcal{P}_{PH} = I\left(1-p^K\right)^{\frac{I}{K}-1}p^{K-1}$. and its second derivative $I(1-p^K)^{\frac{I}{K}-2}p^{K-2}\left[K-1-(I-1)p^K\right]$. Its inflection point is: $\tilde{p}=\left(\frac{K-1}{I-1}\right)^{\frac{1}{K}}$, and is, therefore decreasing in the size of the organization but increasing in the number of hierarchies. It has a unique fixed point in $(0,1)$, $\hat{p}_{PH}$, where it intersects the 45^o from below. A tradeoff emerges between overall probability of acceptance and effectiveness of screening performance, the counterpart to the one for a hierarchy of polyarchies: increasing the number of hierarchy levels reduces the probability that an organization accept any given project while favoring better projects.

3.2 Endogenous Individual Screening Effectiveness

We saw that both hierarchies of polyarchies and polyarchies of hierarchies with identical agents are associated with screening performance functions that are sigmoid in individual screening performance. Therefore, it is the mixing of architectures that is responsible for the sigmoidicity of the performance function and thus the possibility of improvements similar to those enabled by composition. However, even cross-composing once moves the organization toward improved screening performance.

Next we endogenize individual screening performance by optimally setting the wage rate for an organization with a given screening function $\mathcal{P}[\cdot]$. A higher wage sharpens the effectiveness of the typical agent, in that the probability of accepting a good project, $P_g(w)$, is increasing concave, and the probability of accepting a bad project, $P_b(w)$, is decreasing convex in the wage rate. Working from the definition of expected profit (1), the optimal wage rate must satisfy the first- and second-order conditions:

$$\alpha \cdot R_g \cdot \mathcal{P}'\left[P_g(w)\right] P_g' + (1-\alpha) \cdot R_b \cdot \mathcal{P}'\left[P_b(w)\right] P_b' = I, \tag{4}$$

$$\alpha \cdot R_g \cdot \left(\mathcal{P}''\left[P_g(w)\right](P_g')^2 + \mathcal{P}'\left[P_g(w)\right] P_g''\right) + (1-\alpha) \cdot R_b \cdot \left(\mathcal{P}''\left[P_b(w)\right](P_b')^2 + \mathcal{P}'\left[P_b(w)\right] P_b''\right) < 0. \tag{5}$$

An interesting result readily follows. A sufficient condition for the existence of a unique optimal wage rate is that the screening function $\mathcal{P}$ be increasing sigmoid in its argument, being initially convex and then concave. In such a case, a sufficiently high value of the wage rate makes the first term within the first set of parentheses in (5) negative, because of the concavity of $\mathcal{P}[P_g(w)]$ for high values of w. It also makes the first term within the second set of parentheses positive, because of the convexity of $\mathcal{P}[P_b(w)]$ for high values of w, making it more likely that the entire second term be negative. As the wage rate increases, each agent becomes more adept in recognizing good projects, thus increasing $P_g(w)$, and also more adept in recognizing bad projects, thus decreasing $P_b(w)$.

This reveals a critical role of the fact that the screening function is sigmoid when screening performance is endogenized via the wage rate. However, for the optimal wage w^* to also cause the organization to be more effective in screening projects relative to an individual agent, it must be the case that the probability of accepting a bad project should be below p^*, the fixed point of $\mathcal{P}[\cdot]$, and the probability of accepting a good project should be above it, or

$$P_b(w^*) < p^* < P_g(w^*). \tag{6}$$

Practically speaking, the nearer that the fixed and the inflection points, $\hat{p}$ and $\tilde{p}$, of the organization's screening function are to one another, the more likely it is that condition (6) be satisfied.

3.3 Sorting versus Mixing with Heterogeneous Agents and Sah–Stiglitz Architectures

If agents differ in terms of screening effectiveness, how should agents be grouped across firms in terms of their screening effectiveness?[8] We consider two firms, each of which employs two agents and chooses them out of a total of four agents. There are two of each of two different types of agents, with screening effectiveness p_1 and p_2, respectively. Each firm's output is proportional to a firm's probability of approving a project.

