Costly Voting: Voter Composition and Turnout[*]

Jannet Chang[†]
Department of Economics, Virginia Tech

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Abstract

Given voting is costly, we analyze under what conditions rational agents vote and what kinds of agents vote. In a vote on public goods provision, we provide predictions about how idiosyncratic preferences for the public good affect agents' decision to vote. We show that people with stronger preferences for the public good vote, and that a voluntary voting mechanism results in too much participation in terms of social optimality. When voting is no longer binary, i.e., when voters have a continuum of alternatives to vote for, we show that full participation is possible, in contrary to the abstention result in binary voting models. In addition, we show that our model complements binary voting models from a mechanism designer's point of view. By choosing a proper model and tailoring feasible levels of the public good, it is possible to reach a more socially optimal voter turnout.

Keywords: Costly voting; Collective decision-making

JEL Classification: D71, D72

1 Introduction

It has long been considered a paradox that a rational agent chooses to vote voluntarily (Palfrey and Rosenthal, 1985; Downs, 1957; Riker and Ordeshook, 1968; Tullock, 1967; Feddersen, 1992). Given that the probability of being pivotal is small, a rational agent should not have an incentive to vote when voting is costly. However, people do vote. This paper attempts to provide a closer examination of why rational agents vote.

Our point of view is that the subject that agents vote upon matters to participation decisions. When an agent makes a decision whether or not to vote, he considers not only the cost of voting, such as the time taken to travel to the voting

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[†] Department of Economics, 3016 Pamplin Hall, Virginia Tech, Blacksburg, VA 24061. Email: jannet@vt.edu.

booth, but more essentially, the impact and consequence of the voting outcome. For example, in a vote on public good provision, the cost of providing a public good should be a voter's concern if he is subject to sharing the cost. In regard to costly voting, we thus differentiate two types of costs: the cost engaged in voting itself, such as time and effort to vote, and secondly, the cost associated with carrying out a particular voting outcome. Agents have heterogeneous preferences over outcomes and thus take this into account for their participation decision. This separation of cost is in contrast to the assumption in most of the costly voting literature, in which the cost of voting only relates to the voting itself.

We are interested in studying the voter turnout when both types of voting costs are present. We conclude that the choice set of all possible voting outcomes matters to voter turnout. In the voting literature, it is common to assume binary voting: there are only two alternatives to vote for. For example, agents vote for either Candidate A or B, or either undertaking a public project or not. We relax this assumption in this paper. The voting outcome can take on more than two values, in particular, any value in a given interval. This reflects a collective decision-making scenario where voters jointly decide the level of public goods provision. In other words, we endogenize the benefit of voting by viewing voting as a means for collective decision-making. This is particularly the case for small scale voting, such as in a school board or in many committees.

In a model where the utility derived from the public good can differ across agents, we provide predictions about how idiosyncratic preferences for the public good affect agents' decision to vote. Full participation is possible, in contrary to the prediction of abstention in most binary voting models. The coexistence of abstention and full participation in our model is due to the fact that there are more options to vote for. On the one hand, an agent's participation may make the collectively-chosen level of the public good closer to his most desired level. This makes participation attractive and thus full participation is possible. On the other hand, both pivotal effect and smaller expected difference from non-participation can make it unattractive to vote. The chance to be pivotal is small when more people vote. In addition, for a median type, the voting outcome may not be too far off from his ideal level if people with strong preferences, both liking and disliking the public good, participate. Since voting is costly, a median type does not necessarily find it attractive to vote. Abstention thus prevails.

Interestingly, when abstention occurs in our model, the abstention rate can be lower or higher than in a comparable binary voting model, depending on the assumption on the levels of the public good. Intuitively, this is due to a tradeoff a median type faces between participation and non-participation. Comparing with a binary model, the expected difference from non-participation is not as large because the level of the public good can take on various values, for example, any value between 0 and 1 instead of a 0-1 decision in a binary model. This makes participation less attractive for a median type. On the other hand, participation allows him to get closer to his ideal level. In this case, the tradeoff results in higher abstention rate in our model. However, by varying the feasible levels of the public good in the two models, we can reduce such expected difference from non-participation and obtain a converse conclusion. The two models thus complement each other from a policy implication standpoint. A mechanism designer can induce his desired voter turnout by tailoring feasible levels of the public good and picking the right model. For example, when voluntary participation results in over participation, as we find in this model, a social planner can choose a binary voting model and set proper feasible levels of the public good in the model to reach a more socially optimal voter turnout.

