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# Product Differentiation and Upstream-Downstream Relations

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**PRODUCT DIFFERENTIATION AND UPSTREAM-DOWNSTREAM RELATIONS**[1]

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**ABSTRACT**

This paper examines the relationship between a differentiated downstream market and a specialized upstream market. We analyze four different types of vertical relation between the upstream and downstream sectors when the upstream market supplies specialized and complementary inputs to a downstream product differentiated market. The first is the benchmark case of decentralized markets, the second is a network of alliances among upstream suppliers, the third is partial vertical integration and the fourth is an upstream consortium of suppliers. We identify the perfect equilibrium for a symmetric model in each case and show that there is no simple relationship between the degree of connection between upstream and downstream firms and profitability. The key factor affecting prices and the relative profitability of the different market organizations is the degree of product differentiation among the downstream firms because it affects the intensity of competition among upstream suppliers. We show that vertical foreclosure by the partially integrated firms is not an equilibrium strategy.

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## 1. Introduction

*The Wall Street Journal* has noted that "More and more of the work in America is project oriented, with a beginning, a middle and an end. Projects lend themselves to a blend of traditional employees, contract workers and consultants, who combine into teams, do a job and then usually break up, with most of the players looking for their next gig."[2] The project-based approach to doing business has long been practiced in the building trade and in Hollywood. Today it is spreading to a diverse set of downstream markets such as medical care, where general practitioners and specialists combine into preferred provider groups to serve an HMO, and information services, where alliances among high-tech upstream contractors are the norm. Malone and Laubacher (1998), for example, talk of projects undertaken by "e-lancers (who form) fluid and temporary networks…When the job is done … the network disappears, and its members become independent agents again … seeking the next assignment." (1998, p. 146)

Concurrent with the spread of the project based approach, we also see many large companies, such as Shell, Xerox, AMR Corporation and Scandinavian Airlines, spinning off their information technology, training and consulting services into separate divisions. The intent is to market these specialized services to companies other than the parent company. Again this is common in Hollywood where, for example, the first television program developed by ABC and its new studio Dreamworks was a show for CBS.

These changes in the upstream provision of specialized services share some common underlying traits. First, the upstream suppliers who come together to work on a project typically supply specialized services that are complementary to each other. Secondly, any one upstream supplier could be working on more than one project at the same time. Thirdly, when these specialized input services are combined, they produce something that is itself differentiated. These traits suggest a connection between the differentiation of upstream services and the nature of product differentiation downstream. This in turn suggests that the demand for product differentiation in the downstream market could be an important factor affecting the way that specialized upstream services are supplied to the downstream.

[2] Cited in "Flying Solo: High Tech Nomads Write New Program for Future of Work", *The Wall Street Journal*, page A1, August 19 1996.

There is an extensive literature in industrial organization on product differentiation but it has focused almost entirely on price and product competition in the downstream market alone.[3] Similarly there is an extensive literature on vertical relationships between upstream suppliers and downstream firms[4] but the usual assumption is that the upstream firms supply perfect substitutes to downstream firms who also produce perfect substitutes.[5] The connection between differentiation in the upstream market and differentiation in the downstream market has received much less attention. Yet one of the most important means by which downstream firms differentiate their products is through the use of inputs that are differentiated or specialized.

This paper attempts to fill this gap and to explore how consumer attitudes to product differentiation downstream affects vertical relations between the upstream and downstream. We develop a stylized model of an upstream sector that supplies specialized services to a product differentiated downstream sector. Our model captures several important features that are relevant to the increasingly varied vertical relations that we currently observe. Specifically, upstream suppliers supply *complementary* specialized, or differentiated, inputs that are combined to produce goods sold in a set of differentiated downstream markets. Different combinations of these complementary specialized inputs produce different downstream products. In addition, any one upstream supplier may be working with more than one downstream firm at the same time.

We begin our analysis with the benchmark case of a fully decentralized organization of the upstream and downstream markets. For this case, the downstream producers buy their differentiated and complementary inputs from independently owned and managed upstream suppliers. The downstream firms produce their differentiated goods and compete in prices for consumers of their products.

The equilibrium set of upstream and downstream prices for the decentralized case is inefficient due to two well-known complementarities in production. The first of these is the pecuniary externality first noted by Cournot (1838), which here is relevant to the upstream supply

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[3] See Beath and Katsoulacos (1991) for a survey of the economic theory of product differentiation, and Thisse and Norman (1995) for a collection of many of the most important papers.

[4] Some of the notable contributions include Bolton and Whinston (1993), Bonanno and Vickers (1988), Gaudet and Van Long (1996), Hart and Tirole (1990), Ordover, Saloner and Salop (1990) and Salinger (1988).

[5] Exceptions are Bolton and Whinston (1993) who investigate the vertical relationship between an upstream firm who supplies two downstream firms selling in completely differentiated markets and Bonanno and Vickers (1988) who consider two firms which produce differentiated products that compete with each other.

of two complementary inputs.[6] The second type of complementarity is similar to the first but, because it reflects a coordination failure between the upstream and the downstream, is commonly referred to as double marginalization. The coordination failures in decentralized pricing lead to higher prices and loss of potential profits in the upstream and/or downstream sectors.

We then investigate prices and profits under two alternative ways of transacting business between the upstream and the downstream that partially internalize the pecuniary externalities and/or remove the effects of double marginalization.[7] The first of these is the creation of a horizontal network to facilitate alliances among upstream suppliers. The second arrangement is partial vertical integration in which some, but not all, downstream producers own their specialized upstream suppliers. For this case we must also consider how the integrated firm is internally organized, or more specifically, the degree of separation between the downstream and upstream divisions. Bonanno and Vickers (1988) show that some form of vertical separation, which leads to non-marginal cost pricing, may be desirable when it leads to softer downstream price competition. We examine three different internal organizations of the integrated firm with varying degrees of separation, one analogous to the U-form and two variations on the M-form.

In our analysis of these three upstream-downstream arrangements we assume that the upstream firms (or divisions) set linear prices for their inputs. A familiar result in the literature on vertical relations is that two-part pricing between independent firms can often capture the advantages of vertical integration. Why then do we focus on linear pricing? There are two reasons for doing so. First, in the context of our model there is no generally accepted method of determining the Nash equilibrium two-part prices since these are derived from an *N*-firm bargaining game, where *N* is the *total* number of upstream and downstream firms. Secondly, even if we impose what might be argued to be a "reasonable" solution to this bargaining game, there is no reason to believe that two-part pricing is more profitable for the upstream firms than linear pricing.

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[6] The pecuniary externalities associated with the decentralized production and trading of complementary goods originally identified by Cournot (1838) are often called indirect network externalities. Economides and Salop (1992) analyze how indirect network externalities affect competition and integration among the producers of complementary goods in a product network. We also consider multiple producers of differentiated complementary goods. However, in contrast to Economides and Salop (1992), the differentiated complementary goods are intermediate *input* goods demanded by downstream firms to produce a set of differentiated final products.

[7] Our comparison of the different organizational structures is comparative static, in that we identify sufficient but not necessary conditions under which particular organizational structures will emerge. We leave to subsequent analysis a full consideration of the endogenous determination of organizational structure.

All of the analyses of non-linear pricing have the characteristic that a *single* upstream monopolist supplies each downstream firm. Moreover, almost all of the analyses make the further assumption that the downstream firms have no bargaining power relative to the upstream firms: usually by assuming that each downstream firm is selected by its upstream supplier from a large group of potential firms who compete for the supply contract. The two-part price is then simply formulated. Set the unit price to maximize downstream profits and use the franchise fee to extract those profits. Matters are not as simple if the downstream firms have bargaining power. However, since each firm is involved in only one supply relationship this can be treated essentially as a separable set of two-person bargaining games, one for each upstream/downstream pair, with the solution presumably being the Nash bargaining solution for each pair.

Much more complex considerations arise in our model. To anticipate, we consider a situation in which each of four downstream firms relies upon two upstream suppliers, each upstream firm supplies two competing downstream firms and all firms have market power. So with two-part pricing we would have a three-person bargaining game for each downstream product *and* each upstream firm would be bargaining with two downstream firms. So far as we are aware, there is no general solution to this type of eight-person bargaining game.[8] Moreover, the solutions that have been proposed generally involve offers and counteroffers between *all of* the players, a concept that runs into obvious anti-trust problems in our specific context where some of the players - the downstream firms - are in direct competition with each other.

One potentially appealing approach, in that it at least avoids anti-trust complications, might be that we should treat the bargaining game for each downstream product separately. If this were done then the Nash bargaining solution *for each downstream product* would have the two upstream suppliers and their relevant downstream customer share the aggregate profits equally. As a result, each upstream firm would set its unit price to maximize aggregate profit of the two downstream firms it supplies. But then we can show[9] that two-part pricing is more profitable than linear pricing for the upstream firms only for "high" degrees of downstream product differentiation: levels of product differentiation at which the upstream firms would prefer close integration of the upstream

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[8] As Mas-Colell *et al*. (1995) point out, the axiomatic approach underlying the Nash bargaining solution "does not contemplate the possibility of situations intermediate between full cooperation of all agents and the complete breakdown represented by the threat-point outcome. Where there are more than two agents this is definitely restrictive." (1995, p. 846) See also Sutton (1985).

and downstream.[10] At lower degrees of product differentiation, for which it seems reasonable to suppose that some type of arms-length relationships would be preferred, linear pricing would also be preferred by the upstream firms.

The third and final vertical arrangement that we consider is a full consortium of the upstream suppliers for which these bargaining complications disappear. In the consortium the upstream suppliers act as if they are under common ownership and so are able to set a two-part tariff and coordinate the prices charged to each downstream buyer. The result is that the consortium is able to extend its monopoly power into the downstream market.

A central finding of our paper is that the relative profitability of the different vertical relationships between the upstream suppliers and the downstream producers depends critically upon the degree of product differentiation in the downstream sector. Upstream suppliers and downstream producers do not necessarily gain from having closer ties between them because the degree of downstream product differentiation affects the intensity of upstream competition. As a result, organizational structures that strengthen the ties between the upstream and downstream firms have conflicting effects. On the one hand, they weaken the profit-reducing complementarities to which we have referred but on the other, they make the upstream firms more directly subject to downstream competition. The relative strength of these two effects is determined by the degree of downstream product differentiation.

Another important finding is that under partial vertical integration, vertical foreclosure of the non-integrated downstream rivals is never an equilibrium strategy. This result stands in contrast with the analysis of Ordover, Saloner and Salop (1990). The difference is that in our model the exit of the foreclosed downstream firms would so intensify price competition in the downstream market that foreclosure is not a profitable strategy for the integrated firms to pursue.

In the next section we describe our stylized model of upstream and downstream production. The next four sections analyze the different ways in which competition is structured between the upstream and downstream markets. The relative effects of these market structures on prices and profits are detailed in Section 7. Section 8 presents our summary and conclusions.

## 2. THE MODEL

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[9] Details can be obtained from the authors on request.

We assume that the upstream market contains two upstream suppliers each supplying a differentiated or specialized input $j$ = 1, 2 and two upstream suppliers each supplying a differentiated input $k = 1', 2'$. The two inputs are compatible with each other and the marginal cost of each specialized input is assumed to be constant and equal, implying that these costs can be normalized to zero. The unit prices of the inputs are denoted $v_j$ and $v_k$, $j \in \{1, 2\}$, $k \in \{1', 2'\}$ and we denote the vector of unit input prices by $\mathbf{v} = (v_1, v_2, v_{1'}, v_{2'})$.

Each downstream firm produces at most one differentiated product using a (Leontief) constant returns to scale, fixed proportions production technology. Product differentiation arises because each downstream firm uses a different combination of the two upstream inputs $j$ and $k$. Specifically, there are four differentiated downstream goods of type $(j; k)$ = (1;1′), (1;2′), (2;1′) and (2;2′). They are produced in quantities $\mathbf{q} = (q_{(1;1')}, q_{(1;2')}, q_{(2;1')}, q_{(2;2')})$ and sold in the downstream market at prices $\mathbf{p} = (p_{(1;1')}, p_{(1;2')}, p_{(2;1')}, p_{(2;2')})$. We can refer interchangeably to downstream product $(j; k)$ and downstream firm $(j; k)$. Figure 1 illustrates this market structure.