If both firms are organized as hierarchies, then each firm's expected revenue is proportional to product of the screening probabilities of its workers, and thus a convex function of their p's, provided that such screening by each of the two workers is independent. Efficiency then dictates that agents working for hierarchical firms be *sorted.* That is, total expected revenue by two firms that are organized as hierarchies is greater if both agents of the same type work together, $p_1^2 + p_2^2$, than if they work separately, $2p_1p_2$: $p_1^2 + p_2^2 \geq 2p_1p_2$. This follows from the convexity properties of the hierarchy discussed above. This result generalizes readily to the case of many agents. These results carry over to the comparison of expected profits, provided that $R_g > -R_b$ and that agents are paid the same. Therefore, depending upon the sensitivity of individual screening effectiveness to the wage rate, the possibility exists for this sorting result to lead to unequal wages. It is possible

[8] This problem is reminiscent of segregation by skill, discussed by Kremer and Maskin (1996).

to show that more effective workers would be paid more, independently of the wage rate for less effective workers.

If, on the other hand, both firms are organized as polyarchies, then expected revenue is proportional to a concave function of screening probabilities. Efficiency dictates that agents be *mixed.* That is , the concavity properties of the polyarchy discussed above imply that total expected revenue by two firms that are organized as polyarchies is greater if agents of different type work together than if both agents of the same type work together, $2(1-(1-p_1)(1-p_2)) \geq 1-(1-p_1)^2+1-(1-p_2)^2$. Again, this result generalizes readily to the case of many agents. It also carries over to a comparison of expected profits, provided $R_g > -R_b$ and that agents are paid the same. It is possible to show that more effective agents would be paid more than less effective agents but the determination of the wage structure reflects, not surprisingly, both screening functions.

Complicating simple Sah–Stiglitz architectures by means of composition preserves the mixing and sorting properties, because a concave (convex) function of concave (convex) functions is concave (convex). However, it is not particularly interesting to examine such compositions because the simple Sah–Stiglitz architectures do not satisfy the Moore-Shannon criterion, and therefore, such compositions do not improve an organization's screening performance. For this reason, we turn next to apply the Sah–Stiglitz theory to study properties of organizations that are the outcome of cross-composition, that is they are either hierarchies of polyarchies or polyarchies of hierarchies.

For a hierarchy of polyarchies we have:

$$\mathcal{CC}^{\text{HP}}(p_1, I_1; \ldots, p_K, I_K) = [1-(1-p_1)^{I_1}] \times \ldots \times [1-(1-p_K)^{I_K}]. \tag{7}$$

To consider the impact of different probabilities of individual screening effectiveness on organizational architecture, we compare the screening effectiveness for two alternative organizations that are obtained by interchanging an agent from level 1, with an agent from level 2, when a agent with higher performance is interchanged with an agent with lower performance. The comparison depends only upon the value $[1-(1-p_1)^{I_1}] \times [1-(1-p_2)^{I_2}]$ relative to the value of $[1-(1-p_1)^{I_1-1}(1-p_2)] \times [1-(1-p_2)^{I_2-1}(1-p_1)]$. It turns out that if $p_1 < p_2$ and $I_1 \leq I_2$, that is, when there are more higher-ability agents, moving a higher ability agent to the position of a lower ability agent in a polyarchy of lower-ability and replacing that agent with a lower-ability one in a polyarchy of higher-ability (thus reducing size differences) improves screening effectiveness.

What can we say about sorting versus mixing in organizations that consist of hierarchies of polyarchies and of polyarchies of hierarchies? We consider the case of I agents of type 1 and of I agents of type 2 and of 2 firms, and assume the number of agents of each type to be even. If both firms are hierarchies of polyarchies, then

$$2\left(1-(1-p_1)^{\frac{I}{2}}\right)\left(1-(1-p_2)^{\frac{I}{2}}\right) \leq \left(1-(1-p_1)^{\frac{I}{2}}\right)^2 + \left(1-(1-p_2)^{\frac{I}{2}}\right)^2,$$

and it pays on grounds of efficiency for the agents to be *sorted*. Similarly, if both firms are polyarchies of hierarchies, then

$$2\left(1-\left(1-p_1^{\frac{n}{2}}\right)\left(1-p_2^{\frac{n}{2}}\right)\right) \geq \left(1-\left(1-p_1^{\frac{n}{2}}\right)^2\right) + \left(1-\left(1-p_2^{\frac{n}{2}}\right)^2\right),$$

and it pays on grounds of efficiency for the agents to be mixed.

4 A Cognitive Model of Screening Performance

The fact that cross-compositions of pure Sah–Stiglitz architectures are characterized by sigmoid functions of individual performance was shown to be critical in our theory of organizational architecture. We now turn to a model of individual screening behavior that emphasizes cognitive aspects of agents' decision problems. This model allows us to study the performance of general architectures and to pose the problem of optimal organizational design when agents' decisions are interdependent. It also allows us to examine the role of top-to-bottom feedback.