The paper is structured as follows. Section 2 and 3 introduce the model and preliminaries. We discuss full participation in Section 4 and characterize when abstention takes place in the following section. The optimality issue of the equilibrium is studied in Section 6. Lastly, we compare the voter composition in our model with the one in a binary voting model. We make a few concluding remarks in Section 8.

2 Preliminaries

There is a set of $N = \{1, 2, ..., n\}$ individuals. Each individual decides whether or not to participate in a collective decision-making that determines a level of a public good. Whether or not an agent is involved in the collective decision-making, he must pay for carrying out the provision of the public good.

The tradeoff for each agent is as follows: It is costly to participate in the collective decision-making, but if an individual participates, he may be able to influence the collective decision that affects the population at large. If an individual forgoes his right to vote and decides not to participate, he does not have to pay the participation fee but he must accept the collective decision made by the participants and pay an equal share of the amount necessary for carrying out the chosen level of the public good.

Denote by q the collectively chosen level of the public good. We assume that q can take on any value between 0 and 1, i.e., $q \in [0, 1]$. We assume a linear technology: the total production cost is equal to cq where c is the cost per unit. Furthermore, we assume equal sharing of the production cost. Let N be the population size. Equal sharing means that each agent pays $\frac{c}{N}$ per unit of q. Denote by k this amount of per capita unit cost of the public good; $k \equiv \frac{c}{N}$. Every agent must pay an amount of kq once q is chosen. Agents that decide to participate in the collective decision-making also pay an additional flat rate of participation, e. This fee can be viewed as efforts agents make to vote, such as time.

Each agent's preferences are characterized by his type, which measures how he likes the public good. Types are drawn from a uniform distribution with support on the interval, $[1, 2]$, which is common knowledge among agents. Each agent knows his own type but not others'. For a type θ_i agent, his utility function is

$$\begin{tikzcd}
M^- \arrow[d] & M^- \cup \{\theta_i\} \arrow[d] \\
\bar{a} \arrow[d] & \tilde{a} \arrow[d] \\
\bar{q} & \tilde{q}
\end{tikzcd}$$

Figure 1: Equilibrium maps.

quasilinear:

$$\begin{cases} v^i(\theta_i, q) - kq - e, & \text{if } \theta_i \in M; \\ v^i(\theta_i, q) - kq, & \text{if } \theta_i \notin M, \end{cases}$$

where M is the set of agents who participate in voting. The derivatives of v^i with respect to the two arguments are assumed to be positive: $v^i_{\theta_i} \geq 0$ and $v^i_q \geq 0$ for all i. The first property says that higher types, i.e. agents with larger value of θ_i, evaluate the public good more. The second inequality captures the idea that every agent likes the public good and that the more the public good, the better. Furthermore, we assume single-crossing preferences and diminishing marginal utilities, that is, $v^i_{\theta_i q} > 0$ and the second derivatives are non-positive: $v^i_{\theta_i \theta_i} \leq 0$ and $v^i_{qq} < 0$. The single-crossing property allows us to rank voters according to their individual types instead of their preferred policies; concavity implies that each type has a unique most-preferred level of the public good.

A type θ_i agent's most preferred level of the public good is equal to the level of q that maximizes his utility level for any given k. There is a one-to-one increasing map from types to the most preferred levels of the public good, $q^* : [1, 2] \to [0, 1]$, given by $q^*(\theta_i) = \arg\max \ v^i(\theta_i, q) - kq$. Given single-peaked preferences and a single dimensional issue, the median voter theorem implies that the voting outcome is the preferred level of the median type under majority rule. Therefore, if θ_i turns out to be the median type, $q^*(\theta_i)$ amount of the public good will be provided. Denote by M^- an arbitrary voter composition without type θ_i's participation. Furthermore, $\bar{a}$ denotes the median type of M^-. If a θ_i type agent participates in voting, the resulting median type is denoted by $\tilde{a}$. The corresponding chosen levels of q are $\bar{q}$ for M^- and $\tilde{q}$ for $M^- \cup \{\theta_i\}$, respectively. Figure 1 summarizes these notations.