(Figure 1 near here)

The constant-marginal-cost and fixed-proportions-technology assumptions characterize much of the literature investigating the relationship between the upstream and downstream sectors. Suppose now that demand for downstream product $(j; k)$ is given by $q_{(j;k)}(\mathbf{p})$. We can show (see the Appendix) that, provided $q_{(j;k)}(\mathbf{p})$ satisfies standard convexity properties and that $\nabla q_{(j;k)}(\mathbf{p}) < 0$, so that the downstream products are substitutes, any arrangement that reduces the input prices $\mathbf{v}$ also reduces downstream product prices $\mathbf{p}$ and so benefits consumers.

Our focus, however, is not just on the *price* effects of different upstream-downstream arrangements but also on their impacts on *profits.* As a result, in the interests of analytical tractability we need to sacrifice some generality by introducing additional structure to the model. Specifically, we follow Shubik (1980) and Deneckere and Davidson (1985) by assuming that the demand for differentiated good $(j; k)$ in the downstream market is linear and given by:[11]

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[10] Gal-Or (1992) obtains a similar result in a much simpler setting where the determination of the two-part tariff is not subject to the bargaining problems that apply in our analysis.

[11] It is worth noting that since our particular interest is in local neighborhoods where one upstream-downstream arrangement becomes more profitable than another, the linearity assumption can be viewed as a local linear approximation to a non-linear system.

(1) $$q_{(j;k)}(\mathbf{p}) = 1 - p_{(j;k)} - \frac{1}{\gamma}\left(p_{(j;k)} - \bar{p}\right) \qquad (j = 1,2;\ k = 1', 2')$$

where $\bar{p} = \left(p_{(1;1')} + p_{(1;2')} + p_{(2;1')} + p_{(2;2')}\right)/4$

The parameter γ measures the degree of product differentiation, or substitution, among the four products in the downstream market. As γ increases the products become more differentiated in consumption. When $\gamma \to \infty$ the products are completely differentiated. In this case, each firm in the downstream is a single-product monopolist. In contrast, when $\gamma \to 0$ the downstream products are perfect substitutes in consumption. A major advantage of our simplifying assumptions is that the relative merits of the different vertical relationships between the upstream and the downstream can be assessed purely in terms of the parameter γ.

Price competition occurs in two stages. The upstream suppliers set input prices **v** in the first stage. In the second stage, downstream firms, facing these input prices, produce differentiated products and compete in prices **p** for their consumers. We solve for the subgame perfect Nash equilibrium to this two-stage price game for each of the market structures.

## 3. THE BENCHMARK CASE OF DECENTRALIZED PRICING

The first case we consider is the benchmark case of decentralized price setting. In this case the problem facing a downstream firm (*j; k*) in the second stage subgame is to choose the profit-maximizing price $p^*_{(j;k)}$ for its product given the prices it faces for its upstream inputs. The solution to this problem can be written as:

(2) $$p^*_{(j;k)} = \arg\max\left\{q_{(j;k)}(\mathbf{p})\left(p_{(j;k)} - v_j - v_k\right)\right\} \qquad (j = 1,2;\ k = 1', 2')$$

This leads to a best response function in prices from which the set of equilibrium downstream prices in the stage two subgame can be found. The equilibrium prices $\mathbf{p}^*(\mathbf{v}) = \left\{p^*_{(j;k)}(\mathbf{v})\right\}$ are detailed in the Appendix.

Given the set of equilibrium prices **p*(v)** from the second stage subgame, an upstream firm *j* determines the derived demand function $x^*_j(\mathbf{v})$ that it faces in the first stage of the game. Similarly an upstream firm *k* derives its input demand function $x^*_k(\mathbf{v})$. Note that each upstream firm supplies two downstream firms with its specialized input. As a result, the derived demands facing each upstream supplier are:

$$x_j^*(\mathbf{v}) = q_{(j;1')}(\mathbf{p}^*(\mathbf{v})) + q_{(j;2')}(\mathbf{p}^*(\mathbf{v}))$$
(3) $$x_k^*(\mathbf{v}) = q_{(1;k)}(\mathbf{p}^*(\mathbf{v})) + q_{(2;k)}(\mathbf{p}^*(\mathbf{v}))$$ ($j$ = 1,2; $k$ = 1′, 2′)

In the first stage of the game the upstream firms independently choose their input prices to maximize profits defined as[12]:

(4) $$\Pi_u^d = v_u x_u^*(\mathbf{v})$$ $(u \in \{1, 2, 1', 2'\})$

The input price reaction functions (see Appendix) indicate that prices of complementary inputs are strategic substitutes while the prices of competing inputs are strategic complements.

Solving the first-order conditions from (4) gives the Nash equilibrium input prices, which in our symmetric example are identical across all four inputs and are given by:

(5) $$v_u^{*d} = \frac{2\gamma(8\gamma + 7)}{48\gamma^2 + 46\gamma + 3}$$ $(u \in \{1, 2, 1', 2'\})$

Substituting (5) into the reaction functions for output prices gives the downstream Nash equilibrium prices:

(6) $$p_{(j;k)}^{*d} = \frac{8\gamma(40\gamma^2 + 49\gamma + 12)}{(8\gamma + 3)(48\gamma^2 + 46\gamma + 3)}$$ ($j$ = 1,2; $k$ = 1′, 2′)

As we would expect, $\partial v_u^{*d} / \partial\gamma > 0$ and $\partial p_{(j;k)}^{*d} / \partial\gamma > 0$. The Nash equilibrium input and output prices are both increasing functions of the degree of product differentiation in the downstream market. As $\gamma \to 0$, consumers view the products in the downstream market as being less differentiated. Accordingly, decreases in $\gamma$ mean that downstream price competition becomes *tougher* with the result that the equilibrium input price $v_u^{*d} \to 0$, or goes to marginal cost.[13] By contrast, as $\gamma \to \infty$ consumers view the downstream products as being increasingly differentiated. Price competition in the downstream becomes *softer*. As a result the equilibrium input prices $v_u^{*d} \to 1/3$ and the equilibrium output prices $p_{(j;k)}^{*d} \to 5/6$.

The impact of increased product differentiation on upstream and downstream *profits* is less clear-cut as can be seen from Figure 2. On the one hand, softer price competition in the

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[12] The superscript ‘d’ refers to the decentralized case.

[13] It is easily shown that the elasticity of upstream and downstream prices with respect to $\gamma$ is unity when $\gamma = 0$ and tends to zero as $\gamma$ increases. Thus, even a small degree of product differentiation is sufficient to generate a sharp increase in equilibrium downstream prices.

downstream has a favorable effect upon both upstream and downstream profits. On the other hand, the coordination failures that result from noncooperative pricing also become greater the larger is $\gamma$ and adversely affect profitability in both the upstream and downstream. For upstream suppliers the profit-reducing effect of the coordination failures is more than offset by the profit-increasing effect of increased product differentiation for $\gamma < 0.219$. However, as $\gamma$ increases beyond this limit and the downstream becomes more differentiated, the coordination failure dominates, and the profits of the upstream firms fall. The threshold for the downstream sector is even lower. For $\gamma > 0.121$ the profits of the downstream firms fall as the parameter $\gamma$ increases.

The upstream firms are always more profitable than the downstream firms because of the monopoly power they enjoy. The difference in profits between the upstream and downstream is greatest when $\gamma$ is relatively small (equal to 0.112). In the limit as $\gamma \to \infty$ we have that $\Pi_u^{*d} \to 1/9; \Pi_D^{*d} \to 1/36$.

(Figure 2 near here)

## 4. AN UPSTREAM NETWORK OF SUPPLIER ALLIANCES

The coordination failures associated with the decentralized benchmark provide both upstream suppliers and downstream firms with an incentive to find other ways of transacting business. An alliance is one way that two upstream suppliers can coordinate upon the pricing of their complementary services. An upstream supplier *network of alliances* is a structure that facilitates the formation of multiple alliances among the complementary upstream suppliers.[14] When two suppliers create an alliance to work together on producing a specialized input package for a downstream client, they can, at the same time, work in alliance with other upstream partners to produce different specialized input packages for different downstream customers. We illustrate in Figure 3(a) the alliances to which upstream firm 1 belongs and in Figure 3(b) the alliances to which upstream firm 1′ belongs.

(Figure 3 near here)

It should be emphasized that the network of upstream alliances does not eliminate competition since each of the various upstream alliances supplies different downstream firms that are in competition with each other.

[14] Note that we consider only alliances between *complementary* upstream firms i.e. between a *j* firm and a *k* firm.

Consider how alliances among complementary upstream suppliers affect the perfect Nash equilibrium set of prices. In stage two of the game downstream firms, facing the input prices **v**, buy their inputs and compete in price for customers in the downstream market. Hence, the stage two subgame is the same as in the decentralized case [see equation (2)]. However the network of alliances affects stage one since the upstream suppliers are now able to coordinate the pricing of the specialized package of inputs (*j; k*) demanded by their downstream client. The aggregate profit of an alliance between *j* and *k* is:

$$\pi^a_{(j;k)} = (v_j + v_k)q_{(j;k)}(\mathbf{p}^*(\mathbf{v})) \qquad (j = 1,2;\ k = 1', 2') \tag{7}$$

We assume that the upstream suppliers share this profit equally—the Nash bargaining solution for this symmetric game between the upstream members of the alliance.

As we have noted, in belonging to a network of alliances, an upstream supplier *j* is in separate alliances with both of the upstream suppliers *k*. Hence, the maximization problem facing upstream supplier *j* is to choose an input price $v_j$ to maximize its overall return from being a member of its part of the network of alliances. That is, in stage one, the supplier *j* chooses its input price $v_j$ to maximize:

$$\Pi^n_j = \frac{1}{2}\sum_{k=1'}^{2'}(v_j + v_k)q_{(j;k)}(\mathbf{p}^*(\mathbf{v})). \qquad (j = 1, 2) \tag{8}$$

The maximization problems facing the upstream suppliers *k* are similarly defined. The joint solution to these maximization problems yields the input prices for stage one of the game.

As before, the reaction functions for the network alliance are such that the prices of complementary inputs are strategic substitutes while the prices of competing inputs are strategic complements. Simultaneously solving the reaction functions gives the supplier network Nash equilibrium input prices (see Appendix). These are identical across inputs and given by:

$$v^{*n}_u = \frac{\gamma(8\gamma + 7)}{32\gamma^2 + 32\gamma + 3} \qquad (u \in \{1, 2, 1', 2'\}) \tag{9}$$

Substituting in the reaction functions for output prices gives the Nash equilibrium output prices:

$$p^{*n}_{(j;k)} = \frac{2\gamma(96\gamma^2 + 116\gamma + 27)}{(8\gamma + 3)(32\gamma^2 + 32\gamma + 3)} \qquad (j = 1,2;\ k = 1', 2') \tag{10}$$

Comparison with (5) confirms that for all values of the parameter γ the upstream supplier network leads to lower equilibrium input prices and therefore lower equilibrium product prices than does the decentralized benchmark.[15] The network of upstream alliances at least partially removes the coordination failures between upstream suppliers. Note also that $\partial v_u^{*n} / \partial\gamma > 0$ and $\partial p_{(j;k)}^{*n} / \partial\gamma > 0$. The Nash equilibrium input and output prices are both increasing functions of the degree of product differentiation in the downstream market. As $\gamma \to 0$, $v_u^{*n}, p_{(j;k)}^{*n} \to 0$, just as in the decentralized case. By contrast, as $\gamma \to \infty$, the equilibrium input prices $v_u^{*n} \to 1/4$ and the equilibrium output prices $p_{(j;k)}^{*n} \to 3/4$.[16]

As downstream product differentiation increases, the profits of the upstream suppliers and the downstream producers are again affected by the trade-off between softer downstream price competition and coordination failures as can be seen in Figure 4. However, the critical value of γ at which the loss of profit from double marginalization is just offset by the profit increasing effect of softer price competition, is higher than in the decentralized case. The value is γ = 0.24 for the upstream suppliers and γ = 0.55 for the downstream producers.