We model an individual's screening performance in terms of the magnitude of a performance index. For individual i, the components of the index $h_i^j + J_i\Omega_{\nu(i)} + \epsilon_{ij}$ are defined as follows: h_i^j, $j = g, b$, is the individual's assessment of a typical project, with $h_i^g > h_i^b$ representing the fact that other things being equal agent i is more likely to accept a good project rather than a bad project; second is a contribution due to recommendations of an individual's subordinates, $\Omega_{\nu(i)}$, with $\Omega_{\nu(i)} = 1$ (-1), indicating acceptance (rejection), that carries a positive relative weight J_i assigned to the recommendations from subordinates; and a third is a random factor, ϵ_{ij}, to be discussed shortly. If the value of the index $h_i^j + J_i\Omega_{\nu(i)} + \epsilon_{ij}$ exceeds a certain threshold, then agent i accepts a project, $\omega_i = 1$; otherwise, she rejects it, $\omega_i = -1$. Therefore, agent i's decision may be described by a binary indicator, defined in terms of the distribution function of the ϵ_{ij}s. We assume, for convenience, that the random factor in the comparison is extreme-value distributed and therefore the probability that agent i approve of a project of quality j, $j = g, b$, is expressed in terms of the logistic integral:

$$\text{Prob}\left(\omega_i = 1)|\Omega_{\nu(i)}, j\right) = \frac{\exp\left[\beta\left(h_i^j + J_i\Omega_{\nu(i)}\right)\right]}{1 + \exp\left[\beta\left(h_i^j + J_i\Omega_{\nu(i)}\right)\right]},\ j = g, b, \tag{8}$$

where β, is a positive parameter that represents the sensitivity of the evaluation to the random factors, given whether or not a project is good or bad and given the recommendation from subordinates is positive or negative. The case of $\beta \to 0$ implies purely random choice: the two outcomes are equally likely. The higher is β the smaller is the variance of the random, factor associated with the evaluation.

The evaluation probabilities depend, via parameters β, h_i^g, h_i^b, J_i both on the intrinsic quality of a project and on the influence of the other members of the organization that report to agent i. This decomposition is a critical element in that it allows us to examine whether the screening performance of real-life organizations may be improved when the influence on an agent from other members of an organization may be separated from the effect of a project's intrinsic quality and of random factors. It also allows us to incorporate the impact of different incentives, as for example, when an agent's performance is sensitive to remuneration, which is not as flexibly done in the Sah–Stiglitz model. From the results reported in the paper so far, it should be expected that the most critical feature of the cognitive model is that an agent's evaluation is a sigmoid function of the recommendations of her subordinates. Finally, having demonstrated how wages may be endogenized earlier in the paper, we refrain from doing so in this part of the paper.

4.1 General Organizational Architectures

In Section 3 above, Sah–Stiglitz hierarchies and polyarchies were used as building blocks for more complex organizations. Next we examine organizations which are characterized by interdependence among agents that does not presuppose one of these architectures. We turn next to the general notation that we introduced earlier.

Let $\widetilde{\omega}$ denote the $I-$vector containing the decisions of agents $j = 1, \ldots, I$. Agent i receives recommendations from her subordinates $\nu(i)$, which when stacked as a vector are denoted by $\widetilde{\omega}_{\nu(j)}$.

Adapting the cognitive model, we have that agent i makes a decision, taking into consideration her own evaluation and the input from her subordinates $\Omega_{\nu(i)}$, which is a general nonlinear function that aggregates the input from her subordinates $\widetilde{\omega}_{\nu(i)}$, $\Omega_{\nu(i)} = \Omega(\widetilde{\omega}_{\nu(i)})$. Decisions by an agent's subordinates are assumed to be independent, conditional on the information that they possess in common and on whether a project is good or bad. Noteworthy special cases are whether an agent is sensitive to the average of the recommendations from her subordinates, to unanimity among her subordinates (which corresponds to the Sah–Stiglitz hierarchy), and to whether or not the number of positive recommendations exceeds the number of negative ones by at least a given number. The latter could be interpreted as being sensitive to a modified majority of her subordinates; see Sah and Stiglitz (1988).