3 The model and its equilibrium

We propose a two-stage game to study the voter composition when voting is costly. In the first stage of the game, agents decide simultaneously whether or not to participate in voting. In the second stage, the level of the public good, q, is determined by the participants through the simple majority rule. Given the assumption of single-peaked preferences, the majority rule implies that the voting outcome is the preferred level of the median type. In other words, in the second stage of the game, each participant announces his most preferred level of the public good and the median of the proposals wins.

It is worth noticing that no agents have incentives to deviate from reporting their true types, that is, truthful report is a dominant strategy for each participant. To see this, suppose an agent is a lower type than the median type. Since only the order of types matters to the determination of the median type, this low type agent can only alter the median if he reports a type higher than the median, given all other agents report their true types. But this results in a new median larger than the original median, deviating further away from his true type, and thus results in a lower utility. The same logic applies to the case of a high type. Therefore, agents report truthfully in the collective decision-making.

The collectively chosen level of the public good depends on the voter composition. Since the level of the public good matters to each agent's utility and no one knows other agents' types, each agent must form expectation about the voter composition before deciding whether or not to participate in voting. Each agent will only participate if his expected net gain from participating is positive. Let Δ^i denote agent i's net gain from participating. Then the equilibrium for the voting game is defined as follows:

Definition 1 *An equilibrium is defined by a subset of the type space, M, such that*

$$\begin{cases} \Delta^i \geq 0, & \text{if } \theta_i \in M; \\ \Delta^i < 0, & \text{if } \theta_i \notin M. \end{cases}$$

We are interested in characterizing such equilibria. In particular, we would like to know how the participation cost and cost of the public good affect the voter composition.

4 Conditions for full participation

When voting is binary, abstention appears in equilibrium (Ledyard, 1984; Palfrey and Rosenthal, 1985); there is always a set of agents abstaining when voting is costly. This is no longer true when voting is more than a yes-no decision.

We now show that full participation is possible in this model. With only two possible choices, moderates abstain because the cost effect dominates the pivotal effect– it is costly to participate and participation does not guarantee to change the outcome favoring himself. In the current model where the voting outcome can be any value between zero and one, the median's most preferred level is the winning proposal. Consequently, moderates have stronger incentives to participate since their chance of being the median is high. The pivotal effect can thus outweigh the cost effect. However, this does not imply that moderates would always want to participate. This is because they can take advantage of the fact that extreme types always participate. The types preferring a high level of the public good can be offset by the types preferring a low level of it. In the end, the median's favored level may not be too far off from a type that does not have strong preferences about the public good. Hence, such types, or moderates, may not find it worthwhile taking part in the collective decision-making.

To find conditions that yield full participation, we ask the following question: If all types of agents other than i participate, would agent i also participate? If we can find a condition where all agents are willing to participate, given all others participating, we have full participation. Normatively speaking, the condition for existence of such an equilibrium may suggest a way to encourage participation in a voting game, through a mechanism designer or a social planner's eye. In practice, voting is mandatory by law in countries like Italy, Belgium and Australia. A citizen is fined for not voting; the condition for full participation can provide an empirical suggestion as to setting such a penalty.[1]

First consider the case when $N = 3$. Let θ_1 and θ_2 be the types of the other two agents. Furthermore, let $\theta_{(1)}$ and $\theta_{(2)}$ denote these two types when order matters and $\theta_{(1)} \leq \theta_{(2)}$.

