

Partial Licensing of Product Innovations*

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Abstract

We study the incentives of a product innovator to license its product “*partially*” to a potential entrant. In a duopolistic setting we consider product design of a modular nature, which enables the incumbent to license some modules of its innovation. Competition is characterized by the size of the shared component of the product which determines the degree of product differentiation. Therefore, by choosing *how much to license*, i.e., the size of the license, the innovator chooses *how much to compete*. In a general setting, we provide sufficient conditions for partial licensing to occur, and then we use a specific competitive setting and show that the size of the license increases when i) the entrant has a higher development cost to complete the product, and ii) the marginal effect of the size of the license on product differentiation is lower.

Keywords: Licensing, Product Innovation, Modularity.

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1 Introduction

Products consist of a bundle of information embodied in materials, where the information includes product concepts, styling models, specifications, engineering drawings, process designs, etc.¹ When products are designed in a “modular” fashion, i.e., when a product combines distinct building blocks (or modules), a set of those complementary components can be partitioned for the purposes of product development. This mix-and-match feature of modular design allows different product developers to change an arbitrary partition of the functional components of the product without necessarily altering the design of other elements.

Modular product design and its advantages have been extensively discussed in the management science literature. Along with supply side efficiencies like reduced order lead-time, economies of scale (that arise due to modules that are produced in larger quantities), modular product design is argued to provide producers with strategic flexibility. Strategic flexibility allows firms to respond to changing markets and technologies and to develop niche products by rapidly and inexpensively creating product variants derived from different combinations of existing or new modular components. This flexibility, in return, can speed up the rate of technological change and increase product variety.²

In this paper we introduce and study another type of flexibility due to the modular design of products, that is, *flexibility in licensing* product innovations. In a duopolistic setting, we consider an innovator who holds the exclusive rights to its innovation. The innovation has a modular nature and the innovator can decide to license its innovation *partially*, i.e., license an arbitrary partition of it. Our focus is not the stage of modular design, and hence we assume that the innovator has an extensive ex ante product knowledge which is necessary for splitting design into appropriate modules, i.e., for decomposing its product into subsystems. When the entrant buys a partition of the innovator’s innovation it needs to invest in order to “complete” the product. We assume that the possibility of differentiation between the products decreases with the size of the shared component, that is, with the size of the partition of the product innovation that is being licensed. At one extreme, if the product innovator licenses its full product no –or very little– differentiation will occur between the products of the licensor and the licensee. Therefore, by choosing *how much to license*, the innovator chooses *how much to compete*.

In this setting we show that the innovator’s licensing strategy is mainly determined by the entrant’s development cost to complete the product. When the development cost is high the innovator can extract large rents from the entrant by licensing a large portion of its innovation, and hence, when it is sufficiently high, the innovator grants the full license to the entrant. Partial licensing is more likely to occur when the entrant has a low development cost to complete the product, and also when the marginal effect of the size of the license on product differentiation is sufficiently large. Otherwise, the equilibrium

¹See Clark et al. (1987), p. 733.

²See Huang (2000), Schaefer (1999), Baldwin and Clark (1994).

is characterized by full licensing. These results do not depend on the presence of economies of scope in producing modules, because we do not assume any. We also show that the innovator has an incentive to license a larger partition of its innovation i) when it is more efficient than the entrant in production ii) with Cournot competition than with Bertrand competition.

Most products involve some degree of modularity; common examples appear in industries such as automobiles, aircraft, consumer electronics, and personal computers.³ For instance, in the automobile industry two different firms, as well as two different products of the same firm, can share the engine of a car to make up differentiated cars by using different types of vehicle bodies and tires.⁴ Non-physical products like software and computer based information systems are trivial examples of how derivation products can appear around a platform. When a software platform is accessible, a developer can rapidly provide various products without starting from scratch every time. Platform-based approach can even be applied to development of new *services*, e.g., financial services.⁵

When a set of modular components are licensed to a rival firm, that shared subsystem serves as a product platform.⁶ By adding other components to the product platform, the licensee can build a derivative product to the original one. Meyer and Dalal (2002) state that the trend in product and system development over the past several decades is to acquire or license an increasing proportion of the building blocks, be it core product technologies or production process. They also state that incremental product and process innovation usually win over new platform investments because the former carry less short term risk and amortize current capital investments.

The literature on product design does not show any interest in how those platforms or modules are shared between different producers, and how this affects competition. This appears as an important topic, as a flexible product platform allows the licensee to develop a differentiated product from that of the licensor, and hence, constitute a viable alternative to developing a product from scratch. The licensing literature has not analyzed the possibility and the consequences of partial licensing of product innovation, either.⁷

The papers which are most related to ours are Rockett (1990a) and (1990b). Rockett (1990a) analyses the incentives to license a process innovation which is inferior to its own technology. This may occur either when signalling the

³See Sanchez and Mahoney (1996), p. 67, for a list of modular products.

⁴Clark et al. (1987) provide a detailed study on product development in the automobile industry.

⁵For a platform-based approach on services see Meyer and DeTore (2001).

⁶A product platform is a common subsystem and an interface used within and shared across different individual products (see Meyer and Dalal (2002)). Here, “platform” is used in a broader sense than in the Industrial Organization literature. For example in Church and Gandal (2000), hardware is a platform, and software is an application that is complement to a compatible platform; the combination of the two creates a “system”. In our context, *a set of building blocks* that makes a software or a hardware can serve as a platform.

⁷There have been many valuable contributions to the analysis of ex-post and ex-ante incentives for full licensing. See for example, Gallini (1984), Gallini and Winter (1985), Katz and Shapiro (1985) and (1987), Kamien and Tauman (1986). Some of the recent contributions are Yi (1999), van Dijk (2000), and Moldavanu and Aner (2003).

value of the superior technology is too costly, or when the gains from reduced incentives for imitating the inferior technology are sufficiently large. The author considers a continuum of process innovations with different costs of production. Similarly, we consider a continuum of modules for product innovation, and allow the incumbent to decide for the size of the shared component with the entrant's product. However, in our context, a smaller shared component does not imply any quality disadvantage for the entrant.⁸ Rockett (1990b) studies how a monopolist innovator could choose competition with its licensing strategy when it faces heterogeneous potential entrants. She shows that the monopolist may prefer to license its technology to the weaker competitor, in order to prolong its dominant position. In our paper, although the incumbent faces a single entrant, it does, too, choose competition that takes place post-entry, since the shared component determines the degree of product differentiation.

Partial licensing is a real life phenomenon: A leading developer of 3 Dimensional (3D) video games, *id Software*, initially designed and used 3D engines, that provide real-time computing techniques for special effects, for its own 3D video games. Facing the demand of other game publishers and developers, it started to license its 3D engines, so that the game developers could develop their own 3D games without developing their own engines.⁹ Id Software faced the following trade-off in licensing its engines, which is central to this paper: a 3D engine which is initially designed, for example, for 'first person shooting' games, is suitable only for designing similar genre of games, and not for designing 'racing' or 'role-playing' games. Therefore, although licensing generates revenues through the extraction of the savings from the development cost (of the 3D engine), it intensifies competition by leaving too little room for differentiation.