In general, the decision probabilities for agent i may be written in terms of the recommendations of agent i's subordinates by working with (8) and iterating it backwards. This yields:

$$\begin{bmatrix} \text{Prob}\left(\omega_i^j = 1|j\right) \\ \text{Prob}\left(\omega_i^j = -1|j\right) \end{bmatrix}^{\mathtt{T}} = \begin{bmatrix} \text{Prob}\left(\Omega_{\nu(i)}^j = 1|j\right) \\ \text{Prob}\left(\Omega_{\nu(i)}^j = -1|j\right) \end{bmatrix}^{\mathtt{T}} \cdot \mathbf{\Psi}^j, i = 1, \ldots, I, \tag{9}$$

where $\mathbf{\Psi}^j$ is defined as the 2×2, Markov transition matrix as follows:

$$\mathbf{\Psi}^j = \begin{bmatrix} \text{Prob}\left(\omega_i^j = 1|\Omega_{\nu(i)}^j = 1\right) & \text{Prob}\left(\omega_i^j = -1|\Omega_{\nu(i)}^j = 1\right) \\ \text{Prob}\left(\omega_i^j = 1|\Omega_{\nu(i)}^j = -1\right) & \text{Prob}\left(\omega_i^j = -1|\Omega_{\nu(i)}^j = -1\right) \end{bmatrix}, \tag{10}$$

where the conditional choice probabilities are given by (8), adapted in the obvious way. The first (second) row gives the probabilities that an agent will choose to approve, respectively, to reject, a project of quality j, conditional on receiving a positive (negative) recommendation from her subordinates.

Therefore, $\Omega_{\nu(i)}^j$ is defined as a function of $\widetilde{\omega}_{\nu(i)}$, and the probability of agent i's decisions, conditional on a project being good or bad, are given by (9). In order to be able to compute the screening probabilities of the entire organization, we need to specify the aggregator rule $\Omega_{\nu(i)}$, as a function of the set of the recommendations from agent i's subordinates, $\Omega_{\nu(i)}$. Since the organization is defined by means of a directed graph, the probabilities $\text{Prob}\left(\omega_i^j\right)$ may themselves be written by applying (9) recursively in terms of the probabilities describing the decisions of those reporting to each agent i, $\Omega_{\nu(i)}$. We note that (9) may be adapted to apply when screening takes time, as well as when the stochastic dependence occurs contemporaneously and is purely spatial. The tools of Markov random fields may be used in such a case to analyze organizational equilibrium.

Additional results may be obtained for both pure Sah–Stiglitz architectures, when the number of agents is large. The limits for the probabilities of the two possible outcomes for large organizations may be obtained easily. For the hierarchy case, they are given by the invariant distribution of $\Psi^j, j = g, b$. For the polyarchy case, they are given by the first row of Ψ^j. We conclude, therefore, that for a large number of agents, the pattern of interdependence among agents, as represented by the stochastic matrix Ψ^j, determines entirely the decisions of the organization. Of course, Ψ^j encapsulates the fundamentals of the cognitive model. These results are somewhat reminiscent of the properties of informational cascades with large numbers of agents [Bikhchandani, *et al.* (1992)].

4.2 Averaging the Opinions of Subordinates

We demonstrate some of the possibilities afforded by this approach by assuming, for simplicity, that each agent is sensitive to the *average* recommendation of her subordinates. That is, in a static model, $\mathcal{E}\{\Omega_{\nu(i)}|j\} = \frac{1}{|\nu(i)|}\sum_{i'\in\nu(i)}\mathcal{E}\{\omega_{i'}|j\} = \mathcal{E}\{\omega_{i'}|j\}, \ i' \in \nu(i)$. When the number of subordinates, $|\nu(i)|$ is large, then by the law of large numbers the actual magnitude of $|\nu(i)|$ does not matter.[9] Let us assume that agents are labelled so as agent $i = 1$ is the boss. Let the set of agents at level k of the hierarchy be denoted by $\ell(k)$. In particular, $\ell(1) = 1$, and $\ell(K)$ denotes the bottom level of the organization. Thus $\mathcal{I} = \bigcup_{k=1}^{K}\ell(k)$. It then follows, for both $j = g, b$, that:

$$\mathcal{E}\{\omega_i|j\} = \tanh\left(\beta h_i^j + \beta J_i \mathcal{E}\{\omega_{i'}|j\}\right), \ i' \in \nu(i), \ \forall i \in \ell(k), \ k = 1, \ldots, K-1, \tag{11}$$

$$\mathcal{E}\{\omega_i|j\} = \tanh(\beta h_i^j), \ \forall i \in \ell(K). \tag{12}$$