If a type θ_i does not participate in voting, the median type will be the average of the participating types, $\frac{\theta_1+\theta_2}{2}$. Since type θ_i does not know the values of the other two types, the median type is a random variable for a type θ_i agent. Given the probability density functions of θ_1 and θ_2, the expected utility when a type θ_i does not participate is equal to:

$$NJ^i \equiv \int_1^2 \int_1^2 \left[v^i \left(\theta_i, q\left(\frac{\theta_1+\theta_2}{2} \right) \right) - kq\left(\frac{\theta_1+\theta_2}{2} \right) \right] f(\theta_1) f(\theta_2)\ d\theta_1\ d\theta_2,$$

where $f(\theta_j)$ denotes the density function of θ_j, $j = 1, 2$.

If type θ_i participates, his expected utility depends on where he expects the other two types locate relative to his own type in the interval $[1, 2]$. There are three cases to be considered:

1. When $\theta_i \leq \theta_{(1)}$;
2. When $\theta_{(1)} \leq \theta_i \leq \theta_{(2)}$;

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[1] In this case, the penalty for not voting contributes negatively to e, the participation cost. In other words, such a penalty decreases the cost of voting.

3. When $\theta_i \geq \theta_{(2)}$.

The median type for each of the three cases is $\theta_{(1)}$, θ_i, and $\theta_{(2)}$, respectively. The payoff for the second case is straightforward because a type θ_i agent knows his own realization of type. In this case, the payoff is equal to $v^i\left(\theta_i, q(\theta_i)\right) - kq(\theta_i) - e$. Denote this amount by J_2^i. To find the expected payoffs for Case 1 and 3, we employ the order statistics to derive the associated probability density functions.

Definition 2 *Let X_1, X_2,..., $X_{\mathbb{N}}$ be $\mathbb{N}$ independent random variables, each with cumulative distribution function $F(x)$. If these $\mathbb{N}$ random variables are arranged in ascending order of magnitude and then written as*

$$X_{(1)} \leq X_{(2)} \leq ... \leq X_{(\mathbb{N})},$$

we call $X_{(i)}$ the i-th order statistic.

Lemma 1 *Let $X_{(r)}$ be the r-th order statistic when the sample size is $\mathbb{N}$. Let $F_r(x)$ denote the cdf of $X_{(r)}$. Then*

$$\begin{aligned} & Pr\{X_{(r)} \leq x\} \\ = \; & Pr\{\text{at least } r \text{ of the } X_i \text{ are less than or equal to } x\} \\ = \; & \sum_{i=r}^{\mathbb{N}} \binom{\mathbb{N}}{i} F^i(x)[1-F(x)]^{\mathbb{N}-i}. \end{aligned}$$

We now use the above formula to derive the distributions of the first and second order statistics. For the first order statistic X_1, the event that X_1 is less than or equal to x is the union of the following two disjoint events: 1) all the order statistics are less than or equal to x, and 2) all but one of the order statistics are less than or equal to x. By Lemma 1, $F_1(x) = 2F(x) - F^2(x)$ since $\mathbb{N} = 2$; the associated density function, $f_1(\theta_i)$, is equal to $2(1-F(x))f(x)$. Consequently, for the first case when $\theta_i \leq \theta_{(1)}$, the expected payoff for a type θ_i agent is equal to:

$$J_1^i = \int_{\theta_i}^{2} \left[v^i\left(\theta_i, q\left(\theta_{(1)}\right)\right) - kq\left(\theta_{(1)}\right) - e\right] \left[2(1-F(\theta_{(1)}))f(\theta_{(1)})\right] d\theta_{(1)}.$$

Similarly, for the second order statistic X_2, the event that X_2 is less than or equal to x is the same as the event that all the order statistics are less than or equal to x since $\mathbb{N} = 2$. Hence, $f_2(x) = 2F(x)f(x)$ and the expected payoff for the third case is equal to:

$$J_3^i = \int_{1}^{\theta_i} \left[v^i\left(\theta_i, q\left(\theta_{(2)}\right)\right) - kq\left(\theta_{(2)}\right) - e\right] \left[2F(\theta_{(2)})f(\theta_{(2)})\right] \; d\theta_{(2)}.$$

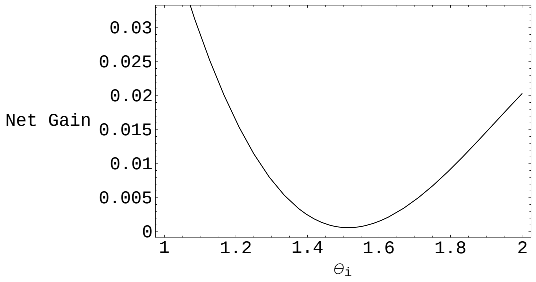

Figure 2: Net gain of participating when $e = 0.05$ and $k = 0.1$.