(Figure 4 near here)

## 5. PARTIAL VERTICAL INTEGRATION

We now consider an alternative way to transact business that is based on vertical integration. Because not all of the downstream firms can integrate upstream (for example, if downstream firm (1;1′) integrates with its two upstream suppliers, then firm (2;1′) cannot do the same) we consider a *partial* framework in which some firms are vertically integrated and others are not. Specifically, we let downstream firms (1;1′) and (2;2′) integrate with their upstream suppliers to form the integrated firms [1;1′] and [2;2′] whereas the downstream firms (1;2′) and (2;1′) remain non-integrated. Figure 5 illustrates the partially vertically integrated structure.

(Figure 5 near here)

We initially assume that there is no strategic attempt at vertical foreclosure. That is, we assume that the upstream divisions of the integrated firms supply their services to the non-

[15] We show in the Appendix that this is also the case for more general demand specifications.
[16] Given our demand system, these are just the prices that would be charged by a monopoly producer of a composite upstream input and a downstream monopolist.

integrated downstream firms. Downstream firm (1;2′) buys input 1 from [1;1′] at price $v_1$, and input 2′ from [2;2′] at price $v_{2'}$. Downstream firm (2;1′) buys input 2 from [1;1′] at price $v_1$ and input 1′ from [1;1′] at price $v_{1'}$.[17] We address the possibility of foreclosure below.

Bonanno and Vickers (1988) show that when downstream firms compete in price for consumers a degree of vertical separation between the upstream and downstream softens price competition and can lead to higher profit. This suggests that marginal cost pricing of the upstream division's services may not necessarily maximize the integrated firm's profit. Accordingly, we consider three different internal organizations of the vertically integrated firms:

(i) Centralized integration in which the upstream division has no pricing autonomy and supplies its services to the downstream division at marginal cost.

(ii) Divisional integration in which the upstream unit is established as a separate division and profit center and sets the prices of its services to maximize its divisional profits.

(iii) Divisional integration in which the upstream unit is established as a separate division and profit center but prices are set to maximize the integrated firm's profits.

For each of the three internal organizations of the integrated firm, we again proceed with a two-stage game. Now, however, the first stage of the game is one in which there are six input prices. First there are the prices $v_j$ and $v_k$ at which the upstream units sells their specialized input goods $j$ and $k$ to non-integrated downstream firms. Then there are the input prices $v_{(i;i')}$ ($i$ = 1, 2) for the combined input packages that are sold internally to the downstream divisions. In the second stage, both the integrated and non-integrated firms choose their downstream product prices $p^*_{(j;k)}$.

Unlike our two previous cases, the partially vertically integrated market case introduces an important asymmetry in the equilibrium prices for products sold in the downstream market.

We begin by looking at the pricing problem facing the integrated and non-integrated downstream firms in stage two of the game. In this stage, the downstream firms choose prices to satisfy the conditions:

(11a) $$p^*_{(i;i')} = \arg\max\left\{q_{(i;i')}(\mathbf{p})\left(p_{(i;i')} - v_{(i;i')}\right)\right\} \qquad (i = 1,2)$$

[17] Upstream alliances and nonlinear pricing can replicate the Nash equilibrium for the partial vertical integration case. This will happen if the upstream supplier 1 forms an alliance solely with upstream supplier 1′ and upstream supplier 2 forms an alliance solely with upstream supplier 2′; and if each of these alliances apply two-part pricing to their captive downstream customer, firm (1;1′) or (2;2′), while supplying its other downstream buyer at a linear price. The *distribution* of profits may differ, however.

(11b) $\quad p^*_{(i;h')} = \arg\max\left\{q_{(i;h')}(\mathbf{p})\left(p_{(i;h')} - v_i - v_{h'}\right)\right\}$ $\qquad$ ($i$, $h$ = 1,2; $i \neq h$)

Each firm's solution to its maximization problem yields a best response function in prices from which the equilibrium downstream prices $\mathbf{p}^*(\mathbf{v}) = \left\{p^*_{(j;k)}(\mathbf{v})\right\}$ can be found, where now $\mathbf{v} = (v_{(1;1')}, v_{(2;2')}, v_1, v_2, v_{1'}, v_{2'})$. These equilibrium prices are derived in the Appendix.

We now turn to characterizing the complete, stage one and stage two Nash equilibria for each of the three different ways vertically integrated firms could be organized. Thereafter, we compare the prices and profits that these equilibria generate for all the firms.[18]

### *5.1 Centralized Integration*

With this organization of an integrated firm we have $v_{(i;i')} = 0$ ($i$ = 1, 2), so that in the first stage of the game the integrated firm [$i; i'$] chooses input prices $v_i$ and $v_{i'}$ to maximize profit:

$$\Pi^{Ic}_{(i;i')} = p^*_{(i;i')}(\mathbf{v})\, q_{(i;i')}(\mathbf{p}^*(\mathbf{v})) + v_i\, q_{(i;h')}(\mathbf{p}^*(\mathbf{v})) + v_{i'}\, q_{h;i'}(\mathbf{p}^*(\mathbf{v})) \qquad (i, h = 1,2;\ i \neq h).$$

Solving the resulting reaction functions gives the equilibrium prices for the traded inputs under centralized integration:

$$v_j^{*cp} = v_k^{*cp} = \frac{2\gamma(7+8\gamma)\left(27+88\gamma+64\gamma^2\right)}{153+1758\gamma+5840\gamma^2+7296\gamma^3+3072\gamma^4} \qquad (j = 1,2;\ k = 1', 2') \tag{12}$$

Substituting (12) into the reaction functions for output prices yields the following Nash equilibrium prices which hold for $\gamma > 0.0297$:

$$p^{*cp}_{(i;i')} = \frac{4\gamma(7+8\gamma)\left(15+62\gamma+48\gamma^2\right)}{153+1758\gamma+5840\gamma^2+7296\gamma^3+3072\gamma^4} \qquad (i = 1,2) \tag{13a}$$

$$p^{*cp}_{(i;h')} = \frac{8\gamma\left(93+463\gamma+688\gamma^2+320\gamma^3\right)}{153+1758\gamma+5840\gamma^2+7296\gamma^3+3072\gamma^4} \qquad (i, h = 1,2;\ i \neq h) \tag{13b}$$

For $\gamma \leq 0.0297$ downstream price competition is very tough and the prices set for the inputs sold externally to the non-integrated downstream firms are such that the non-integrated firms cannot sell positive quantities at the Nash equilibrium downstream prices. They exit from the market. We refer to this as non-predatory vertical foreclosure since it arises as a natural consequence of Nash equilibrium pricing on the part of the upstream units of the integrated firms

[18] The intriguing but complicated question remains as to which organizational form is most likely to emerge. We hope to pursue this question in subsequent analysis.

rather than from a deliberate attempt by these firms to foreclose by refusing to supply the independent downstream firms.

Eliminating firms (1;2′) and (2;1′), deriving and solving the reaction functions for firms [1;1′] and [2;2′] gives the Nash equilibrium output prices for the two integrated firms:

(13c) $$p^{*cp}_{(i;i')} = \frac{2\gamma}{1+4\gamma} \qquad \text{for } \gamma \leq 0.0297$$

Comparison of (13a) and (13c) indicates that the exit of the high-cost downstream firms further toughens price competition in the downstream market, reducing prices and profitability of the two remaining integrated firms.

It is easy to show that $\partial v^{*cp}_{\bullet} / \partial\gamma > 0$ and $\partial p^{*cp}_{\bullet} / \partial\gamma > 0$ for both the upstream divisions and their downstream clients. Once again, the Nash equilibrium input and product prices are increasing functions of the degree of product differentiation in the downstream market. Because the integrated firms acquire their specialized input package at marginal cost equal to zero, whereas the non-integrated firms do not, there is, of course, an asymmetry in downstream prices with $p^{*cp}_{(i;i')} < p^{*cp}_{(i;h')}$. As $\gamma \to \infty$, the equilibrium input price $v^{*cp}_j, v^{*cp}_k \to 1/3$ as in the decentralized case, while the equilibrium output prices $p^{*cp}_{(i;i')} \to 1/2$; $p^{*cp}_{(i;h')} \to 5/6$.

### 5.2 *An Upstream Profit Maximizing Division*

Under this organizational form the integrated firm establishes the upstream unit as a divisional profit center, charged with the goal of maximizing its own profit. This implies that in the first stage of the game the input prices $v_{(i;i')}$, $v_j$ and $v_k$ are set to maximize profit:

$$\Pi^{Id}_{(i;i')} = v_{(i;i')} q_{(i;i')}(\mathbf{p}*(\mathbf{v})) + v_i q_{(i;h')}(\mathbf{p}*(\mathbf{v})) + v_{i'} q_{(h;i')}(\mathbf{p}*(\mathbf{v})) \qquad (i, h = 1,2;\ i \neq h).$$

Solving the resulting reaction functions gives the Nash equilibrium upstream prices:

(14) $$v^{*dp}_{(i;i')} = \frac{4\gamma(7+8\gamma)}{(3+60\gamma+64\gamma^2)};\ v^{*dp}_j = v^{*dp}_k = \frac{8\gamma(7+8\gamma)}{3(3+60\gamma+64\gamma^2)} \qquad (i = 1, 2;\ j = 1,2;\ k = 1', 2'))$$

As can be seen, $v^{*dp}_{(i;i')} < v^{*dp}_j + v^{*dp}_k$, indicating that the upstream division of each integrated firm discriminates against its non-integrated downstream buyers. Moreover, the upstream division will never adopt marginal cost pricing internally when $\gamma > 0$, and as $\gamma \to \infty$, the price of the internal package of services $v^{*dp}_{(i;i')} \to 1/2$ and the price of a service traded externally $v^{*dp}_j, v^{*dp}_k \to 1/3$. These make sense. When γ is small, competition in the downstream market is fierce, constraining the price-setting power of an upstream division. As γ rises, the upstream profit center behaves increasingly like an upstream monopolist with respect to its captive downstream customers.

Substituting (14) into the downstream price reaction functions gives the Nash equilibrium downstream prices:

(15a) $$p^{*dp}_{(i;i')} = \frac{8\gamma\left(39+172\gamma+144\gamma^2\right)}{3(3+8\gamma)\left(3+60\gamma+64\gamma^2\right)} \qquad (i = 1, 2)$$

(15b) $$p^{*dp}_{(i;h')} = \frac{4\gamma\left(87+380\gamma+320\gamma^2\right)}{3(3+8\gamma)\left(3+60\gamma+64\gamma^2\right)} \qquad (i, h = 1, 2;\; i \neq h)$$

In this case, the downstream prices are such that the non-integrated firms sell a positive amount no matter the degree of product differentiation. Note too that, as we would expect, $p^{*dp}_{(i;i')} < p^{*dp}_{(i;h')}$. The input cost advantage enjoyed by the downstream units translates into lower downstream prices. As $\gamma \to \infty$ we have $p^{*dp}_{(i;i')} \to 3/4;\; p^{*dp}_{(i;h')} \to 5/6$.