These results allow us to adapt the expression for expected profit from considering a randomly chosen project, according to (1). In the simplest possible case when each level of the hierarchy

[9]The case when an agent is sensitive to the actual sample mean of the responses of her subordinates is of particular interest because it models the case of modified majority. In fact, the probability distribution for the sample mean may expressed in terms of those for the number of favorable (or of unfavorable) recommendations. Specifically, let Y_i denote the sample mean: $Y_i = \sum_{i\in\nu(i)}\omega_i$. Then, using the fact that favorable recommendations are coded by 1 and unfavorable ones by -1, implies that the number of favorable recommendations is equal to $\frac{Y_i+|\nu(i)|}{2}$. However, the probability distribution for this random variable may be expressed in terms of the probability for success in $|\nu(i)|$ Bernoulli trials.

contains a single agent, $K = I$, and all agents are identical, expected profit from (1),

$$R_g \cdot \alpha \cdot \text{Prob}\{\omega_1^g = 1|g\} + R_b \cdot (1-\alpha) \cdot \text{Prob}\{\omega_1^b = 1|b\} - Kw, \tag{13}$$

may be computed by computing recursively the probabilities attached to recommendations by all agents. That is, the acceptance probability by agent 1, given quality j, may be obtained recursively according to $\text{Prob}\{\omega_1 = 1|j\} = \frac{1}{2}\left[1 + \tanh\left(\beta h^j + \beta J E\{\omega_2|j\}\right)\right]$, and so on down the hierarchy according to (11) and (12), separately for $j = g, b$. Given wages, then the optimal number of hierarchy levels is defined in terms of the trade-off between the advantage of additional workers' improving the organization's screening performance and their cost. It is straightforward to establish that $\text{Prob}\{\omega_1 = 1|g\}$ is an increasing concave function of K, and $\text{Prob}\{\omega_1 = 1|b\}$ is decreasing convex function of K, the number of hierarchies (and agents). Therefore, $R_b < 0$ suffices for expected revenue to be concave in the number of agents and therefore for the optimal number of agents to be well defined, given w.

Next we demonstrate how adding levels in the hierarchy improves organizational screening performance. We gauge the improvement in performance as the number of agents increases in terms of reduction in the probability that the organization reject good projects, given by

$$\text{Prob}\{\omega_1^j = -1|g\} = \frac{1}{2}\left[1 - \tanh\left(\beta h^g + \beta J E(\omega_2)\right)\right], \tag{14}$$

and in the probability that it accept bad projects, given by

$$\text{Prob}\{\omega_1^j = 1|b\} = \frac{1}{2}\left[1 + \tanh\left(\beta h^b + \beta J E(\omega_2)\right)\right]. \tag{15}$$

As we see shortly, both these conditional acceptance probabilities decrease as we increase the number of levels of the hierarchy. In fact, the limits of these probabilities as the number of levels tends to infinity may be characterized precisely as follows.

It is easiest to do this graphically. Let us assume, without loss of generality, that the contribution of project quality to the performance index satisfies that $h^b < 0 < h^g$. Starting with the agent at level K,the bottom of the hierarchy, the probability of rejecting a good project is given by $\frac{1}{2}\left[1 - \tanh\left(\beta h^g\right)\right]$, and the probability of accepting a bad project is given by $\frac{1}{2}\left[1 + \tanh\left(\beta h^b\right)\right]$. These probabilities are equal $\frac{1}{2}|G_1 G^1|$ and $\frac{1}{2}|B_1 B^1|$, respectively, in Figure 2. The rejection probability for a good project after adding a second agent is given by:

$$\text{Prob}\{\omega_1 = 1|g\} = \frac{1}{2}\left[1 - \tanh\left(\beta h^g + \beta J\left(\tanh\left(\beta h^g\right)\right)\right)\right],$$

and similarly for the acceptance probability for a bad project. Referring to Figure 2, we mark the point with horizontal coordinate equal to $|O_1 g_1| = \beta h^g$ and then obtain the value of $\tanh\left(\beta h^g + \beta J \tanh\left(\beta h^g\right)\right)$. By construction, this quantity, marked by $|G_2 G^2|$, is less than $|G_1 G^1| = \tanh\left(\beta h^g\right)$. It then follows that as the number of levels K increases tending to infinity, the acceptance probability for a good project tends asymptotically to 1 minus the value of the upper fixed point of $\varpi = \tanh\left[\beta h^g + \beta J \varpi\right]$. This fixed point, to be referred to as $\overline{\varpi}$, always exists and is stable if $h^g > 0$ [Ioannides (2001), Proposition 1]. We may work in the same manner for the acceptance probability for a bad project.