The overall expected payoff for participating is the weighted sum of J_l^i, $l = 1, 2, 3$. The weight is the probability associated with each case: $(2-\theta_i)^2, 2(\theta_i - 1)(2-\theta_i)$, and $(\theta_i - 1)^2$:

$$\begin{aligned}J^i \equiv \sum_{l=1}^{3} J_l^i &= 2\left(\theta_i - 1\right)\left(2-\theta_i\right)\left[v^i\left(\theta_i, q\left(\theta_i\right)\right) - kq\left(\theta_i\right) - e\right] + \\ &+ 2\int_{\theta_i}^{2}\left[v^i\left(\theta_i, q\left(\theta_{(1)}\right)\right) - kq\left(\theta_{(1)}\right) - e\right]\left(2-\theta_{(1)}\right)d\theta_{(1)} + \\ &+ 2\int_{1}^{\theta_i}\left[v^i\left(\theta_i, q\left(\theta_{(2)}\right)\right) - kq\left(\theta_{(2)}\right) - e\right]\left(\theta_{(2)} - 1\right)d\theta_{(2)}\end{aligned}$$

Let Δ^i denote the net gain of participating. Then $\Delta^i = J^i - NJ^i$. The necessary and sufficient condition for full participation is a relationship of e and k such that $\Delta^i \geq 0$ for all i.

Example 1 *First note that when the distribution of types is uniform in* $[1,2]$, $F(x) = \frac{x-\theta_i}{2-\theta_i}$ *for Case 1 and* $F(x) = \frac{x-1}{\theta_i - 1}$ *for Case 3.*

Consider a concave utility function, $v^i = q - \frac{q^2}{2\theta_i}$. *The first-order condition implies that the unique* $q^* = x(1-k)$ *if* x *is the median type. By substitution, we can compute the net gain from participating for a type* θ_i. *The condition for full participation will be a relationship of the two costs such that* $\min_{\theta_i} \Delta^i \geq 0$. *For the considered utility function, the minimal net benefit of participating is that of* $\theta_i = \frac{6+\sqrt{3\left(12-\sqrt{78}\right)}}{6}$ *for any given admissible* $k \in (0,1)$. *Figure 2 depicts one such case. Hence, the condition for full participation becomes*

$$e \leq \frac{\left(-\sqrt{312 - 26\sqrt{78}} + 2\sqrt{36 - 3\sqrt{78}} + 2\left(-7 + \sqrt{78}\right)\right)(-1+k)^2}{12\left(6 + \sqrt{36 - 3\sqrt{78}}\right)},$$

or approximately, $e \leq 0.00691258\,(-1+k)^2$. *Since* $0 < k < 1$, *which is implicitly imposed by the utility functional form, there is an upper bound for the participation cost. It is also worth noting that there exists a trade-off between the scale of* e *and* k. *For a larger amount of* k, *the participation cost has to be smaller to guarantee full participation.*

The same reasoning allows us to formulate the expected net benefit of participating for an arbitrary population size N. The question becomes: Given all other $N-1$ agents participating, what is type θ_i's expected benefit of participating? Same as in the three-agent case, a type θ_i agent forms his expectation by computing probabilities and expected outcomes for all possible combinations of types. For computation purposes, we rank types from low to high: $\theta_{(1)} \leq \theta_{(2)} \leq ... \leq \theta_{(N-1)}$. Furthermore, we denote by n the number of types smaller than θ_i; $n \leq N-1$. Figure 3 visualizes such a distribution.