*5.3 AN UPSTREAM PROFIT CENTER MAXIMIZING GROUP PROFIT*

For this internal organization, we assume that, although the upstream division of an integrated firm is established as a profit center, it is allocated a profit share determined by the profitability of the integrated firm [*i*;*i′*]. This means that the input prices $v_{(i;i')}$, $v_j$ and $v_k$ are set to maximize the integrated firm's profit:

$$\Pi^{Ig}_{(i;i')} = p^*_{(i;i')}(\mathbf{v})q_{(i;i')}(\mathbf{p}^*(\mathbf{v})) + v_i q_{(i;h')}(\mathbf{p}^*(\mathbf{v})) + v_{i'} q_{(i;i')}(\mathbf{p}^*(\mathbf{v})) \qquad (i, h = 1,2;\; i \neq h).$$

We again derive the Nash equilibrium prices from simultaneous solution of the implied reaction functions. These are:

(16) $$v^{*gp}_{(i;i')} = \frac{4\gamma(7+8\gamma)(21+16\gamma)}{(3+4\gamma)\left(51+896\gamma+2304\gamma^2+1536\gamma^3\right)};\quad v^{*gp}_j = v^{*gp}_k = \frac{32\gamma(7+8\gamma)(1+2\gamma)}{\left(51+896\gamma+2304\gamma^2+1536\gamma^3\right)}$$

This form of divisionalization also leads to price discrimination against the non-integrated downstream buyers. However, in contrast with the previous cases, we find that now there is a non-monotonic relationship between the internal input price and the degree of product differentiation. The internal input price initially increases with γ, but then as $\gamma \to \infty$, $v^{*gp}_{(i;i')} \to 0$. In other words, when the upstream unit sets prices to maximize the integrated firm's profit rather than its own divisional profit, a high degree of product differentiation in the downstream market will lead it to set its internal price "close" to marginal cost. It should be emphasized, however, that it is *only* when the downstream firms are effectively monopolists that this form of vertical integration leads to marginal cost pricing of the internally supplied input services. When there is competition in the downstream market, it is desirable for the integrated firm to have some degree of vertical separation between its upstream and downstream divisions, even though this separation introduces double marginalization between them.[19]

Substituting (16) into the downstream reaction functions gives the equilibrium prices:

(17a) $$p^{*gp}_{(i;i')} = \frac{48\gamma(7+8\gamma)(1+2\gamma)}{(51+896\gamma+2304\gamma^2+1536\gamma^3)} \qquad (i = 1, 2)$$

(17b) $$p^{*gp}_{(i;h')} = \frac{4\gamma(111+408\gamma+320\gamma^2)}{(51+896\gamma+2304\gamma^2+1536\gamma^3)} \qquad (i, h = 1, 2;\ i \neq h)$$

As in the centralized form of integration, there is a degree of product differentiation, in this case $\gamma = 0.0175$, below which the non-integrated downstream firms choose to exit the market. In this range of $\gamma$ the Nash equilibrium price $v_{(i;i')}$ for the input is:

(18) $$v^{*gp}_{(i;i')} = \frac{2\gamma}{(1+2\gamma)(1+12\gamma+16\gamma^2)} \qquad \text{for } \gamma \leq 0.0175 \qquad (i = 1, 2)$$

The Nash equilibrium output prices for the two integrated firms are now:

(19) $$p^{*gp}_{(i;i')} = \frac{4\gamma(1+2\gamma)}{(1+12\gamma+16\gamma^2)} \qquad \text{for } \gamma \leq 0.0175 \qquad (i = 1, 2)$$

### *5.4 Relative Prices and profitability with Partial Vertical Integration*

Input and product prices under the three different organizational forms of partial vertical integration are illustrated in Figure 6 and point to a simple relationship. Centralized integration gives the lowest prices, whereas the divisional form in which the upstream unit is motivated only by its own profit gives the highest prices. The intuition behind this outcome is straightforward. Supplying the specialized input package internally at marginal cost toughens price competition in

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[19]A model of two-part pricing between non-integrated firms gives the same result that the unit price of the input should be set to marginal cost only if the downstream market is monopolized. In any other situation (other than $\gamma$ = 0) the unit price should exceed marginal cost.

the downstream market, and so constrains the prices of upstream services that can be charged to the non-integrated downstream firms. By contrast, when an upstream division is concerned purely with its own profits, the division has the incentive to maintain a higher internal price for its services, which in turn allows it to charge higher external prices for its services. The monopoly power of the autonomous upstream division is, of course, still constrained by competition in the downstream market.

(Figure 6 near here)

Not surprisingly, the profit of a non-integrated downstream firm increases with the degree of product differentiation in the downstream market (see Figure 7(a)). The internal organization of the integrated firm also affects a non-integrated firm's profitability. Centralized integration gives the lowest profit to the non-integrated downstream firms and the divisional form in which the upstream unit is motivated only by its own profit gives the non-integrated firms the highest profit. This ranking reflects a trade-off between two forces. On the one hand, an internal organizational form that leads to lower external input prices benefits the non-integrated firms by reducing their costs. On the other hand, the corresponding lower *internal* input prices adversely affect the non-integrated firms by making them less price competitive with their integrated competitors. What is quite clear from Figure 7(a) is that the detrimental price competitive effect more than offsets the beneficial cost effect.

(Figure 7 near here)

No such simple relationship characterizes the profitability of an integrated firm, as can be seen from Figure 7(b). As $\gamma$ increases, the integrated firm's profit is affected by the trade-off between softer downstream price competition and the coordination failures that remain with partial vertical integration. Of much more interest, however, is the result that the *relative* profitability of the different ways of internally organizing the integrated firm also varies with the degree of product differentiation. With the exception of the polar cases where $\gamma = 0$ or $\gamma = \infty$, centralized integration, or the traditional U-form, is always less profitable than divisional integration in which the upstream division's concern is with group profits. It is true that divisional integration reintroduces the efficiency losses associated with double marginalization. However, setting a higher internal input price softens price competition among the integrated firms and therefore increases the external input price that can be charged to non-integrated firms. The gain from softening downstream competition more than offsets the efficiency loss of double marginalization.

Yet divisional integration in which the upstream unit is concerned with group profits is *not* always more profitable than divisional integration in which the upstream unit is a profit center concerned solely with its own profit. Specifically, if the degree of product differentiation in the downstream market is less than $\gamma = 0.459$, the integrated firm earns greater aggregate profits if the upstream division sets prices to maximize the upstream division's own profit. In short, a divisional structure is preferable to a centralized one, but the form of divisionalization "matters".

The intuition behind this initially surprising result lies in issues that we have already discussed. If an integrated firm establishes its upstream division as a divisional profit center it reintroduces all of the efficiency losses associated with double marginalization. However, such a structure insulates the upstream unit from downstream price competition much more effectively than if the upstream division aims to maximize group profits. When the degree of downstream product differentiation is low it is more profitable to adopt the structure that softens downstream price competition than the structure that increases pricing efficiency.

*5.5 VERTICAL FORECLOSURE*

We have already noted that when the degree of downstream differentiation is low, it is possible that the non-integrated rivals are not able to survive at the Nash equilibrium prices at which integrated firms sell their upstream services. This occurs when $\gamma < 0.029$ in the case of the centralized U form of integration, and when $\gamma < 0.0175$ in the case of organizing the upstream unit as a profit center maximizing the integrated firm's profit. Exit of the non-integrated firms in such cases is the outcome of equilibrium behavior, and cannot be considered to be predatory.

We now consider whether it is in the interests of an integrated firm *deliberately* to foreclose on one or both of the nonintegrated downstream firms. This type of foreclosure is a predatory act that is taken by an integrated firm to drive a non-integrated downstream firm from the market. For example, since there is no other supplier of upstream product 1, the refusal by firm [1;1′] to supply downstream firm (1;2′) is a predatory act because it forces firm (1;2′) to exit the market.

Consideration of all possible downstream configurations that such foreclosure might generate yields the pay-off matrix of Table 1.[20]

Each of the strategy combinations on the main diagonal of Table 1 constitutes a Nash equilibrium. Note, however, that "no foreclosure" is a weakly dominant strategy for both partially vertically integrated firms. These firms never have an incentive to cut off downstream rivals from the necessary inputs because such a predatory action intensifies competition between the integrated firms. Rather, the integrated firms have an incentive to keep the non-integrated, high-cost downstream firms in operation in order to soften downstream price competition.

(Table 1 near here)

## 6. AN UPSTREAM CONSORTIUM

The final vertical arrangement that we consider is one in which the upstream suppliers are able fully to coordinate their prices by forming an upstream consortium. For example, professional organizations in the health sector could form such a consortium. The advantage of the consortium is that it eliminates the inefficiencies that characterize the other cases. Now the upstream suppliers are able to implement a two-part tariff of the form $T_{(j;k)} + v_{(j;k)}q_{(j;k)}$, where $v_{(j;k)}$ is the unit price charged for the combined input package (*j; k*) used by downstream firm (*j; k*).

Given the symmetry of our model, we know that the upstream consortium will charge the same unit price *v* for the combined input package to each of the downstream firms. In the second stage price game the downstream firms choose the profit-maximizing prices $p^*_{(j;k)}$ for their products sold in the downstream market. These prices satisfy:

(20) $$p^*_{(j;k)} = \arg\max\left\{q_{(j;k)}(\mathbf{p})\left(p_{(j;k)} - v\right)\right\} \qquad (j = 1,2;\ k = 1', 2')$$

Again, the equilibrium prices $\mathbf{p}^*(v) = \left\{p^*_{(j;k)}(v)\right\}$ are detailed in the Appendix. In the first stage, the upstream consortium sets *v* to maximize aggregate downstream profits because this permits setting the highest fixed charge $T_{(j;k)}$. The resultant Nash equilibrium input price for the combined input package is:

(21) $$v^* = \frac{3}{2(3+4\gamma)}.$$

[20] We present specific results only for centralized partial integration. A sketch of the calculations is presented in the Appendix. The same conclusions apply to either of the divisional forms of partial integration. Further details are

Substituting (21) into the downstream price reaction functions gives the Nash equilibrium product prices:

(22) $p^{*uc}_{(j;k)} = 1/2$ . $(j = 1,2;\ k = 1', 2')$

As we might expect, the input price falls from the price $v^* = 1/2$ when $\gamma = 0$, in which case the downstream market is effectively perfectly competitive, to $v^* = 0$ when $\gamma = \infty$, in which case each downstream firm is effectively a monopolist. It turns out, however, that these price changes are exactly offsetting. That is, the equilibrium prices (21) and (22) generate an aggregate profit of $\Pi^a = 1$ *no matter the degree of product differentiation in the downstream market*.[21] The ability of the upstream consortium to apply a two-part tariff insulates them from the effects of product differentiation and allows the consortium to extend its monopoly power into the downstream market.

## 7. COMPARISON OF UPSTREAM AND DOWNSTREAM MARKET STRUCTURES

There are important but far from obvious differences in prices and profitability among the three different methods of transacting business between the upstream suppliers and their downstream customers. We summarize the main elements of this comparison in Table 2 and illustrate the profit comparisons in Figure 8.

Consider first the impact of different vertical relationships on prices. Compared to the decentralized benchmark, coordination of pricing decisions either among the upstream suppliers or between upstream suppliers and downstream producers generally benefits consumers via lower downstream prices. However, whether downstream prices are lower under the upstream network or under partial vertical integration depends as well upon how the integrated firms are internally organized. When the upstream divisions are established as divisional profit centers, the input prices they charge lead to product prices in the downstream market that are higher than the prices set by downstream firms when they buy from an upstream network of suppliers. Under other forms of vertical integration, the lower input costs of the integrated firms intensify downstream competition and yield downstream prices that are, on average, lower than under an upstream network of supplier alliances. Finally, only when the degree of product differentiation downstream is "large

available from the authors on request.

[21] If the upstream consortium did not apply two-part pricing it would set an upstream price for the combined input of $v^* = 1/2$.

enough" is the upstream consortium better for consumers than decentralized markets, the network of alliances, or the divisional profit center. When the products marketed downstream are not too differentiated, the upstream consortium's ability to exploit its monopoly power leads to higher downstream prices.

Comparisons of profits under different vertical relationships shed some light on the varied relationships that we might actually observe in different sectors. Consider first the profitability of the downstream firms. Because their profits are always greater with an upstream network of supplier alliances than with decentralization, we should expect downstream firms to support any effort to create or to facilitate the formation of such a network of alliances so long as it falls short of a full consortium. Not all of the downstream firms fare equally well under partial vertical integration. Because a non-integrated downstream firm is adversely affected by having to compete with a low-cost integrated rival, a non-integrated firm has incentive to block the backward integration of its rivals.