We conclude that since the upper and lower fixed points of the respective screening functions are well defined, they provide the lower bounds for the screening probabilities. These fixed points can always be computed in an iterative fashion from the above formulas. They may not be reduced further by increasing the number of agents, but the optimal size of the firm is well defined.

4.3 Organizations with Top-to-Bottom Feedback

We next turn to examining screening performance when there is feedback from higher levels to lower levels of the hierarchy. We examine an extreme example, that is when the boss directs the lower level staff to projects after her own preliminary assessment of their quality. As we see shortly, this model introduces endogenous social interactions that affect organizational screening performance. Such feedbacks from agents at higher levels of an organization to those at lower levels organizational performance are reminiscent of a model of social interactions in organizations like that of DeMarzo, Vayanos and Zwiebel (2003). [10] A boss's influencing the evaluation process at the lowest level of the hierarchy is a special case of the listening structure in *ibid.*. This would be a way to model the role of an organization's principal in setting an organization's agenda. A key consequence of

[10] In DeMarzo, Vayanos and Zwiebel (2003), individuals update their own beliefs by weighting the beliefs of the other agents they "listen to" in terms the respective relative precisions. The resulting dynamic model in implies that in the long run beliefs converge to a set of consensus beliefs which depend only on the spectral properties of the matrix that describes the listening structure. In the special case of hierarchical organizations possessing a "standard listening structure," that is where every agent except the chief has a single boss and each agent listens to her subordinates and to all of those located higher up than herself, the relative beliefs over the frequency distribution that characterizes consensus beliefs of an agent and a subordinate depend only on the numbers of her subordinates and on her level in the hierarchy [*ibid.*, Theorem 10].

this assumption is, as we see shortly, that the principal's input to entry level decisions influences all decisions and by cascading back up to her cause the equilibrium outcome to be independent of the number of agents.

Suppose that agents at level K expect agent 1's average decision to be $\hat{\omega}_1^j$, conditional on a project of quality j. Then the average decision for agents at level K is $\mathcal{E}\left\{\omega_K\right\} = \tanh\left(\beta h^j + \beta J \mathcal{E}\left\{\hat{\omega}_1^j\right\}\right).$ It follows then that the expectation of agent 1's decision satisfies

$$\mathcal{E}\left\{\omega_1^j\right\} = \mathcal{T}^K\left(\beta h^j + \beta J \hat{\omega}_1^j\right),$$

where $\mathcal{T}^K[\cdot]$ denotes the $K-$th order iterate of the function $\tanh\left(\beta h^j + \beta J \varpi\right)$. Therefore, the principal's decision as an organizational outcome at equilibrium satisfies $\mathcal{E}\left\{\omega_1^j\right\} = \hat{\omega}_1^j$, and is given a fixed point of the function $\mathcal{T}^K$. The fixed points of this function coincide with the fixed points of the function $\varpi = \tanh(\beta h^j + \beta J \varpi)$. Ioannides (2001) shows that there may either one or three fixed points, depending upon parameter values. Therefore, we conclude that due to "closure" in the listening structure (made possible by the top level's providing feedback directly to the lowest level of the hierarchy, the screening probabilities at equilibrium are independent of the number of levels of the hierarchy. The screening probabilities are given by $\frac{1}{2}\left[1 + \tanh\left(\beta h^g + \beta J \overline{\varpi}^g\right)\right]$, for good projects, and $\frac{1}{2}\left[1 + \tanh\left(\beta h^b + \beta J \underline{\varpi}^b\right)\right]$, for bad projects, where $\overline{\varpi}^g$ is the upper fixed point of $\varpi = \tanh(\beta h^g + \beta J \varpi)$, and $\underline{\varpi}^b$ is the lower fixed point of $\varpi = \tanh(\beta h^b + \beta J \varpi)$. This also means that the number of hierarchy levels, and thus the number of agents as well, is indeterminate.