Although not through licensing, another example in which an innovator shares some modules of its product with its competitors is that of ETA Sa., a subsidiary of the Swatch group.¹⁰ ETA supplies mainly two types of watch movement mechanisms, "raw" and "integrated", to other high-end watch manufacturers. A raw mechanism is the core of the movement, and does not contain any other functionalities, e.g., chronograph or time zone indicator. Third-party manufacturers can develop and add functionalities to a raw mechanism and differentiate their high-end watches. Integrated mechanisms contain built-in functionalities, and hence, leave less room for differentiation among the products that carry this large common component.

The remainder of the paper is organized as follows. In the following section we set up a general model and provide sufficient conditions under which partial licensing occurs. In Section 3, we illustrate our finding of the general model with

⁸Similarly, a recent work by Anton and Yao (2003) studies the amount of an innovation (process innovation) that should be disclosed in a duopoly setting when disclosures make imitation feasible.

⁹Note that 3-D engines and game design are complement inputs in the product development, and are not sold separately at the retail level.

¹⁰ETA Sa. is a company of Swatch group which is specialized in the production of watch components and watch movements. It provides 75 % of the movement mechanisms that are used for making high-end watches. See an article from Liberation at <http://www.liberation.fr/page.php?Article=161543&AP>

a specific competitive setting, which enables us to provide some comparative statics and extensions to the model. Finally, we conclude.

2 The Model

We consider an innovator (Firm 1) and a potential entrant (Firm 2). The innovation is composed of a continuum of modules (divisible layers).¹¹ The innovator decides to license a portion $\alpha \in [0, 1]$ of its innovation to the potential entrant. When $\alpha = 0$, there is *no* licensing and when $\alpha = 1$, there is *full* licensing. For all $\alpha \in (0, 1)$, there is *partial* licensing. We assume that the portion α that is to be licensed consists of the initial α layers, which implies that a partial license is viable only if the entrant has access to all layers that are lower than α . In the rest of the paper we refer α as the "size of the license". If the entrant acquires the license, it decides on the modules that it uses in its product, $\hat{\alpha} \in [0, \alpha]$, which implies that the size of the shared component in both products is $\hat{\alpha}$. When Firm 1 does not license its innovation, the entrant's sole possibility to enter in the market is by developing its own product from scratch.¹²

When there is entry, competition is characterized by the degree of differentiation, $s \in [\underline{s}, \bar{s}]$, between the two firms. We assume that the duopoly profits gross of development costs, $\pi(s)$, are non-decreasing with product differentiation, i.e., $\partial\pi(s)/\partial s \geq 0$. If entry occurs via acquiring a license, the degree of differentiation depends on the size of the shared component of the innovation. In particular, we assume that $\partial s(\hat{\alpha})/\partial \hat{\alpha} < 0$, that is, the degree of differentiation decreases with $\hat{\alpha}$. If entry occurs via development of a new product (firms have no shared component), we assume that the products of the firms have a maximum degree of differentiation, $s = \bar{s}$.¹³

If the entrant acquires a partial license of α modules and uses $\hat{\alpha}$ of it, then it needs to develop the remaining $(1 - \hat{\alpha})$ modules in order to "complete" the product, which entails a development cost denoted by $d(\hat{\alpha})$. If the entrant develops its product from scratch, it has to invest $d(0)$. There are no economies (or diseconomies) of scope in developing modules since in this formulation the cost of developing the first $\hat{\alpha}$ modules, $(d(0) - d(\hat{\alpha}))$, plus the cost of completing the product, $d(\hat{\alpha})$, is equal to $d(0)$, that is, to the cost of developing the product from scratch.

In Section 3, we provide an example of a competitive setting, which satisfies

¹¹Considering discrete layers would be more realistic, however, it would not add much to our analysis.

¹²We believe that this might be feasible in some product markets and not in some others, and hence, we also discuss the case when the entrant has no other entry option than acquiring a license. However, from the theoretical point of view this case is not interesting since the innovator could replicate the duopoly outcome when the entrant has no other entry option.

¹³We treat the degree of differentiation as exogenously determined in this subgame where the entrant develops its product from scratch. However, if s were a strategic variable for the entrant, given the innovator's product design, it would choose to differentiate its product as much as possible since we have assumed that $\partial\pi/\partial s \geq 0$.

the conditions stated for the general model.

The timing The timing of the game is as follows.

1. The innovator decides on α and sets a fixed licence fee $f \geq 0$ for it.¹⁴
2. The entrant decides whether or not to buy the license.
3. If it buys the license, it decides on $\hat{\alpha} \in [0, \alpha]$, i.e., the modules it uses for its product. Finally, it develops the remaining $(1 - \hat{\alpha})$ parts. If it does not buy the license, it develops its product from scratch.
4. Competition and profits are realized.

The Equilibrium Let Π_i denote the net profits of firm i , with $i = 1, 2$, when entry occurs with licensing (firms have common component $\hat{\alpha}$ in their product design). Then, firms obtain

$$\Pi_1(\hat{\alpha}, f) = \pi(s(\hat{\alpha})) + f, \quad (1)$$

and

$$\Pi_2(\hat{\alpha}, f) = \pi(s(\hat{\alpha})) - f - d(\hat{\alpha}). \quad (2)$$

Let $\tilde{\Pi}_i$ denote the net profits of firm i , when entry occurs with new product development (firms have no common component in their product design, i.e., $\hat{\alpha} = 0$). Then, firms obtain

$$\tilde{\Pi}_1 = \pi(\bar{s}),$$

and

$$\tilde{\Pi}_2 = \pi(\bar{s}) - d(0).$$

Entry with no licensing is viable only if and only if $\tilde{\Pi}_2 \geq 0$, and we assume that this is the case.¹⁵ We also assume that profit functions Π_1 and Π_2 are continuous and strictly concave, which also implies that, for both firms, second order condition for profit maximization is satisfied. In Section 3 we provide an example in which this assumption is satisfied.

¹⁴We do not consider the possibility of subsidized entry.

¹⁵When $\tilde{\Pi}_2 < 0$, then the entrant has no viable outside option, and hence remains outside the market if there is no licensing. Such a situation creates a discontinuity in the profit function of the innovator, and the innovator has to compare its profit under optimal partial licensing to its profit with no licensing. In Appendix A we provide the analysis for $\tilde{\Pi}_2 < 0$, and show that the decision on *how much to license* does not differ in this case (at the licensing equilibrium).

Stage 3 – Entrant’s product design If the entrant buys the license (in stage 2), for a given size of license, α , set by the innovator, it has the following problem,

$$\max_{\hat{\alpha} \in [0, \alpha]} \Pi_2(\hat{\alpha}, f) = \pi(s(\hat{\alpha})) - f - d(\hat{\alpha}).$$

To begin with, assume that the innovator provides full licensing, i.e., $\alpha = 1$. In this case, the first order condition of the entrant’s problem is

$$\frac{\partial \pi(s)}{\partial s} \frac{\partial s(\hat{\alpha})}{\partial \hat{\alpha}} - \frac{\partial d(\hat{\alpha})}{\partial \hat{\alpha}} = 0. \quad (3)$$

Note that it is as if the size of the license were determined by the entrant. Let $\hat{\alpha}_1 \geq 0$ define the solution to (3).