Turning to the profits of the upstream suppliers, clearly the upstream consortium, which confers upon them the greatest monopoly power, is the most profitable. It may however be the most difficult to implement. If it cannot be established, then we are more likely to observe upstream firms seeking to integrate vertically, particularly when such integration leads to the formation of divisional profit centers since this internal organization generates the greatest upstream profits. Somewhat more surprisingly, upstream suppliers do not always do well in a network of alliances. Specifically, if the degree of downstream differentiation is relatively low, or if $\gamma < 0.55$, then the upstream suppliers' profits are higher in the decentralized benchmark case. The upstream suppliers compete indirectly through the price competition of their clients in the downstream market. What the network of alliances does is to increase the degree to which the upstream and downstream markets are connected. This benefits the upstream suppliers at reasonably high levels of downstream product differentiation by reducing the profit dissipation effects of coordination failures. But it *harms* the upstream firms at low levels of downstream product differentiation by increasing the extent to which the upstream firms are directly subject to competitive forces downstream in setting their prices.

In order to compare the profits of the integrated firms with the corresponding profits under either the decentralized case or the network of upstream alliances, we compare $\Pi^{*p}_{(i;i')}$ with

$\Pi_D^{*d} + 2\Pi_u^{*d}$ and $\Pi_D^{*n} + 2\Pi_u^{*n}$, that is with the combined profits of one downstream and two upstream firms in either case. Given our discussion of the relative profitability of different forms of vertical integration, it should not be surprising to find that here again divisional form "matters".

With either centralized integration or the group profit center, there is a trade off between the profit-increasing effect of removing, at least partially, pecuniary externalities and double marginalization, and the profit-reducing effect of making the upstream suppliers more directly subject to competitive pressures in the downstream market. When $\gamma$ is "low" and downstream price competition is toughest the decentralized benchmark gives the greatest overall profitability. For intermediate values of $\gamma$ the greatest overall profit is obtained under a network of upstream alliances. Only for $\gamma$ "high" is it the case that the two internal forms of integration offer greater overall profitability than the network of upstream alliances.

A different picture emerges however when vertical integration establishes upstream divisional profit centers. In this case, partial vertical integration gives the greatest profit for "low" values of $\gamma$, whereas the network of upstream alliances does better for "high" values. The decentralized benchmark is now never the most profitable arrangement. The reasoning behind this ranking has already been discussed. This form of vertical integration reintroduces the efficiency losses of complementary pricing but also gives the upstream divisions the ability to soften downstream competition. The competitive effect dominates the efficiency loss so long as the downstream market is not too highly differentiated.

(Table 2 and Figure 8 near here)

## 8. SUMMARY AND CONCLUSIONS

It is well known that market transactions between upstream suppliers of specialized or differentiated services and their downstream customers are affected by two types of market failure: lack of price coordination between upstream suppliers of complementary inputs and double marginalization. These market failures are greatest in a fully decentralized market setting as a result of which a move to coordinate pricing between upstream and downstream markets generally benefits consumers through lower downstream prices.

What has emerged from our analysis, however, is that improved upstream/downstream co-ordination will not necessarily result in increased profits for either the downstream firms or the upstream suppliers. On the one hand, improved coordination at least partially corrects market failure but on the other it exposes the upstream firms much more directly to downstream competition. To illustrate, we have shown that the profits of downstream firms increase when their complementary upstream suppliers form a network of alliances, but such alliances increase the profits of the upstream suppliers only if the degree of product differentiation downstream is sufficiently large.

Similar forces are at work in understanding the profit impact of partial vertical integration. Moreover, we have seen that whether integrated firms are more profitable depends not only upon the degree of product differentiation in the downstream market, but also upon the organizational form that integration takes. When the integrated firm is organized with the goal of maximizing the firm's aggregate profit, partial vertical integration is more profitable provided that there is a "sufficient" degree of downstream product differentiation. By contrast, if the upstream division of an integrated firm is established as a divisional profit center, partial vertical integration is more profitable provided that the degree of product differentiation is "low". And whatever the form of integration, the integrated firms do not find it in their interest to force the exit of their non-integrated rivals.

This raises a number of important strategic issues to which we hope to return in subsequent analysis. It is tempting to believe that partial vertical integration, or its non-linear pricing interpretation, will always be attractive *provided that* the integrating firms can be flexible with respect to the internal structure they adopt. What is not clear is first, whether competing integrated firms will adopt the same internal structure and second, whether firms will be driven to integrate even though this could lead to profit reducing downstream competition.

The final important lesson that we draw from our analysis is that the way in which upstream firms supply their specialized inputs to downstream firms will be characterized by a considerable degree of diversity across different industries. This occurs because the relative profitability of different relationships between the upstream and downstream sectors depends in a non-trivial and not necessarily obvious way upon the nature and strength of competition in the downstream market. The recent attention in the business press on the project-based approach to supply relationships is suggestive of a trend toward flexible specialization upstream and a move away from complete vertical integration. This paper suggests why this trend might be expected to strengthen in some sectors while being resisted in others.

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## MATHEMATICAL APPENDIX.

### 1. A Generalization

Assume the basic model presented in the body of the paper with the generalization that that demand in the downstream market for product $(j;\ k)$ is $q_{(j;k)}(\mathbf{p})$, where $\mathbf{p} = (p_{(1;1')}, p_{(1;2')}, p_{(2;1')}, p_{(2;2')})$. Define $\nabla q_{(j;k)}(\mathbf{p}) = \left[\partial q_{(j,k)} / \partial p_{(1,1')}, \partial q_{(j,k)} / \partial p_{(1,2')}, \partial q_{(j,k)} / \partial p_{(2,1')}, \partial q_{(j,k)} / \partial p_{(2,2')}\right]$ and assume that $\nabla q_{(j;k)}(\mathbf{p}) < 0$. Then profit to downstream firm $(j;\ k)$ is:

(A.1) $\pi_{(j;k)} = \left(p_{(j;k)} - v_j - v_k\right) q_{(j;k)}(\mathbf{p})$.

The price reaction function for firm $(j;k)$ is:

(A.2) $R_{(j;k)}:\ \dfrac{\partial \pi_{(j;k)}}{\partial p_{(j;k)}} = \left(p_{(j;k)} - v_j - v_k\right) \dfrac{\partial q_{(j;k)}}{\partial p_{(j;k)}} + q_{(j;k)}(\mathbf{p}) = 0$ $\qquad (j = 1,2;\ k = 1', 2')$

The comparative statics of the solution to these reaction functions are then:

(i) $$\frac{\partial p_{(j;k)}}{\partial p_{(m;n)}} = -\frac{\left(p_{(j;k)} - v_j - v_k\right) \partial^2 q_{(j;k)} / \partial p_{(j;k)} \partial p_{(m;n)} + \partial q_{(j;k)} / \partial p_{(m;n)}}{\partial^2 \pi_{(j;k)} / \partial p_{(j;k)}^2} \qquad (j,m = 1,2;\ k,n = 1', 2')$$

The denominator is negative so long as second order conditions are satisfied. The numerator is also negative provided that demand for product $(j;\ k)$ is a decreasing function of its own and the other product prices. We can, therefore, expect the price reaction functions to be upward sloping.

(ii) $$\frac{\partial p_{(j;k)}}{\partial v_u} = \frac{\partial D_{(j;k)} / \partial p_{(j;k)}}{\partial^2 \pi_{(j;k)} / \partial p_{(j;k)}^2} > 0 \qquad (u \in \{1, 2, 1', 2'\})$$

Any arrangement upstream that reduces the prices charged to the downstream firms will result in lower downstream prices.

Solving the downstream reaction functions gives the downstream prices $\mathbf{p}(\mathbf{v})$ and gives upstream demand for inputs $j$ and $k$ as a function of the upstream prices. We can then investigate the maximizing problem for an upstream firm in the first-stage price game. The profit function for an upstream firm $j$, for example, is:

(A.3) $\Pi_j = v_j \sum_{k=1'}^{2'} q_{(j;k)}\left(\mathbf{p}(\mathbf{v})\right)$ $\qquad (j = 1, 2)$

with similar profit equations for a firm $k$. We can then compare the decentralized case with the upstream alliance. To do so, first define $D_u \mathbf{p}(\mathbf{v}) = \left[\partial p_{(1;1')} / \partial v_u, \partial p_{(1;2')} / \partial v_u, \partial p_{(2;1')} / \partial v_u, \partial p_{(2;2')} / \partial v_u\right]$ for $u \in \{1, 2, 1', 2'\}$.

### 1.1 The Decentralized Case

The upstream price reaction function for upstream firm *j* is:

$$\text{(A.4)}\quad R_j : \frac{\partial \Pi_j^d}{\partial v_j} = \sum_{k=1'}^{2'} q_{(j;k)}(\mathbf{p}(\mathbf{v})) + v_j \sum_{k=1'}^{2'} \nabla q_{(j;k)}(\mathbf{p}(\mathbf{v})).D_j \mathbf{p}(\mathbf{v}) = 0. \qquad (j = 1, 2)$$

and similarly for the upstream firms *k*.

1.1 **The Network Case**

Now the profit function for an upstream firm *j* is:

$$\text{(A.5)}\quad \Pi_j^n = \frac{1}{2}\left( \sum_{k=1'}^{2'} (v_j + v_k) q_{(j;k)}(\mathbf{p}(\mathbf{v})) \right) \qquad (j = 1, 2)$$

This gives the upstream price reaction function:

$$\text{(A.6)}\quad R_j : \frac{\partial \Pi_j^n}{\partial v_j} = \sum_{k=1'}^{2'} q_{(j;k)}(\mathbf{p}(\mathbf{v})) + \sum_{k=1'}^{2'} (v_j + v_k) \nabla q_{(j;k)}(\mathbf{p}(\mathbf{v})).D_j \mathbf{p}(\mathbf{v}) = 0 \qquad (j = 1, 2)$$

Similar profit and reaction functions hold for upstream firms *k*. Comparison with the decentralized case confirms that (A.6) is negative at the solution for the decentralized case. In other words, the network alliance always leads to lower input prices than the decentralized case and so to lower final product prices.

No similar general comparison is possible for partial vertical integration since such an arrangement is likely to have asymmetric effects on integrated and non-integrated downstream firms.

## 2. The Linear Demand Case

### 2.1 Downstream prices for decentralized markets or an upstream network alliance:

The downstream prices are the solutions to the following problem:[22]

$$\text{(A.7)}\quad p^*_{(j;k)} = \arg\max\left\{ q_{(j;k)}(\mathbf{p})\left(p_{(j;k)} - v_j - v_k\right)\right\} \qquad (j = 1, 2,\ k = 1', 2¢)$$

where: $q_{(j;k)}(\mathbf{p}) = 1 - p_{(j;k)} - \left(p_{(j;k)} - \overline{p}\right)/\gamma$.

The solution is:

$$\text{(A.8a)}\quad p^*_{(i;i')} = \frac{28\gamma + 32\gamma^2 + \left(15 + 44\gamma + 32\gamma^2\right)(v_i + v_{i'}) + (6 + 8\gamma)(v_h + v_{h'})}{\left(21 + 80\gamma + 64\gamma^2\right)} \qquad (i, h = 1,2;\ i \text{ ¹ } h)$$

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[22] These and all other calculations were performed using *Mathematica*. Details are available from the authors on request.

(A.8b) $$p^*_{(i;h')} = \frac{28\gamma + 32\gamma^2 + \left(15 + 44\gamma + 32\gamma^2\right)\left(v_i + v_{h'}\right) + \left(6 + 8\gamma\right)\left(v_h + v_{i'}\right)}{\left(21 + 80\gamma + 64\gamma^2\right)} \quad (i,\ h = 1,2;\ i \neq h)$$

2.2 **Upstream prices for the decentralized case:**

The reaction functions corresponding to the stage one game played in input prices are then:

(A.9a) $$v_i^{*d} = \frac{(4\gamma + 3)v_h - \gamma(8\gamma + 7)(v_{1'} + v_{2'} - 2)}{2\left(16\gamma^2 + 18\gamma + 3\right)} \quad (i,\ h = 1,2;\ i \neq h)$$

(A.9b) $$v_{i'}^{*d} = \frac{(4\gamma + 3)v_{h'} - \gamma(8\gamma + 7)(v_1 + v_2 - 2)}{2\left(16\gamma^2 + 18\gamma + 3\right)} \quad (i,\ h = 1,2;\ i \neq h)$$

It follows immediately that $\partial v_i^{*d} / \partial v_h > 0$ and $\partial v_j^{*d} / \partial v_k < 0$.