Suppose first, that N is an even number. Consequently, $N-1$ is odd and the median of the $N-1$ agents is the $\frac{N}{2}^{th}$ type. There are two cases to be considered: $n < \frac{N}{2}$ and $n \geq \frac{N}{2}$. In the former case, the expected median type falls on the right hand side of θ_i. Hence, if the θ_i type agent abstains, the median is the $\left(\frac{N}{2}-n\right)^{th}$ order statistic in the interval $[\theta_i, 2]$. By applying the order statistics formula (1), the distribution function associated with the median type becomes

$$F_{E1} \equiv \sum_{i=\frac{N}{2}-n}^{N-n-1} \binom{N-n-1}{i} \left(\frac{x-\theta_i}{2-\theta_i}\right)^i \left[1-\frac{x-\theta_i}{2-\theta_i}\right]^{N-n-1-i}.$$

Thus, the expected utility from not participating is equal to

$$NJ_{E1} \equiv \int \left[v(\theta_i, q(x)) - kq(x)\right] dF_{E1}(x).$$

Because $n < \frac{N}{2}$, the θ_i agent's participation shifts the median to the left to become the average amount of the $\frac{N}{2}^{th}$ and $\frac{N}{2}-1^{th}$ types. His expected utility of participating becomes

$$J_{E1} \equiv \int\int \left[v\left(\theta_i, q\left(\frac{x+y}{2}\right)\right) - kq\left(\frac{x+y}{2}\right)\right] dF_{E1}(x) d\tilde{F}_{E1}(y),$$

if $n \neq \frac{N}{2}-1$.[2] The net gain from participating in this case is equal to $J_{E1} - NJ_{E1}$. If instead, $n = \frac{N}{2}-1$, then the agent expects himself to become one of the two types that determine the median. Thus the expected utility of participating is equal to

$$J_{E1'} \equiv \int \left[v\left(\theta_i, q\left(\frac{x+\theta_i}{2}\right)\right) - kq\left(\frac{x+\theta_i}{2}\right)\right] dF_{E1}(x).$$

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[2] $\tilde{F}_{E1}(y)$ denotes the distribution function of the $\left(\frac{N}{2}-n-1\right)^{th}$ order statistic. It is equal to $\sum_{i=\frac{N}{2}-n-1}^{N-n-1} \binom{N-n-1}{i} \left(\frac{x-\theta_i}{2-\theta_i}\right)^i \left[1-\frac{x-\theta_i}{2-\theta_i}\right]^{N-n-i}$.

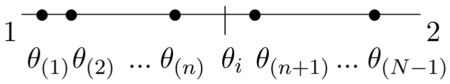

Figure 3: A distribution of types

The net gain in this case is equal to $J_{E1'} - NJ_{E1}$.

If instead, $n \geq \frac{N}{2}$, then abstention implies that the median is the $\frac{N}{2}^{th}$ order statistic in the interval $[1, \theta_i]$. Denote by F_{E2} its distribution function; it is equal to

$$F_{E2} \equiv \sum_{i=\frac{N}{2}}^{n} \binom{n}{i} \left(\frac{x-1}{\theta_i - 1}\right)^i \left[1 - \frac{x-1}{\theta_i - 1}\right]^{n-i}.$$

If the type θ_i agent participates and $n > \frac{N}{2}$, the median type moves to the right to become the average of the $\frac{N}{2}^{th}$ and $\left(\frac{N}{2}+1\right)^{th}$ order statistics. The net gain looks very similar to the case of $E1$; we denote it by $J_{E2} - NJ_{E2}$. In a special case when $n = \frac{N}{2}$, participation implies that the agent expects himself to become one of the two types that determine the median. Hence, his expected utility becomes

$$J_{E2'} \equiv \int \left[v\left(\theta_i, q\left(\frac{x+\theta_i}{2}\right)\right) - kq\left(\frac{x+\theta_i}{2}\right)\right] dF_{E2}(x).$$