Now consider any α , with $\alpha \in [0, 1]$. Since the entrant is constrained by the size of the license, that is by $\hat{\alpha} \in [0, \alpha]$, the entrant’s optimal product design decision, $\hat{\alpha}^*(\alpha)$, is given by

$$\hat{\alpha}^*(\alpha) = \begin{cases} \alpha & \text{if } \alpha \leq \hat{\alpha}_1 \\ \hat{\alpha}_1 & \text{if } \alpha > \hat{\alpha}_1 \end{cases}.$$

If the entrant does not acquire the license, it invests $d(0)$ for its product design, and obtains $\tilde{\Pi}_2$.

Stage 2 – Entrant’s decision to acquire license The entrant acquires the license if and only if

$$\Pi_2(\hat{\alpha}^*(\alpha), f) \geq \tilde{\Pi}_2. \quad (4)$$

Stage 1 – Licensing Decision The innovator has the following problem

$$\max_{\alpha, f} \Pi_1(\alpha, f) = \max_{\alpha, f} \{\pi(s(\hat{\alpha}^*(\alpha))) + f\}$$

subject to (4), which can be rewritten as¹⁶

$$f \leq \pi(s(\hat{\alpha}^*(\alpha))) - \pi(\bar{s}) + d(0) - d(\hat{\alpha}^*(\alpha)).$$

Since for any α the innovator’s profits are increasing with f , the constraint is binding, and the innovator maximizes the following

$$\bar{\Pi}_1(\alpha) = 2\pi(s(\hat{\alpha}^*(\alpha))) - \pi(\bar{s}) + d(0) - d(\hat{\alpha}^*(\alpha)) \quad (5)$$

with respect to α . Note that

$$\bar{\Pi}_1(0) = \tilde{\Pi}_1,$$

¹⁶Note that if the entrant had no option of entry other than acquiring the license, the constraint would not include the term $\pi(\bar{s}) - d(0) = \tilde{\Pi}_2$, that is, the opportunity cost of acquiring a license when developing the product from scratch is possible. Since $\tilde{\Pi}_2$ is independent of $\hat{\alpha}$, the innovator’s optimal choice of α would also be valid if entry was possible only via licensing.

therefore, the optimal licensing strategy (no licensing, full licensing or partial licensing) maximizes $\bar{\Pi}_1(\alpha)$, defined in (5).¹⁷

Licensing a larger portion of the innovation yields higher profits to the incumbent due to the entrant's higher willingness to pay for the license (because of lower development cost to complete the product), but at the same time lowers profits due to the reduced degree of differentiation (which intensifies competition). With the following Lemma, we show that if the innovator licenses its product, the optimal size of the license is such that the entrant uses all of it.

Lemma 1 *If there is licensing at the equilibrium, the entrant uses all the licensed modules, i.e., $\hat{\alpha}^* = \alpha^*$.*

Proof. First, if there is full licensing at the equilibrium, then we have

$$\left. \frac{\partial(2\pi(s))}{\partial s} \frac{\partial s(\alpha)}{\partial \alpha} - \frac{\partial d}{\partial \alpha} \right|_{\alpha=1} > 0,$$

which implies that

$$\left. \frac{\partial \pi(s)}{\partial s} \frac{\partial s(\hat{\alpha})}{\partial \hat{\alpha}} - \frac{\partial d(\hat{\alpha})}{\partial \hat{\alpha}} \right|_{\hat{\alpha}=1} > 0,$$

as $\frac{\partial \pi(s)}{\partial s} \frac{\partial s(\hat{\alpha})}{\partial \hat{\alpha}} < 0$. This proves that when there is full licensing, the entrant uses all the modules.

Now consider partial licensing. Remark that when $\alpha \geq \hat{\alpha}_1$, the entrant uses only $\hat{\alpha}_1$ modules, i.e., $\hat{\alpha}^*(\alpha) = \hat{\alpha}_1$. This implies that $\bar{\Pi}_1(\alpha)$ is constant in this range, and hence it suffices to determine the optimum of $\bar{\Pi}_1(\alpha)$ for $\alpha \leq \hat{\alpha}_1$. For $\alpha \leq \hat{\alpha}_1$, we have $\hat{\alpha}^* = \alpha$, hence the entrant uses all the modules in the license. The optimal size of the license, α^* , is a solution to

$$\frac{\partial(2\pi(s))}{\partial s} \frac{\partial s(\alpha)}{\partial \alpha} - \frac{\partial d}{\partial \alpha} = 0. \quad (6)$$

Since $\hat{\alpha}_1$ satisfies

$$\frac{\partial \pi(s)}{\partial s} \frac{\partial s(\hat{\alpha})}{\partial \hat{\alpha}} - \frac{\partial d(\hat{\alpha})}{\partial \hat{\alpha}} = 0,$$

we have

$$2 \left. \frac{\partial \pi(s)}{\partial s} \frac{\partial s(\alpha)}{\partial \alpha} - \frac{\partial d(\alpha)}{\partial \alpha} \right|_{\alpha=\hat{\alpha}_1} < 0.$$

This implies that $\Pi_1(\alpha, f)$ is strictly decreasing at $\hat{\alpha}_1$ in α . Since $\Pi_1(\alpha, f)$ is concave in α , this proves that $\alpha^* < \hat{\alpha}_1$. Therefore, α^* is the global optimum for the innovator's profit. Furthermore, when the incumbent provides α^* modules, since $\alpha^* < \hat{\alpha}_1$, the entrant uses all of it, i.e., $\hat{\alpha}^* = \alpha^*$. ■

What we have just shown is, the entrant prefers to use more modules than what is licensed, i.e., $\hat{\alpha}_1 > \alpha^*$. This is because when the innovator sets α^* , it

¹⁷If the entrant had no other entry option other than licensing, before concluding for the optimal licensing strategy, one would need to compare the innovator's monopoly profits (no licensing) to its duopoly profits (with licensing).

internalizes the effect of its decision on the entrant's gross profits through the license fee, and hence is concerned with the negative marginal effect of α on the *industry* profits. The entrant encounters this effect only on its gross profits, therefore, it has a preference over a larger-sized license than the innovator.