2.3 **Upstream prices for the network alliance**

The reaction functions corresponding to the stage one game for the network alliance are:

(A.10a) $$v_i^{*n} = \frac{2(4\gamma + 3)v_h - \left(32\gamma^2 + 32\gamma + 3\right)\left(v_{1'} + v_{2'}\right) + 4\gamma(8\gamma + 7)}{4\left(16\gamma^2 + 18\gamma + 3\right)} \quad (i,\ h = 1,2;\ i \neq h)$$

(A.10b) $$v_{i'}^{*n} = \frac{2(4\gamma + 3)v_{h'} - \left(32\gamma^2 + 32\gamma + 3\right)\left(v_1 + v_2\right) + 4\gamma(8\gamma + 7)}{4\left(16\gamma^2 + 18\gamma + 3\right)} \quad (i,\ h = 1,2;\ i \neq h)$$

As in the decentralized case we have $\partial v_i^{*n} / \partial v_h > 0$ and $\partial v_j^{*n} / \partial v_k < 0$.

2.4 **Partial Vertical Integration**

The downstream prices are the solutions to the following:

(A.11a) $$p^*_{(i;i')} = \arg\max\left\{q_{(i;i')}(\mathbf{p})\left(p_{(i;i')} - v_{(i;i')}\right)\right\} \quad (i = 1,2)$$

(A.11b) $$p^*_{(i;h')} = \arg\max\left\{q_{(i;h')}(\mathbf{p})\left(p_{(i;h')} - v_i - v_{h'}\right)\right\} \quad (i,\ h = 1,2,\ i \neq h).$$

This gives:

(A.12a) $$p^*_{(i;i')} = \frac{\left(28\gamma + 32\gamma^2\right) + (3 + 4\gamma)\left(4v_{(i;i')}(1 + 2\gamma) + v_{(h;h')} + v_1 + v_2 + v_{1'} + v_{2'}\right)}{\left(21 + 80\gamma + 64\gamma^2\right)} \quad (i,\ h = 1,2;\ i \neq h)$$

(A.12b) $$p^*_{(i;h')} = \frac{\left(28\gamma + 32\gamma^2\right) + \left(12 + 40\gamma + 32\gamma^2\right)\left(v_i + v_{h'}\right) + (3 + 4\gamma)\left(v_{(1;1')} + v_{(2;2')} + v_h + v_{i'}\right)}{\left(21 + 80\gamma + 64\gamma^2\right)} \quad (i,\ h = 1,2;\ i \neq h).$$

### 2.4.1 **Centralized integration**

With centralized partial vertical integration we know that $v_{(i;i')} = 0$. The reaction functions corresponding to the stage one pricing game are then:

(A.13a) $$v_i^* = \frac{A(\gamma) + B(\gamma)v_h + C(\gamma)v_{i'} - E(\gamma)v_{h'}}{D(\gamma)}$$ ($i$, $h$ = 1,2; $i \neq h$)

(A.13b) $$v_{i'}^* = \frac{A(\gamma) + B(\gamma)v_{h'} + C(\gamma)v_i - E(\gamma)v_h}{D(\gamma)}$$ ($i$, $h$ = 1,2; $i \neq h$)

where the positive coefficients are:

$A(\gamma) = 756\gamma + 3328\gamma^2 + 4608\gamma^3 + 2048\gamma^4$; $B(\gamma) = 81 + 372\gamma + 544\gamma^2 + 256\gamma^3$;

$C(\gamma) = 144 + 696\gamma + 1056\gamma^2 + 512\gamma^3$; $D(\gamma) = 360 + 3072\gamma + 8864\gamma^2 + 10240\gamma^3 + 4096\gamma^4$;

$E(\gamma) = 171 + 1512\gamma + 4416\gamma^2 + 5120\gamma^3 + 2048\gamma^4$.

### 2.4.2 **A divisional upstream profit center**

When the upstream division aims to maximize its own divisional profits we have the upstream reaction functions:

(A.14a) $$v_{(i;i')}^* = \frac{(28\gamma + 32\gamma^2) + (3 + 4\gamma)(v_{(h;h')} + 2(v_i + v_{i'}) + v_h + v_{h'})}{2(9 + 40\gamma + 32\gamma^2)}$$ ($i$, $h$ = 1,2; $i \neq h$)

(A.14b) $$v_i^* = \frac{(28\gamma + 32\gamma^2) + (3 + 4\gamma)(2v_{(i;i')} + v_{(h;h')} + v_h + 2v_{i'}) - (9 + 40\gamma + 32\gamma^2)v_{h'}}{2(9 + 40\gamma + 32\gamma^2)}$$ ($i$, $h$ = 1,2; $i \neq$

$h$)

(A.14c) $$v_{i'}^* = \frac{(28\gamma + 32\gamma^2) + (3 + 4\gamma)(2v_{(i;i')} + v_{(h'h')} + 2v_i + v_{h'}) - (9 + 40\gamma + 32\gamma^2)v_h}{2(9 + 40\gamma + 32\gamma^2)}$$ ($i$, $h$ = 1,2; $i \neq h$)

### 2.4.3 **An upstream profit center maximizing group profits**

When the upstream division is established as a profit center concerned with the aggregate profits of the integrated firm the upstream reaction functions are:

(A.15a) $$v_{(i;i')}^* = \frac{(v_{(h;h')} + v_h + v_{h'})F(\gamma) + (v_i + v_{i'})G(\gamma) + H(\gamma)}{J(\gamma)}$$ ($i$, $h$ = 1,2; $i \neq h$)

(A.15b) $$v_i^* = \frac{v_{(i;i')}K(\gamma) + (v_{(h;h')} + v_h)L(\gamma) + v_{i'}M(\gamma) + N(\gamma) - v_{h'}O(\gamma)}{P(\gamma)}$$ ($i$, $h$ = 1,2; $i \neq h$)

(A.15c) $$v_{i'}^* = \frac{v_{(i;i')}K(\gamma) + (v_{(h;h')} + v_{h'})L(\gamma) + v_iM(\gamma) + N(\gamma) - v_hO(\gamma)}{P(\gamma)}$$ ($i$, $h$ = 1,2; $i \neq h$)

where the positive coefficients are:

$F(\gamma)=(9+12\gamma)$; $G(\gamma)=72+336\gamma+512\gamma^2+256\gamma^3$; $H(\gamma)=84\gamma+96\gamma^2$;

$J(\gamma)=8(27+210\gamma+56\gamma^2+640\gamma^3+256\gamma^4)$; $K(\gamma)=72+336\gamma+512\gamma^2+256\gamma^3$;

$L(\gamma)=81+372\gamma+544\gamma^2+256\gamma^3$; $M(\gamma)=144+696\gamma+1056\gamma^2+512\gamma^3$;

$N(\gamma)=756\gamma+3320\gamma^2+4608\gamma^3+2048\gamma^4$; $O(\gamma)=1512\gamma+4416\gamma^2+5120\gamma^3+2048\gamma^4$;

$P(\gamma)=8(45+384\gamma+1108\gamma^2+1280\gamma^3+512\gamma^4)$.

## 2.5 The upstream consortium

The problem facing the downstream firms is to choose prices to solve:

(A.16) $p^*_{(j;k)}=\arg\max\{q_{(j;k)}(\mathbf{p})(p_{(j;k)}-v)\}$ ($j$ = 1, 2, $k$ = 1′, 2′).

The solutions are:

(A.17) $p_{(j;k)}=\dfrac{3v+4\gamma(1+v)}{3+8\gamma}$ ($j$ = 1, 2, $k$ = 1′, 2′).

The upstream consortium chooses $v$ to maximize aggregate profit of the upstream and downstream firms. This is:

(A.18) $\Pi(v,\gamma)=\dfrac{4(3+4\gamma)(1-v)(3v+4\gamma(1+v))}{(3+8\gamma)^2}$.

Maximizing this with respect to $v$ gives:

(A.19) $v^*=3/2(3+4\gamma)$.

## 2.6 Vertical Foreclosure

We provided the analysis for the centralized partial vertical integration case. Assume that integrated firm [1;1′] forecloses on downstream firm (1;2′) while integrated firm [2;2′] either remains willing to supply both downstream firms or also forecloses on firm (1;2′). In either case, firm (1;2′) is forced to exit the market, The downstream demand functions become:

(A.20) $q_{(j;k)}(\mathbf{p})=1-p_{(j;k)}-\dfrac{1}{\gamma}(p_{(j;k)}-\bar{p})$ ($j;k$) $\in$ {(1;1′), (2;1′), (2;2′)},

where $\bar{p}=\dfrac{1}{3}(p_{(1;1')}+p_{(2;1')}+p_{(2;2')})$.

As above, the downstream prices satisfy:

(A.21) $p^*_{(i;i')}=\arg\max\{q_{(i;i')}(\mathbf{p})p_{(i;i')}\}$ ($i$ = 1,2)

(A.22) $p^*_{(2;1')} = \arg\max\{q_{(2;1')}(\mathbf{p})(p_{(2;1')} - v_2 - v_{1'})\}$.

The solutions are:

(A.23a) $$p^*_{(i;i')} = \frac{3\gamma(5+6\gamma)+(2+3\gamma)(v_h + v_{i'})}{(5+6\gamma)(2+6\gamma)}$$ ($i$, $h$ = 1,2; $i \neq h$)

(A.23b) $$p^*_{(2;1')} = \frac{3(\gamma(5+6\gamma)+(2+3\gamma)(1+2\gamma)(v_2 + v_{1'}))}{(5+6\gamma)(2+6\gamma)}.$$

Substituting these prices into the profit functions for firms [1;1′] and [2;2′], which are:

(A.24a) $$\Pi^I_{(1;1')} = p^*_{(1;1')}(\mathbf{v})q^*_{(1;1')}(\mathbf{p}(\mathbf{v})) + v_{1'}q^*_{(2;1')}(\mathbf{p}(\mathbf{v}))$$

(A.24b) $$\Pi^I_{(2;2')} = p^*_{(2;2')}(\mathbf{v})q^*_{(2;2')}(\mathbf{p}(\mathbf{v}))$$

deriving the reaction functions for $v_2$ and $v_{1'}$ and solving gives the input prices to firm 21:

(A.25) $$v_2^* = v_{1'}^* = \frac{3\gamma(5+6\gamma)(7+24\gamma+18\gamma^2)}{(4+27\gamma+27\gamma^2)(13+48\gamma+36\gamma^2)}.$$

Substituting (A.25) into (A.23) and (A.24) gives the profits to the upstream firms with this partial foreclosure.

Symmetry implies that the same solution applies if either or both integrated firms foreclose on firm (2;1′) but not (1;2′). Finally, foreclosure by the integrated firms on different downstream firms is equivalent to their foreclosing on both downstream firms. This gives the pay-off matrix for the integrated firms of Table 2. Straightforward but tedious analysis confirms that for any value of $\gamma > 0$ the only Nash equilibrium for this game is “no foreclosure” by either integrated firm.