The symmetry with which all agents are treated and the fact that the model rests only the average decision are clearly responsible for the resulting independence from the number of agents. We recall that the model without feedback is inherently asymmetric. By slightly modifying the above model, that is by allowing for an influence coefficient from agent 1 to agent K, J_{1K}, that differs from J, we end up with explicit dependence of the screening probabilities on the number of hierarchy levels and of agents. The dependence of the decision of agent 1 on that of the agents at level K is expressed by:

$$E\left\{\omega_1\right\} = \mathcal{T}^{K-1}\left(\beta h^j + \beta J E\left\{\omega_K\right\}\right), \tag{16}$$

and the dependence of the decision of agent K on that of agent 1 is expressed by:

$$E\left\{\omega_K\right\} = \tanh\left(\beta h^j + \beta J_{1K} E\left\{\omega_1\right\}\right). \tag{17}$$

Proposition 1 in Ioannides (2001) again applies with minimal modification and ensures that there exist an upper fixed point $\overline{\varpi}^g$ of Equ. (16) – (17) for $j = g$, and a lower fixed point $\underline{\varpi}^b$ of Equ. (16) – (17) for $j = b$. The screening probabilities for good, respectively, bad projects are given by the same formulas as above and are concave increasing, respectively, convex decreasing functions of the number of hierarchy levels. Therefore, the optimal size of the organization is well defined. The quantities $\underline{\varpi}^b$ and $\overline{\varpi}^b$ coincide with those of the symmetric case above, when $J_{1K} = J$.

This result allows us to conclude that the cognitive model of individual behavior does not allow us to avoid the complexity of the Von Neumann–Moore–Shannon solution. Still, it is interesting that top-to-bottom feedback improves screening performance over an finite-sized hierarchy, by allowing an organization to attain immediately the best possible values for the likelihood of rejecting good projects and of accepting bad ones as equilibrium outcomes.

The results of this example and of the earlier one on averaging over the opinions of subordinates both demonstrate an important property of the cognitive model: organizational screening performance is sigmoid function of $h^j, j = g, b$, each agent's contribution to her own evaluation of a typical project. With the cognitive model, adding agents improves over individual screening performance even with pure Sah–Stiglitz architecture, and even *without* mixing (that is, cross-composing) of architectures.

5 Conclusion

This paper models organizations whose members are engaged in screening projects. It examines how architectural features of organizations affect organizational screening performance. It shows that for an organization of several agents to be able to improve upon individual performance, the organization's screening function must be sigmoid in individual performance. Without a cognitive model of individual performance, organizations with mixed Sah–Stiglitz architectures, such as hierarchies made up of components that are polyarchies and polyarchies made up of components that are hierarchies, do give rise to such functions. This property is in turn critical for the determination of optimal firm size and levels of hierarchy, and the endogenous determination of individual screening performance. They are examined in detail in this paper. The paper also introduces a cognitive model that allows for individuals' own screening efforts to combine with influence from others, subordinates and superiors. Such a model makes an agent's screening performance be sigmoid with respect to the influence of others. The paper examines the implications of such interactions for the limits to organizational performance.

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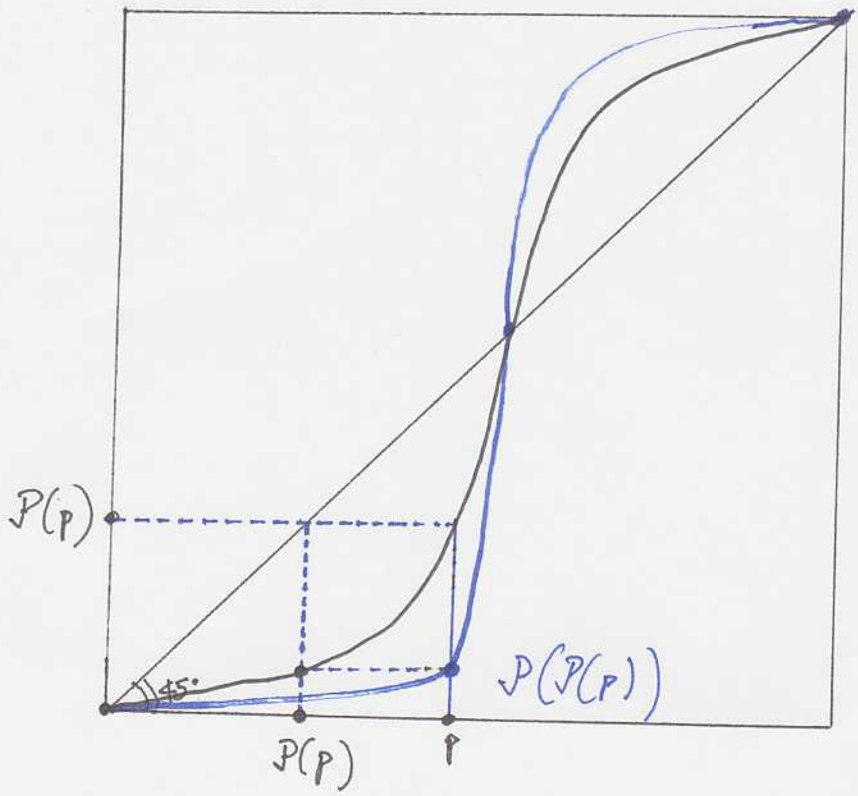