Proposition 1 *There is partial licensing at the equilibrium, that is, $\alpha^* \in (0, 1)$ if and only if*

$$\left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=1} < 0 < \left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=0}.$$

Proof. $\partial \bar{\Pi}_1(s(\alpha))/\partial \alpha$ is determined in (6), and its sign is indeterminate since we have $\partial s(\alpha)/\partial \alpha < 0$, $\partial \pi(s)/\partial s \geq 0$ and $\partial d/\partial \alpha \leq 0$. Since $\partial \bar{\Pi}_1(s(\alpha))/\partial \alpha$ decreases with α (implied by the strict concavity of $\bar{\Pi}_1$) we have

$$\left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=1} < \left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=0}.$$

We may find ourselves in one of the three following cases:

1. If $\left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=1} \geq 0$, when α increases, the rate at which industry profits decrease is always lower than the rate at which the development cost decreases. Then, $\alpha^* = 1$, i.e., there is *full licensing*.
2. If $\left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=0} \leq 0$, when α increases, the rate at which industry profits decrease is always higher than the rate at which the development cost decreases. Then, $\alpha^* = 0$, i.e., there is *no licensing*.
3. If $\left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=0} > 0 > \left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=1}$, then the profit of Firm 1 is maximized at $\alpha^* \in (0, 1)$, which implies that there is *partial licensing*.

■

In our basic static model, we did not consider how licensing could affect the timing of entry, and in return how this could influence the licensing strategy of the innovator. Time-to-market for new products is another important component that characterizes competition. With the following exercise, we introduce a timing game for the entrant in order to investigate this issue.

Time-to-Market The size of the shared component of the licensed innovation may affect the time-to market (the timing of entry), which introduces an additional trade-off the innovator faces when it determines its licensing strategy.

We assume that the innovator launches its product as soon as product innovation takes place. However, the entrant needs to develop its product before commercializing it. Acquiring a license from the innovator accelerates the introduction of its product, since it entitles him with an access to α components of the product and it only remains to develop $(1 - \alpha)$ modules. The entrant pays the license fee when it acquires the license, and it incurs the development

cost at the beginning of the time. We analyze this case by assuming that the entrant uses all the modules for its product design when it acquires the license. In Appendix B, we prove that this is indeed the case.

Formally, assume that when the entrant uses α modules of the innovator's product, the entrant introduces its product at $T(\alpha)$, with $T(\alpha) \geq 0$ and $\partial T(\alpha)/\partial\alpha \leq 0$. Hence, full licensing allows the entrant to introduce its product at the earliest possible date, whereas the time lag in introducing its product is highest with no licensing. Finally, let λ denote the discount factor.

When the innovator licenses α modules, it obtains a discounted profit of

$$\Pi_1(\alpha) = \int_0^{T(\alpha)} \pi_1^m e^{-\lambda t} dt + \int_{T(\alpha)}^{\infty} \pi(s(\alpha)) e^{-\lambda t} dt + f,$$

where π_1^m denotes the monopoly profit flows. If the entrant acquires the license, it enters in the market at $T(\alpha)$, and obtains

$$\Pi_2(\alpha) = \int_{T(\alpha)}^{\infty} \pi(s(\alpha)) e^{-\lambda t} dt - d(\alpha) - f.$$

whereas when it develops its product from scratch it enters in the market at $T(0)$ and obtains

$$\tilde{\Pi}_2(\alpha) = \int_{T(0)}^{\infty} \pi^d(0) e^{-\lambda t} dt - d(0).$$

The innovator's problem is the following

$$\max_{\alpha, f} \Pi_1(\alpha)$$

subject to

$$\tilde{\Pi}_2(\alpha) \leq \Pi_2(\alpha),$$

which can be rewritten as

$$f \leq \int_{T(\alpha)}^{\infty} \pi(s(\alpha)) e^{-\lambda t} dt - d(\alpha) - \left(\int_{T(0)}^{\infty} \pi(0) e^{-\lambda t} dt - d(0) \right).$$

For any α , the innovator's profit is increasing with f , hence the constraint is binding and the innovator maximizes the following

$$\bar{\Pi}_1(\alpha) = \int_0^{T(\alpha)} \pi_1^m e^{-\lambda t} dt + \int_{T(\alpha)}^{\infty} 2\pi(s(\alpha)) e^{-\lambda t} dt - d(\alpha) - \left(\int_{T(0)}^{\infty} \pi(0) e^{-\lambda t} dt - d(0) \right).$$

The marginal effect of α on the innovator's profit is composed of two terms:

$$\left\{ \frac{e^{-\lambda T(\alpha)}}{\lambda} \frac{\partial(2\pi)}{\partial s} \frac{\partial s}{\partial \alpha} - \frac{\partial d}{\partial \alpha} \right\} + \left\{ (\pi_1^m - 2\pi(s(\alpha))) e^{-\lambda T(\alpha)} T'(\alpha) \right\}.$$

The first term represents the variation of profit with respect to α when there is no time-to-market effect, i.e., when $T'(\alpha) = 0$. This term is similar to the one in the static model except that $\partial(2\pi)/\partial\alpha$ is multiplied by $e^{-\lambda T(\alpha)}/\lambda$, which depends on α . The higher the discount rate, λ , the lower $e^{-\lambda T(\alpha)}/\lambda$, hence the higher $\partial\bar{\Pi}_1/\partial\alpha$ (as $\partial(2\pi)/\partial s \times \partial s/\partial\alpha$ is negative). This means that a higher discount rate implies higher incentives to license. Indeed, while the innovator obtains the license fee at the beginning of time, entry has a negligible effect on discounted profits.

The second term represents the time-to-market effect. It has the sign of $\pi_1^m - 2\pi(\alpha)$. When $\pi_1^m < 2\pi(\alpha)$, this effect tends to increase the optimal α . Indeed, as entry increases industry profits, the innovator is willing to *accelerate* entry by increasing α . When $\pi_1^m > 2\pi(\alpha)$, this effect works in the opposite direction; the time-to-market effect tends to decrease the optimal α . This is because entry decreases industry profits, which provides the innovator with incentives to *delay* entry by decreasing α .

3 An Illustration

In this section we provide an example of a competitive setting, and, following the steps of the general proof, we show that partial equilibrium occurs at the equilibrium. Then, we study how the optimal licensing strategy of the innovator varies with the model parameters, and then we provide a numerical example. Finally, we study some extensions to the model.

Following Dixit (1979) and Singh and Vives (1984), let the utility of the representative consumer be defined by

$$u(q_1, q_2, y) = a(q_1 + q_2) - (q_1^2 + q_2^2)/2 - (1 - s)q_1q_2 + y, \quad (7)$$

where y is the numeraire good. Then the demand is

$$p_i = a - q_i - (1 - s)q_j,$$

where q_i and p_i denote the quantity and price of firm i , with $i, j = 1, 2$ and $i \neq j$, and $s \in [\underline{s}, \bar{s}]$ the degree of differentiation, with $\underline{s} \in [0, \bar{s})$ and $\bar{s} \in (0, 1]$. We assume that the degree of differentiation between the two final products is the sum of the degree of differentiation between the corresponding modules in each product. Given that there is a continuum of modules of mass 1, the degree of differentiation is

$$s = \int_0^1 \sigma(x) dx,$$

where $\sigma(x)$ is the degree of differentiation between the x th module of the entrant and that of the innovator. We assume that when firms have a common component α , the differentiation for those modules is \underline{s} . For the remaining modules, the degree of differentiation is \bar{s} .¹⁸ Therefore,

$$s = \underline{s}\alpha + \bar{s}(1 - \alpha), \quad (8)$$

¹⁸We do not formalize the differentiation stage. However, since we assume that there is no

and $\partial s/\partial\alpha < 0$ as in the general model. When $\alpha = 1$ (full licensing), there is minimum differentiation, and when $\alpha = 0$ (no licensing), there is maximum differentiation. We denote $\Delta s = \bar{s} - \underline{s}$. Remark that $\Delta s = -\partial s/\partial\alpha$, that is, Δs represents the marginal effect of the size of the license on the degree of differentiation.