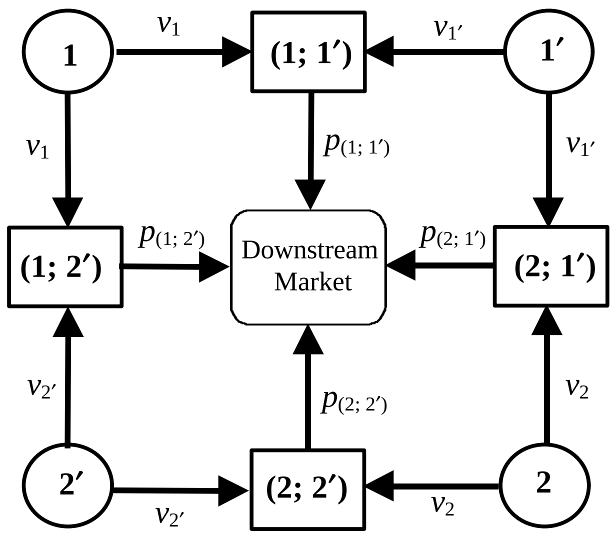



**Figure 1: The Basic Market Structure**

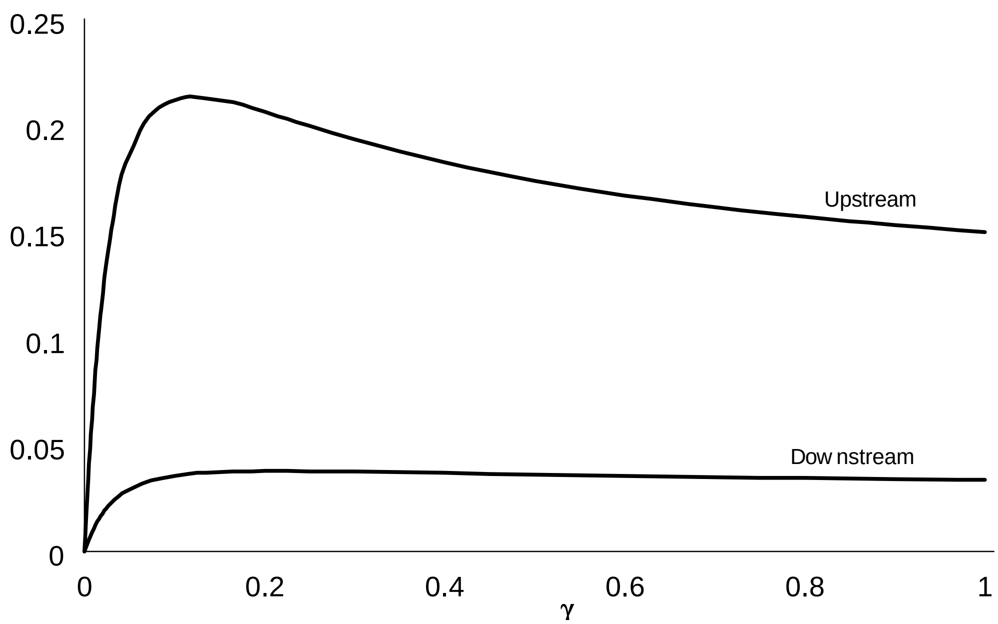
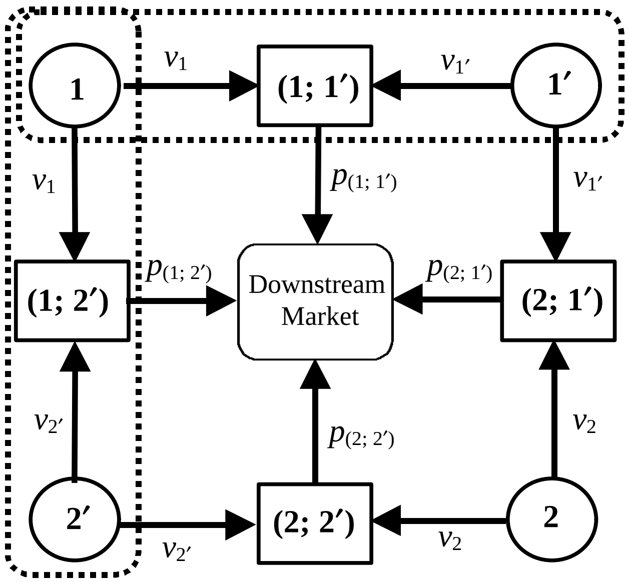
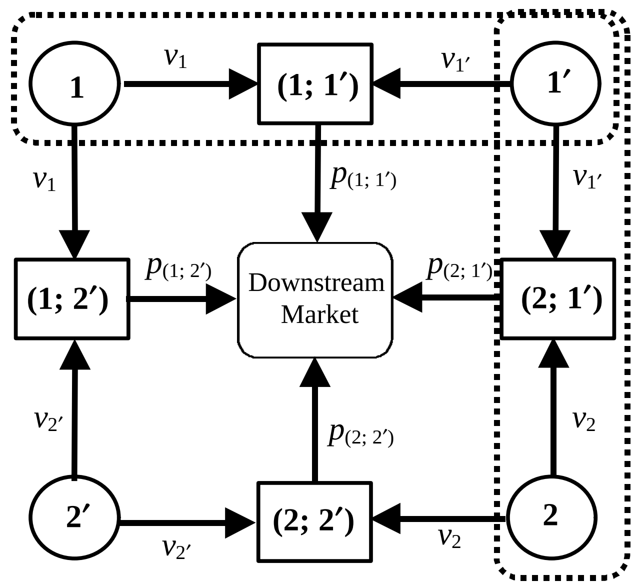



**Figure 2: Decentralized Profits**




(a) Alliances for Firm 1




(b) Alliances for Firm 1′

**Figure 3: Network of Upstream Alliances**

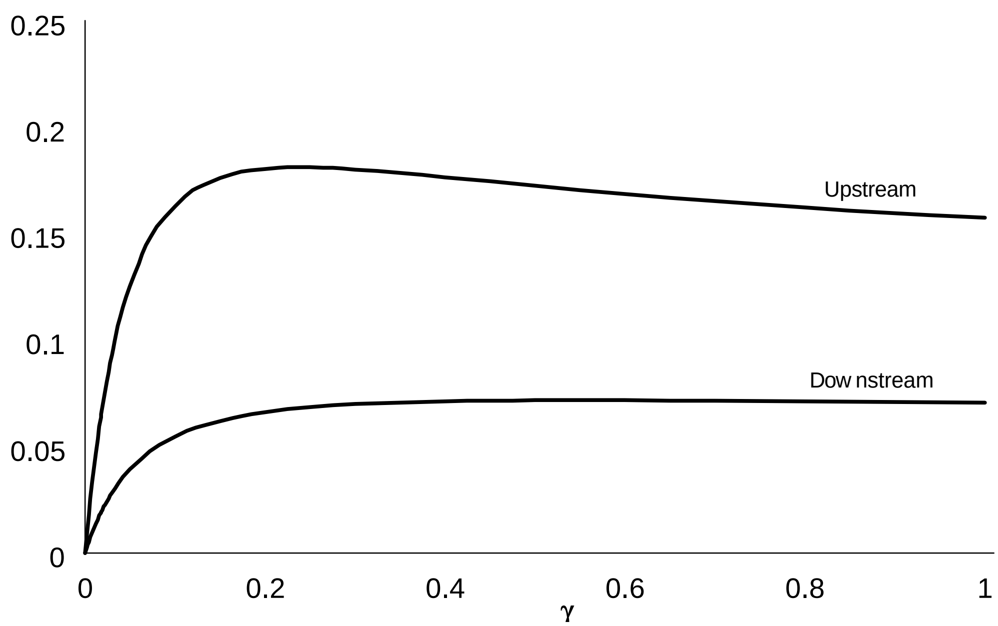



**Figure 4: Network Alliance Profits**

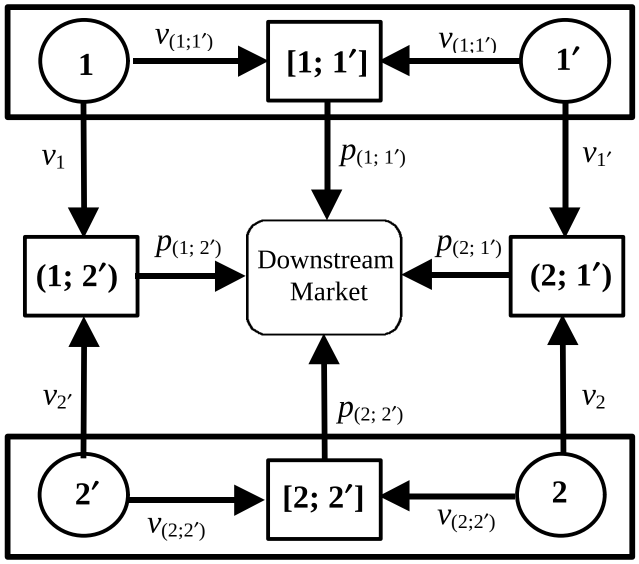
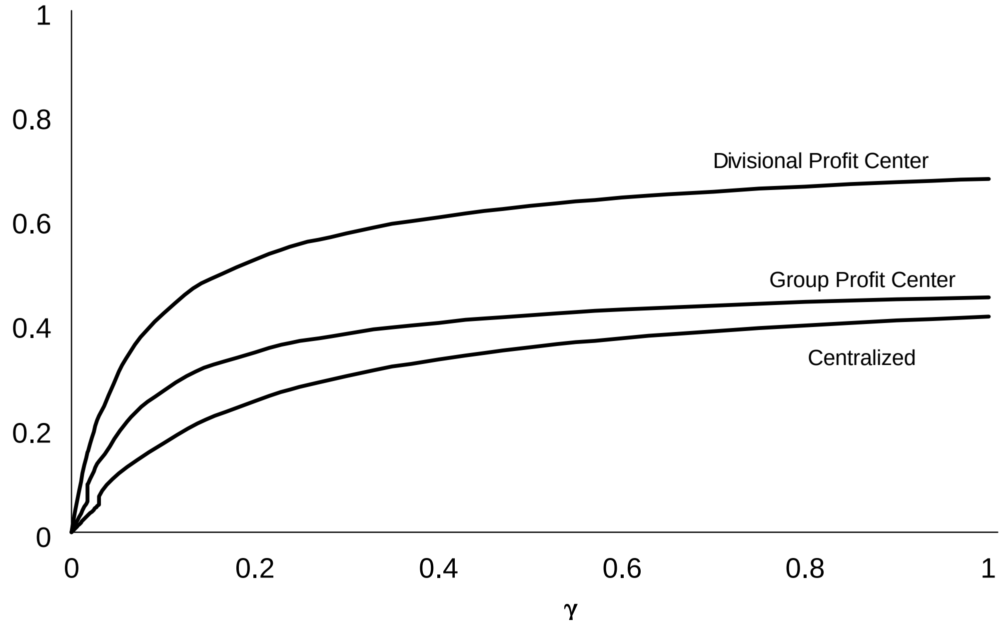
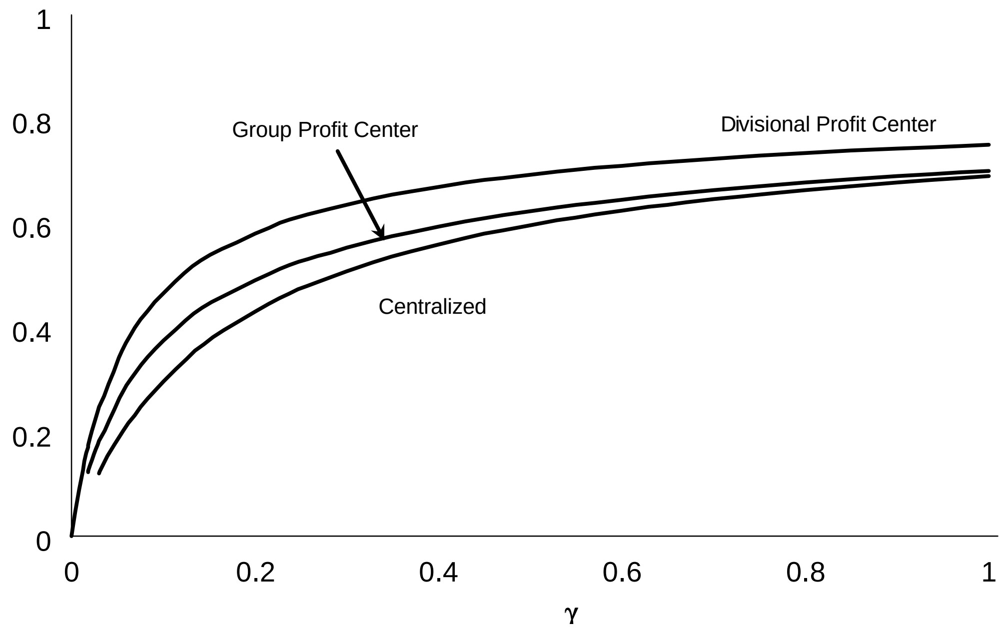