FIGURE 1

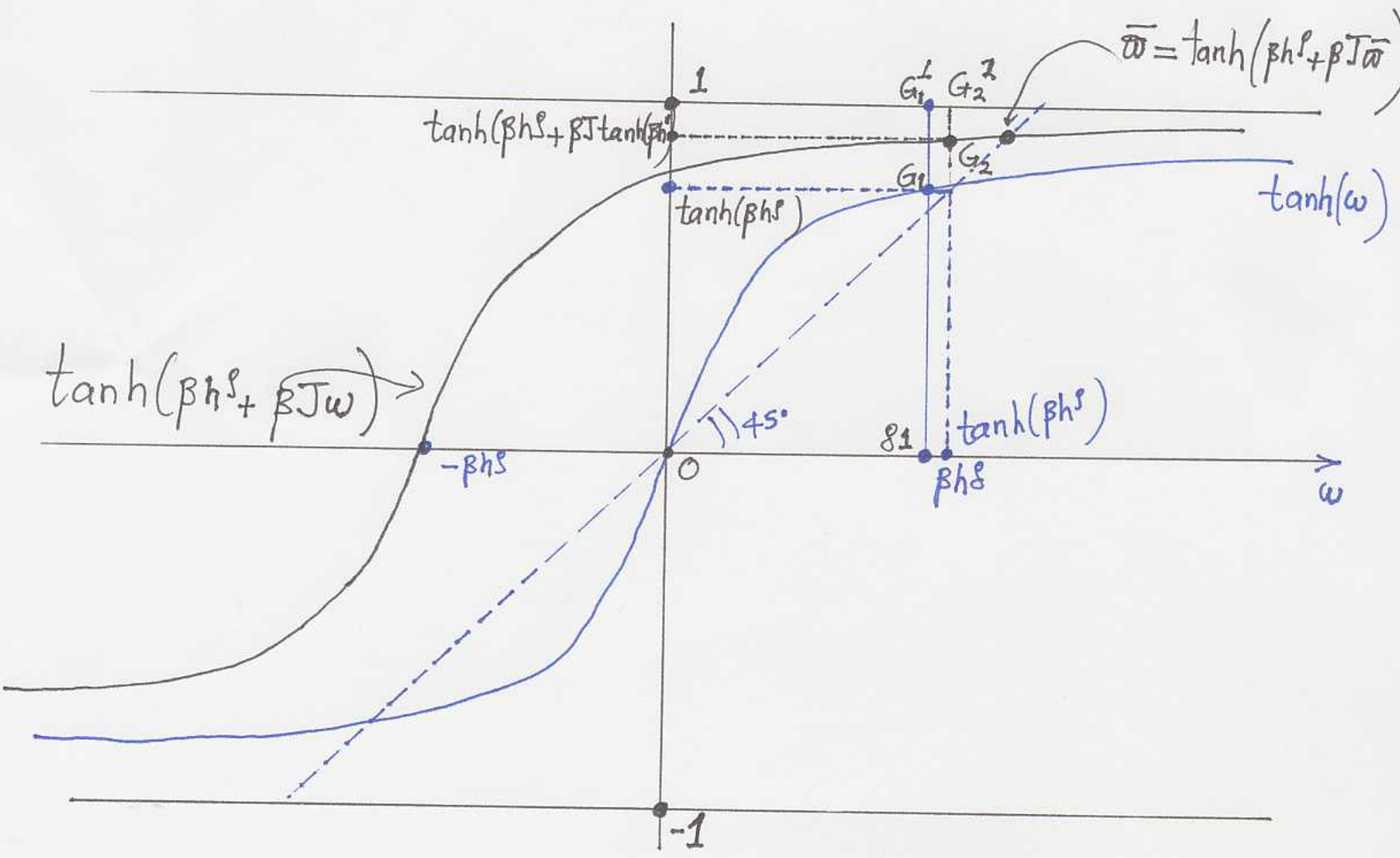

FIGURE 2

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