Let the marginal development cost of the α th module be defined as

$$m(\alpha) = \frac{\delta}{n\alpha^{1-1/n}}.$$

with $\delta > 0$, and $n \geq 2$. Remark that the marginal cost of developing modules is decreasing. When the firm has α modules, the development cost of completing the product is

$$d(\alpha) = \int_{\alpha}^1 m(x) dx = \delta \left(1 - \alpha^{1/n}\right).$$

Once the licensing and entry decisions are made, firms compete in prices. For simplicity, assume that firms have symmetric unit costs of production, c .

In this setting, duopoly profits gross of development cost are defined as

$$\pi(s) = \frac{(a-c)^2 s}{(2-s)(1+s)^2}. \quad (9)$$

Entry with no licensing is viable if and only if $d(0) \leq \pi(\bar{s})$, and following the general setting, we assume that this is indeed the case.

The innovator has the following problem

$$\max_{\alpha} \bar{\Pi}_1(\alpha),$$

where $\bar{\Pi}_1(s)$ is defined in (5).

Proposition 2 *The optimal licensing strategy is partial licensing if*

$$\frac{\delta}{n} < 4(a-c)^2 \Delta s \gamma(\underline{s}), \quad (10)$$

with

$$\gamma(s) = \frac{s^2 - s + 1}{(1+s)^3 (2-s)^2},$$

and it is full licensing, otherwise.

Proof. Since the illustrative model, by construction, satisfies the properties of the general model, we basically proceed with the steps of the proof provided for the general model.

cost of differentiation, and since the gross duopoly profits are increasing with the degree of differentiation, given the innovator's product design, the entrant would indeed choose maximum differentiation for the remaining $(1-\alpha)$ modules, that is it would set $\sigma(x) = \bar{s}$ for all $x > \alpha$.

First, we find that $\bar{\Pi}_1$ is strictly concave for all α , if \bar{s} is not too high ($\bar{s} \in (0, 0.63)$). Indeed, we have $\partial^2 d / \partial \alpha^2 > 0$, and $\partial^2 s / \partial \alpha^2 = 0$, and for $\bar{s} \in (0, 0.63)$, we have $\partial^2 \pi / \partial \alpha^2 \leq 0$. Second, we have

$$\frac{\partial \bar{\Pi}_1}{\partial \alpha} = -4(a-c)^2 \Delta s \gamma(s) + \frac{\delta}{n} \alpha^{1/n-1},$$

with

$$\gamma(s) = \frac{s^2 - s + 1}{(1+s)^3 (2-s)^2},$$

and we find that $\left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=0} = +\infty$ and that $\left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=1} = -4(a-c)^2 \Delta s \gamma(\underline{s}) + \delta/n$. In the general model, we have shown that the optimal licensing strategy is partial licensing if and only if

$$\left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=1} < 0 < \left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=0}.$$

Since the inequality on the right hand side holds always, when the second order condition is satisfied for all α (for $\bar{s} < 0.63$), the optimal licensing strategy is partial licensing if and only if $\left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=1} < 0$, that is, if and only if

$$\frac{\delta}{n} < 4(a-c)^2 \Delta s \gamma(\underline{s}).$$

For $\bar{s} \in [0.63, 1]$, we show that the optimal licensing strategy is partial licensing if and only if inequality (10) holds (see Appendix C). This concludes that inequality (10) is the necessary and sufficient condition for partial licensing. If it does not hold, we have full licensing, since $\left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=1} \geq 0$. Furthermore, “no licensing” is not an equilibrium outcome since we always have $\left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=0} > 0$. ■

It is important to note that as we do not consider prohibitively high development costs such that the entrant obtains negative profits when it develops its own product from scratch, the analysis excludes the cases in which the innovator may choose not to license its innovation. The inequality (10) can be read in the following way.

- Partial licensing occurs when the development cost is not too high. When the development cost is sufficiently high, the innovator can extract larger rents by full licensing.
- Partial licensing occurs when the marginal effect of α on product differentiation, Δs , is sufficiently large. For small Δs , the innovator prefers full licensing. Consider the limit case $\underline{s} = \bar{s}$, in which this marginal effect is zero. Then, the innovator would grant the full license to the entrant. This is because whether the entrant develops its own product from scratch or acquires the license, the gross duopoly profits do not differ, as the degree of differentiation is the same in both cases. Since the innovator gets a license fee which increases with α , the innovator would set $\alpha = 1$.

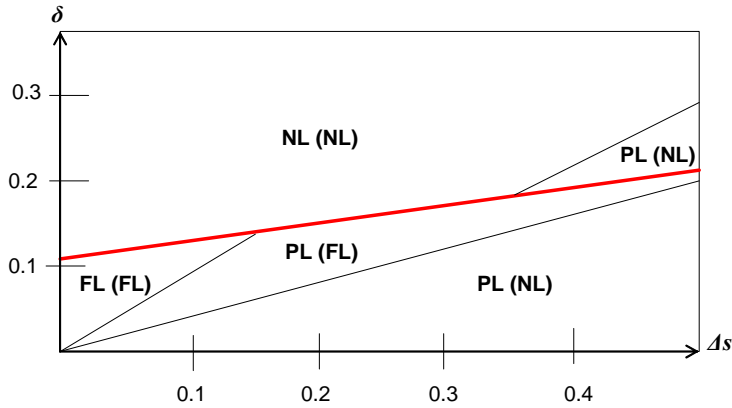
Proposition 3 *The size of the optimal license*

- (i) *increases with the development cost, δ , and*
- (ii) *decreases with the marginal effect of α on product differentiation (i.e., with Δs).*

Proof. See Appendix D. ■

We show that up to the threshold δ for which higher development costs leads to full licensing, the optimal α increases with the development cost. When the development cost is higher, the innovator can extract more rents from the entrant by licensing a larger portion of its innovation, and hence, when δ is sufficiently high, the innovator grants a full license to the entrant. At the same time, by choosing α , the innovator chooses the degree of competition. Hence, a higher adverse marginal effect of α on the degree of differentiation yields a lower equilibrium value of α . This concludes that the impact of partial licensing on softening competition through differentiation is an important determinant of the optimal licensing strategy.

A Numerical Example We now provide a numerical example, to illustrate how the licensing strategy of the innovator changes with respect to δ and Δs . We consider both cases when the entrant has an outside option and when it has not.



The Innovator's Equilibrium Licensing Strategy, for $a = 1$, $c = 0$, $\underline{s} = 0.35$, $n = 2$.