**Figure 5: Partial Vertical Integration**

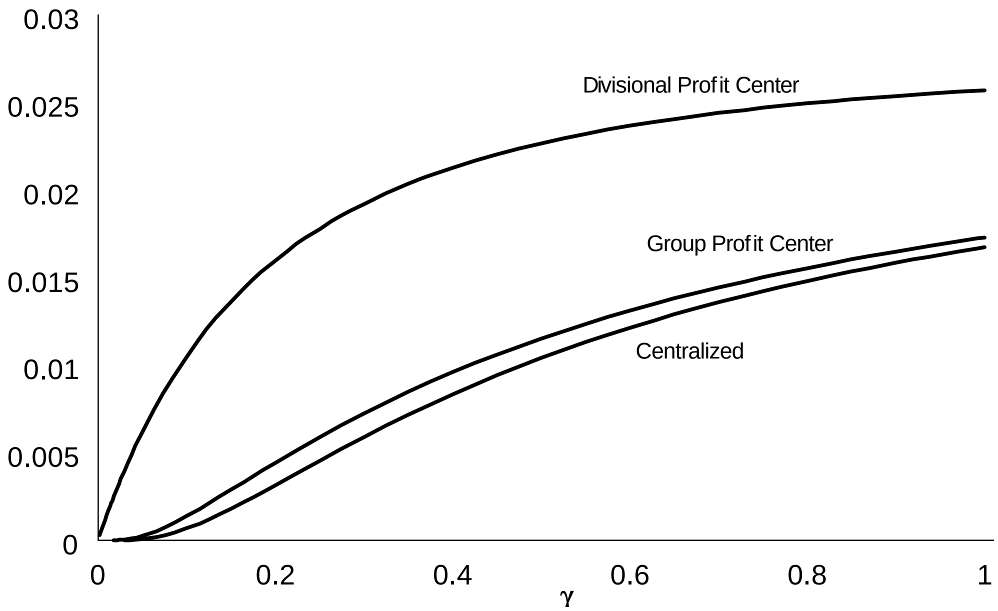



**(a) Integrated Firms**

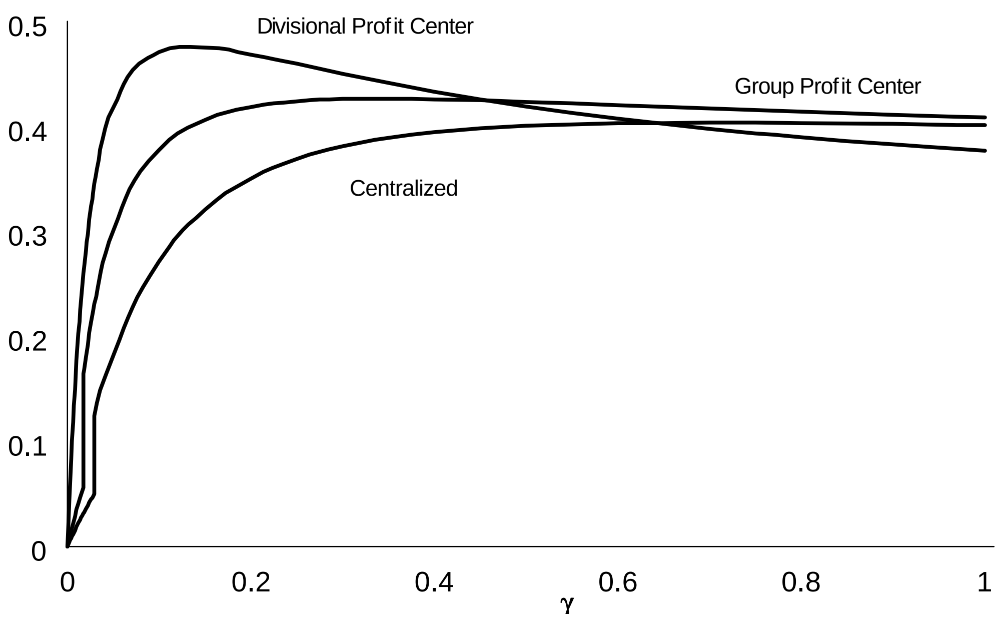



**(b) Non-Integrated Firms**

**Figure 6: Downstream Prices with Partial Vertical Integration**




**(a)**
**(b) Non-Integrated Firms**




**(c) Integrated Firms**

**Figure 7: Profits with Partial Vertical Integration**

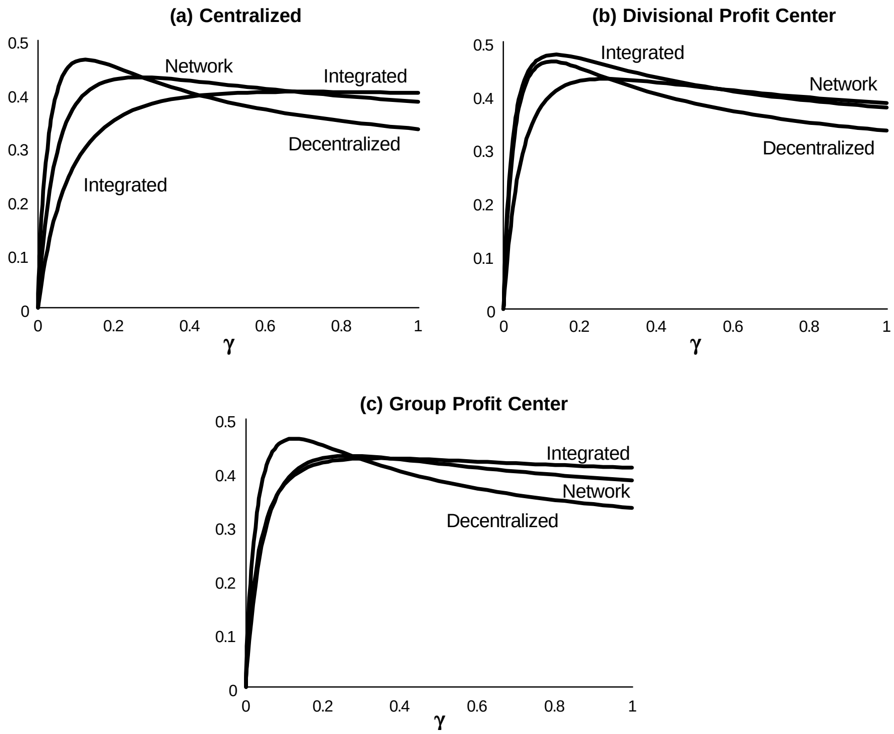



**Figure 8: Profit Comparisons**

## Table 1: Pay-Off Matrix for Partially Vertically Integrated Firms

| | | Firm 22 | | | |
|---|---|---|---|---|---|
| | | **no foreclosure** | **foreclose 12** | **foreclose 21** | **foreclose both** |
| **Firm 11** | **no foreclosure** | $\Pi_{11}^{nf}, \Pi_{22}^{nf}$ | $\Pi_{11}^{pf}, \Pi_{22}^{pf}$ | $\Pi_{11}^{pf}, \Pi_{22}^{pf}$ | $\Pi_{11}^{ff}, \Pi_{22}^{ff}$ |
| | **foreclose 12** | $\Pi_{11}^{pf}, \Pi_{22}^{pf}$ | $\Pi_{11}^{pf}, \Pi_{22}^{pf}$ | $\Pi_{11}^{ff}, \Pi_{22}^{ff}$ | $\Pi_{11}^{ff}, \Pi_{22}^{ff}$ |
| | **foreclose 21** | $\Pi_{11}^{pf}, \Pi_{22}^{pf}$ | $\Pi_{11}^{ff}, \Pi_{22}^{ff}$ | $\Pi_{11}^{pf}, \Pi_{22}^{pf}$ | $\Pi_{11}^{ff}, \Pi_{22}^{ff}$ |
| | **foreclose both** | $\Pi_{11}^{ff}, \Pi_{22}^{ff}$ | $\Pi_{11}^{ff}, \Pi_{22}^{ff}$ | $\Pi_{11}^{ff}, \Pi_{22}^{ff}$ | $\Pi_{11}^{ff}, \Pi_{22}^{ff}$ |

$$\Pi_{ii}^{nf} = \frac{8\gamma(7+8\gamma)\left(747+8547\gamma+32998\gamma^2+56560\gamma^3+44672\gamma^4+13312\gamma^5\right)}{\left(153+1758\gamma+5840\gamma^2+7296\gamma^3+3071\gamma^4\right)^2};$$

$$\Pi_{ii}^{nf} = \frac{9\gamma(2+3\gamma)(5+6\gamma)\left(408+4450\gamma+17127\gamma^2+29610\gamma^3+23652\gamma^4+7128\gamma^5\right)}{4\left(4+27\gamma+27\gamma^2\right)^2\left(13+48\gamma+36\gamma^2\right)^2}$$

$$\Pi_{ii}^{ff} = \frac{2\gamma(1+2\gamma)}{(1+4\gamma)^2}.$$

**Table 2: Comparison of Prices and Profits**

| | |
|---|---|
| 1. Prices | |
| Upstream | $v^{*d} > v^{*p} > v^{*n}$ |
| Downstream | Centralized partial vertical integration |
| | $p^{*d} > p^{*n} > p_{ij}^{*p} > p_{ii}^{*p}$ for $\gamma \leq 0.275$ |
| | $p^{*d} > p_{ij}^{*p} > p^{*n} > p_{ii}^{*p}$ for $0.275 < \gamma$ |
| | Divisional profit center |
| | $p_{ij}^{*p} > p^{*d} > p_{ii}^{*p} > p^{*n}$ for $g \leq 0.264$ |
| | $p^{*d} > p_{ij}^{*p} > p_{ii}^{*p} > p^{*n}$ for $0.264 < \gamma$ |
| | Group profit center |
| | $p^{*d} > p_{ij}^{*p} > p^{*n} > p_{ii}^{*p}$ |
| 2. Profits | |
| Upstream | $\pi_U^{*d} > \pi_U^{*n}$ for $\gamma \leq 0.551$ |
| | $\pi_U^{*n} > \pi_U^{*d}$ for $0.551 < \gamma$ |
| Downstream | $\pi_D^{*n} > \pi_D^{*d} > \pi_D^{*p,ni}$ |
| Combined[1] | Centralized integration |
| | $\pi_D^{*d} > \pi_D^{*n} > \pi_D^{*p}$ for $\gamma \leq 0.275$ |
| | $\pi_D^{*n} > \pi_D^{*d} > \pi_D^{*p}$ for $0.275 < \gamma \leq 0.428$ |
| | $\pi_D^{*n} > \pi_D^{*p} > \pi_D^{*d}$ for $0.428 < \gamma \leq 0.676$ |
| | $\pi_D^{*p} > \pi_D^{*n} > \pi_D^{*d}$ for $0.676 < \gamma$ |
| | Divisional profit center |
| | $\pi_D^{*p} > \pi_D^{*d} > \pi_D^{*n}$ for $\gamma \leq 0.275$ |
| | $\pi_D^{*p} > \pi_D^{*n} > \pi_D^{*d}$ for $0.275 < \gamma \leq 0.562$ |
| | $\pi_D^{*n} > \pi_D^{*p} > \pi_D^{*d}$ for $0.562 < \gamma$ |
| | Group profit center |
| | $\pi_D^{*d} > \pi_D^{*n} > \pi_D^{*p}$ for $\gamma \leq 0.275$ |
| | $\pi_D^{*n} > \pi_D^{*d} > \pi_D^{*p}$ for $0.275 < \gamma \leq 0.294$ |
| | $\pi_D^{*n} > \pi_D^{*p} > \pi_D^{*d}$ for $0.294 < \gamma \leq 0.369$ |
| | $\pi_D^{*p} > \pi_D^{*n} > \pi_D^{*d}$ for $0.369 < \gamma$. |

Notes: [1] Combined profit is the sum of profits of one downstream and two upstream firms.

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