In Figure 1, NL, PL, and FL stand for no licensing, partial licensing and full licensing, respectively. The equilibrium strategy of the innovator when partial licensing is not a possibility is indicated in parenthesis. The thick line represents the threshold values of δ above which the entrant has no viable outside option. As it can be seen from the figure, for a given Δs , the entrant has a viable entry

option if the development cost is sufficiently low. Likewise, for a given δ , the entrant is more likely to have a viable outside option for higher values of Δs .

When partial licensing is a possibility, the set of parameters, for which there is no licensing at the equilibrium, shrinks. This is also true for full licensing. Partial licensing is likely to occur when Δs is sufficiently large and/or when δ is sufficiently small.

Partial Licensing and Social Welfare Now we briefly discuss the impact of modular product design, and hence the possibility of partial licensing, on social welfare. The utility function of the representative consumer is defined as in (7). Let I denote the initial endowment of the consumer of the numeraire good. Then, the consumer utility is $u(q_1, q_2, (I - p_1 q_1 - p_2 q_2))$. Replacing for the equilibrium prices and quantities yields the surplus of the representative consumer,

$$CS = \frac{(a - c)^2}{(2 - s)(1 + s)^2} + I.$$

Since the consumer surplus decreases with s , consumers are better off with a lower degree of differentiation, and hence with a higher α .¹⁹ This implies that consumers would prefer full licensing of the product innovation.

When partial licensing is not a possibility, we have argued that the innovator prefers no licensing for low development costs, and full licensing for high development costs. As it can be seen from Figure 1, when partial licensing is an option for the innovator, no licensing never occurs. This implies that for low development costs, partial licensing improves social welfare. However, for some range of higher development costs, for which the innovator would grant a full license in the absence of possibilities of partial licensing (and licenses its innovation partially otherwise), the total effect on social welfare is ambiguous. For this range of development costs, consumer welfare under partial licensing is lower whereas industry profits are higher compared to the case in which partial licensing is not possible.

Cournot versus Bertrand Competition The incentives regarding partial licensing differ under price and quantity competition, since licensing shapes competition through the degree of differentiation. The nature (and intensity) of competition in the underlying industry is important, since the innovator can soften competition by providing a smaller component of its innovation to the entrant.

In our competitive setting, we have assumed that once the decision for licensing and entry is made, firms compete in prices. In the following, we repeat the same exercise with quantity competition and compare the incentives for partial licensing under Bertrand and Cournot competition. Recall from equation (5)

¹⁹In this setting, consumers do not derive utility from variety, and hence, consumers are better off when products have a higher degree of substitutability.

that by licensing its innovation, the innovator obtains

$$\bar{\Pi}_1^\tau(\alpha) = 2\pi^\tau(s) - \pi^\tau(\bar{s}) + d(0) - d(\alpha),$$

with $\tau = B, C$ denoting Bertrand and Cournot competition respectively, with $\pi^B(s)$ defined in (9), and with $\pi^C(s) = (a - c)^2 / (3 - s)^2$. Assuming that the second order conditions are satisfied, the optimal licensing strategy is determined by $\partial \bar{\Pi}_1^\tau(\alpha) / \partial \alpha = 0$, which is equivalent to

$$2 \frac{\partial \pi^\tau}{\partial s} \frac{\partial s}{\partial \alpha} - \frac{\partial d}{\partial \alpha} = 0.$$

As $\partial \pi^B / \partial s \geq \partial \pi^C / \partial s$, we have $\partial \bar{\Pi}_1^B(\alpha) / \partial \alpha \leq \partial \bar{\Pi}_1^C(\alpha) / \partial \alpha$. Firms' gross profits are more responsive to differentiation under Bertrand competition than under Cournot competition. This is because, for a given degree of differentiation, Cournot competition is softer than Bertrand competition. Since a larger (smaller) common component implies less (more) differentiation, the innovator licenses a larger (smaller) portion of its innovation under Cournot (Bertrand) competition. Therefore, the incentives to license are higher with Cournot competition than with Bertrand competition.

Asymmetric efficiencies in production Asymmetries in production costs can also affect the innovator's licensing strategy. In particular, we look at the case where the incumbent is more efficient in production (the results would be reversed if the entrant were more efficient). Let c_i denote the constant marginal cost of production of firm i , with $c_2 \geq c_1$, and let π_i denote duopoly profits.

If the innovator licenses α modules of its product, it obtains

$$\bar{\Pi}_1(\alpha) = \pi_1(s) + \pi_2(s) - d(\alpha) - (\pi_2(\bar{s}) - d(0)).$$

The marginal effect of α on $\bar{\Pi}_1(\alpha)$ is

$$\frac{\partial \bar{\Pi}_1(\alpha)}{\partial \alpha} = \frac{\partial(\pi_1 + \pi_2)}{\partial s} \frac{\partial s}{\partial \alpha} - \frac{\partial d}{\partial \alpha}.$$

Since $\partial s / \partial \alpha$ is negative and $\partial(\pi_1 + \pi_2) / \partial s$ is positive, then the higher $\partial(\pi_1 + \pi_2) / \partial s$, the greater α . Simulating our example with both Bertrand and Cournot competition, we find that $\partial^2(\pi_1 + \pi_2) / \partial s \partial c_2 \leq 0$, which implies that the industry profits become less sensitive to product differentiation, when the entrant's marginal cost increases (for any given c_1). The reason is that efficiency differences mitigate the effect of product differentiation which is determined by the licensing strategy. Hence, the innovator licenses a larger component of its innovation when the entrant is less efficient.

4 Conclusion

We have shown that to the extent that the size of the common component in competing products determines the degree of differentiation, partial licensing can be used by a product innovator to shape post-entry competition. Our model predicts that partial licensing is more likely to occur when development cost are low and/or the marginal effect of the size of the license on the degree of differentiation is sufficiently large. The optimal size of the partition to be licensed increases with the development cost, and hence, for sufficiently high development costs, full licensing dominates partial licensing. We also show that when firms compete in quantities, the innovator has incentives to license a larger partition of its innovation. This is because under price competition, profits are more responsive to product differentiation. The same result holds when the innovator is more efficient in producing the good.

Partial licensing may also have important implications in terms of timing of entry. When entrants have access to a larger partition of a product innovation, it may require less time to develop and introduce their own product to the market. If the innovator discounts the future with a sufficiently low rate, this might lead it to license a smaller partition of its product.

In this paper we did not consider the possibility of imitation. Partial licensing arrangements might facilitate imitation compared to no licensing, since they transfer the product information *partially* to the entrant, which may reveal some information on modules that are not subject to the license. The entrant who infers this additional information may use it for imitating other modules of the innovator's product, instead of developing them independently. Our analysis shows that the entrant would have such an incentive (given that imitation is less costly than development), since the entrant prefers to have a larger common component with the innovator's product, compared to what it can with the size of the license. Hence, the innovator's strategy for partial licensing would be altered in the presence of imitation, in particular if possibilities of imitation are related to the size of the license.

We have restricted our attention to a duopoly setting. With a larger number of entrants the innovator is likely to license a smaller portion of its product, to soften competition through higher degree of differentiation. However, other interesting dynamics can arise with multiple entrants; for example, the innovator can adopt an exclusive licensing strategy and license out a larger portion of its product than it would under non-exclusivity, and deter entry if the development cost of the next potential entrant in line is sufficiently high.

Furthermore, different licensing regimes may lead to different size of licenses. For example, the innovator might license more modules of its product when it sets a per unit royalty, r , instead of a fixed fee. This is because a per unit royalty increases the marginal cost of the entrant and hence, the competitive advantage of the innovator may compensate it for licensing more modules, i.e., for a smaller degree of differentiation.

Last but not least, our simple theoretical framework can also be used for other product development related questions, e.g., cooperation in product R&D

(*how much* to cooperate: too much cooperation may lead to too similar products), product proliferation (*how different* the new product should be: it may more costly to introduce more differentiated products), and also for infrastructure development questions (*how much* to build and *how much* to buy: relying largely on the existing facilities may reduce possibilities for differentiation).

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Appendix

A

In this section, we first show that, given that there is licensing at the equilibrium, the size of the license is the same with and without the presence of an outside option. Then, we study the optimal licensing strategy of the innovator when the entrant has no outside option ($\tilde{\Pi}_2 < 0$), and compare it to the optimal licensing strategy when there is an outside option ($\tilde{\Pi}_2 \geq 0$).

If $\tilde{\Pi}_2 < 0$, the only possibility of entry is through licensing. If the innovator decides to license its product, it sets α such that its profits are maximized, subject to

$$f \leq \pi(s(\alpha)) - d(\alpha).$$

Then, we have

$$\bar{\Pi}_1(s(\alpha)) = 2\pi(s(\alpha)) - d(\alpha). \quad (11)$$

Remark that maximizing (11) with respect to α is equivalent to maximizing (5), since $\pi(\bar{s})$ and $d(0)$, that are in (5), are both independent of α . Therefore, given that there is licensing, when the entrant has no other entry option than licensing, the optimal α is the same as that of our main analysis.

Let π_1^m denote the monopoly profits. There are three cases.

Case 1 $\left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=1} > 0$: *There is either no licensing or full licensing.*

In Case 1, the optimal size of the license (if there is licensing) is 1. Hence, in this case the innovator compares its profits with no licensing, π_1^m , and full licensing, $2\pi(\underline{s})$. This proves that there is full licensing if and only if

$$2\pi(\underline{s}) \geq \pi_1^m, \quad (12)$$

and there is no licensing, otherwise.

Case 2 $\left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=0} < 0$: *There is no licensing.*

In this case when $\alpha = \epsilon$, with $\epsilon > 0$, and very small, we have

$$\bar{\Pi}_1 \leq 2\pi(\bar{s}) - d(0).$$

We have $\pi(\bar{s}) < d(0)$ by definition (no outside option), hence for $\alpha = \epsilon$, we have $\bar{\Pi}_1 < \pi(\bar{s})$. Since $\pi(\bar{s}) \leq \pi_1^m$, it proves that the innovator prefers no licensing.

Case 3 $\left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=1} < 0 < \left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=0}$: *There is either no licensing or partial licensing.*

If the innovator licences its product partially, it gets

$$2\pi(s(\alpha^*)) - d(\alpha^*).$$

First, in this case, we know that partial licensing is preferred to full licensing (since α^* remains unchanged as long as licensing is preferred to no licensing). Hence, we have

$$2\pi(s(\alpha^*)) - d(\alpha^*) \geq 2\pi(\underline{s}). \quad (13)$$

The innovator prefers partial licensing to no licensing if and only if

$$2\pi(s(\alpha^*)) - d(\alpha^*) > \pi_1^m. \quad (14)$$

Now we compare the innovator's licensing strategy with and without the outside option. In Case 2, we have no licensing, regardless of the presence of an outside option. In Case 1 and Case 3 we have licensing (full or partial) with outside option, whereas we may have no licensing when there is no outside option. This proves that incentives for licensing (either fully or partially) are lower when the entrant has no outside option.

Finally, note that, if partial licensing was not a possibility, for a wider range of parameters we would find ourselves under no licensing. This is because since we have inequality 13 satisfied, then the condition (12) implies the condition (14), whereas the reverse is not true.

B

To show that the entrant does not use less modules than what is provided by the innovator, let's assume that it is the entrant who chooses α .

Assuming that there is an interior solution, the optimal α for the entrant, $\hat{\alpha}^*$, satisfies

$$\frac{\partial \Pi_2}{\partial \alpha} = \frac{e^{-\lambda T(\alpha)}}{\lambda} \frac{\partial \pi}{\partial s} \frac{\partial s}{\partial \alpha} - \frac{\partial d}{\partial \alpha} - \pi(s(\alpha)) e^{-\lambda T(\alpha)} T'(\alpha) = 0$$

whereas, the optimal for the incumbent, α^* , satisfies

$$\frac{\partial \Pi_1}{\partial \alpha} = \frac{e^{-\lambda T(\alpha)}}{\lambda} \frac{\partial (2\pi)}{\partial s} \frac{\partial s}{\partial \alpha} - \frac{\partial d}{\partial \alpha} + (\pi_1^m - 2\pi(s(\alpha))) e^{-\lambda T(\alpha)} T'(\alpha) = 0.$$

We have

$$\frac{\partial \Pi_1}{\partial \alpha} = \frac{\partial \Pi_2}{\partial \alpha} + \frac{e^{-\lambda T(\alpha)}}{\lambda} \frac{\partial \pi}{\partial s} \frac{\partial s}{\partial \alpha} + (\pi_1^m - \pi(s(\alpha))) e^{-\lambda T(\alpha)} T'(\alpha),$$

and the second and third terms on the right-hand side of the equality are both negative, which implies that

$$\frac{\partial \Pi_1}{\partial \alpha} < \frac{\partial \Pi_2}{\partial \alpha}.$$

Since the optimal license for the incumbent satisfies

$$\left. \frac{\partial \Pi_1}{\partial \alpha} \right|_{\alpha=\alpha^*} = 0,$$

we have

$$\left. \frac{\partial \Pi_2}{\partial \alpha} \right|_{\alpha=\alpha^*} > 0,$$

which proves that $\hat{\alpha}^* > \alpha^*$.

C

With the following, we show that for all $\bar{s} \in (0, 1]$ and $\underline{s} \in [0, \bar{s})$, we have $\alpha^* \in (0, 1)$ (partial licensing) if and only if

$$\frac{\delta}{n} < 4(a-c)^2 \Delta s \gamma(\underline{s}). \quad (15)$$

To prove that, we proceed in four steps. First, we show that if the first-order condition is satisfied, the second-order condition is also satisfied. Second, we show that there exists some $\hat{\alpha}$ that satisfies the first-order condition if (15) holds. Third, we show that if there is partial licensing at the equilibrium, then (15) is satisfied. Fourth, we show that the solution to the first-order condition, if it exists, is unique.

1. The first-order condition for profit maximization is

$$4(a-c)^2 \Delta s \gamma(s) = \frac{\delta}{n} \alpha^{1/n-1}, \quad (16)$$

with

$$\gamma(s) = \frac{s^2 - s + 1}{(1+s)^3 (2-s)^2}.$$

Assume that $\hat{\alpha}$ is a solution of (16). The second-order condition for profit maximization is

$$\left. \frac{\partial^2 \bar{\Pi}_1}{\partial \alpha^2} \right|_{\alpha=\hat{\alpha}} \leq 0, \quad (17)$$

with

$$\frac{\partial^2 \bar{\Pi}_1}{\partial \alpha^2} = 12 \Delta s^2 (a-c)^2 \phi(s) - \delta \frac{n-1}{n^2} \alpha^{1/n-2},$$

and

$$\phi(s) = \frac{s^3 - 2s^2 + 4s - 2}{(1+s)^4 (2-s)^3}.$$

Using (16), the second-order condition becomes

$$3 \Delta s \frac{n}{n-1} \leq \frac{\gamma(s)}{\alpha \phi(s)}.$$

Replacing $\alpha = (\bar{s} - s) / (\bar{s} - \underline{s})$, we have

$$3 \frac{n}{n-1} \leq \frac{\gamma(s)}{(\bar{s} - s) \phi(s)}.$$

First, remark that the left-hand side decreases with n . Hence, if this condition is true for a given n , it is true for $n + 1$, which means that it is sufficient to show that it holds for $n = 2$. Second, the right-hand side decreases with \bar{s} , hence it suffices to show that the condition holds when $\bar{s} = 1$. We have

$$6 < \frac{\gamma(s)}{(1-s)\phi(s)},$$

or

$$\frac{(1-s)\phi(s)}{\gamma(s)} < \frac{1}{6}.$$

As the left-hand side is a function of s only, and s is bounded, we revert to a numerical analysis, which shows that this condition is always satisfied.

2. Now, we show that there exists some $\hat{\alpha}$ that satisfies (16) if (15) holds. Indeed, we have $\left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=0} > 0$ and $\left. \frac{\partial \bar{\Pi}_1}{\partial \alpha} \right|_{\alpha=1} < 0$ if (15) holds. Since $\partial \bar{\Pi}_1 / \partial \alpha$ is continuous, there exists $\hat{\alpha}$ such that (16) is satisfied.
3. With the following, we show that if there is partial licensing, then (15) holds. Assume that it is not true, i.e., that

$$\frac{\delta}{n} \geq 4(a-c)^2 \Delta s \gamma(\underline{s}).$$

Since there is partial licensing, then (16) holds, that is,

$$\frac{\delta}{n} = 4(a-c)^2 \Delta s \gamma(s) \alpha^{1-1/n},$$

therefore we should have

$$\gamma(s) \alpha^{1-1/n} \geq \gamma(\underline{s}).$$

Replacing for $\alpha = (\bar{s} - s) / (\bar{s} - \underline{s})$, we have

$$\gamma(s) (\bar{s} - s)^{1-1/n} \geq \gamma(\underline{s}) (\bar{s} - \underline{s})^{1-1/n}.$$

Let $Y(n, \bar{s}, s) = \gamma(s) (\bar{s} - s)^{1-1/n}$. The above condition can be rewritten as $Y(n, \bar{s}, s) \geq Y(n, \bar{s}, \underline{s})$. We are going to show that $Y(n, \bar{s}, s)$ is decreasing in s , hence that this condition is wrong. Remark that

$$Y(n, \bar{s}, s) = Y(2, 1, s) \times \frac{(1-s)^{1-1/n}}{(1-s)^{1-1/2}} \times \left(\frac{\bar{s}-s}{1-s} \right)^{1-1/n},$$

and that the second and third on the right-hand side are decreasing in s . Since $Y(2, 1, s)$ is a function of s only, we use a numerical analysis to show that $Y(2, 1, s)$ is strictly decreasing in s for $s \in [0, 1]$, which implies that $Y(n, \bar{s}, s)$ is decreasing in s . This proves that if there is partial licensing, then (15) holds.

4. The solution to (16) is unique as $Y(n, \bar{s}, s) = \gamma(s)(\bar{s} - s)^{1-1/n}$ is strictly decreasing in s .

D

Proof of Proposition 3.

- (i) Assume that there is partial licensing at the equilibrium. Then we have

$$\frac{\partial \bar{\Pi}_1}{\partial \alpha} = 0,$$

which implies

$$\frac{\delta}{4(a-c)^2 n \Delta s} = \frac{\gamma(s)}{\alpha^{1/n-1}}.$$

The left-hand side of this equality is independent of α , and it is increasing with δ . To show that α^* is increasing with δ , it suffices to show that the right-hand side of the equality is increasing with α . The derivative of the term on the right-hand side with respect to α is

$$\frac{\alpha^{-1/n}}{(1+s)^3 (2-s)^2} \left(\left(3\Delta s \alpha \frac{-s^3 + 2s^2 - 4s + 2}{(1+s)(2-s)} \right) + \left(1 - \frac{1}{n} \right) (s^2 - s + 1) \right).$$

For all $s < 0.63$ the above expression is positive, since we have $-s^3 + 2s^2 - 4s + 2 > 0$. It remains to verify that it is also positive for all $s \geq 0.63$. Remarking that

$$3\Delta s \alpha \frac{-s^3 + 2s^2 - 4s + 2}{(1+s)(2-s)} + \left(1 - \frac{1}{n} \right) (s^2 - s + 1)$$

is increasing with n , shows that if it is positive for $n = 2$, then it is positive for all $n \geq 2$. Replacing for $n = 2$ and for $\Delta s \alpha = \bar{s} - s$, this expression is positive if and only if

$$\bar{s} < \frac{(s^2 - s + 1)(1+s)(2-s)}{6(-s^3 + 2s^2 - 4s + 2)} + s = z(s).$$

This condition is always verified for $s \geq 0.63$, as $z(s) > 1$.

- (ii) Remark that $\partial s/\partial \alpha = -\Delta s$. To show that α^* is decreasing with Δs , we show that for a given \underline{s} , α^* is decreasing with \bar{s} , and that for a given \bar{s} , α^* is increasing with \underline{s} . Let

$$g(s) = \gamma(s) (\bar{s} - s)^{1-1/n},$$

and

$$h = \frac{\delta}{4n(a-c)^2 (\bar{s} - \underline{s})^{1/n}}.$$

$\partial \bar{\Pi}_1/\partial \alpha = 0$ implies $g(s) = h$. First remark that $g(s)$ is independent of \underline{s} , whereas h is increasing with \underline{s} , hence, everything else remaining constant, a larger \underline{s} implies an upwards shift in h , whereas $g(s)$ remains unchanged. Since $g(s)$ is decreasing with s , this implies a lower $s(\alpha^*)$, and hence a higher α^* . Second, remark that $g(s)$ is increasing in \bar{s} , whereas h is decreasing in \bar{s} . A larger \bar{s} implies an upwards shift in $g(s)$, and a downward shift in h , which in turn implies a higher $s(\alpha^*)$, and hence a lower α^* .

■