Given the four possible outcomes discussed above, we can derive agent i's net gain from participating. Denote by $\Delta^i(N)$ the net gain of agent i when the population size is N. Then $\Delta^i(N)$ consists of N terms of products. Each product equals the likelihood of the event where n out of $N-1$ types are smaller than θ_i, times the net benefit associated with the event. For an event where exactly n types are smaller than θ_i, the binomial formula shows that the probability associated with it is

$$\mathbb{P}(n) \equiv \binom{N-1}{n} (\theta_i - 1)^n (2 - \theta_i)^{N-n-1}.$$

Hence, if N is even, then

$$\begin{aligned}\Delta^i(N) \quad = \quad & \sum_{n<\frac{N}{2}, n \neq \frac{N-1}{2}} \mathbb{P}(n)[J_{E1} - NJ_{E1}] + \sum_{n>\frac{N}{2}} \mathbb{P}(n)[J_{E2} - NJ_{E2}] \\ & + \mathbb{P}\left(\frac{N}{2} - 1\right)[J_{E1'} - NJ_{E1}] + \mathbb{P}\left(\frac{N}{2}\right)[J_{E2'} - NJ_{E2}]. \qquad (1)\end{aligned}$$

The reasoning for the case when N is odd is very similar. Note first that the median of $N-1$ types is the average of the $\frac{N}{2} - 1^{th}$ and $\frac{N}{2} + 1^{th}$ types if the agent abstains. There are again two cases to be considered: $n < \frac{N-1}{2}$ and $n \geq \frac{N-1}{2}$.

In the former case, the agent's participation shifts the median type to the left and the median becomes the $\frac{N}{2}-1^{th}$ type. Hence, if the type θ_i participates, the median becomes the $\left(\frac{N-1}{2}-n\right)^{th}$ order statistic in the interval $[\theta_i, 2]$, and the expected utility for participating is equal to

$$J_{O1} \equiv \int [v(\theta_i, q(x)) - kq(x)]\, dF_{O1}(x),$$

where F_{O1} is the distribution function of the $\left(\frac{N-1}{2}-n\right)^{th}$ order statistic. If the agent abstains, the expected median type is the average of the $\left(\frac{N-1}{2}-n\right)^{th}$ and $\left(\frac{N+1}{2}-n\right)^{th}$ order statistics. His expected utility function in this case becomes

$$NJ_{O1} \equiv \int\int \left[v\left(\theta_i, q\left(\frac{x+y}{2}\right)\right) - kq\left(\frac{x+y}{2}\right)\right] dF_{O1}(x)d\tilde{F}_{O1}(y),$$

where $\tilde{F}_{O1}(y)$ denotes the distribution function of the $\left(\frac{N+1}{2}-n\right)^{th}$ order statistic. The net gain of participating becomes $J_{O1} - NJ_{O1}$.

If $n \geq \frac{N-1}{2}$, we have two cases to consider: If the inequality is strict, i.e., $n > \frac{N-1}{2}$, and agent i abstains, then the median type is the average of the $\left(\frac{N}{2}-1\right)^{th}$ and $\left(\frac{N}{2}+1\right)^{th}$ order statistics in the interval $[1, \theta_i]$. If he participates, the median type shifts to the right to become the $\left(\frac{N}{2}+1\right)^{th}$ order statistic. Denote by J_{O2} and NJ_{O2} the expected utility functions associated with this case. If instead, $n = \frac{N-1}{2}$, participating implies that the agent expects himself to be the median type; denote by $J(\theta_i)$ the associated expected utility. In this case, abstaining implies that the median type is the average of the two closest types to the agent's type, one smaller and one larger than his own type. In particular, the smaller one is the n^{th} order statistic in the interval $[1, \theta_i]$ and the larger one is the 1^{st} order statistic in the interval $[\theta_i, 2]$. Denote by $NJ_{O2'}$ the associated expected utility. All cases considered, if N is odd, the net gain from participating is

$$\begin{aligned}\Delta^i(N) &= \sum_{n<\frac{N-1}{2}} \mathbb{P}(n)[J_{O1} - NJ_{O1}] + \sum_{n>\frac{N-1}{2}} \mathbb{P}(n)[J_{O2} - NJ_{O2}] \\ &\quad + \mathbb{P}\left(\frac{N-1}{2}\right)[J(\theta_i) - NJ_{O2'}]. \qquad (2)\end{aligned}$$

Given a level of N, agent i would participate in the collective decision-making if $\Delta^i(N)$ in either (1) or (2), depending on N, is greater than or equal to zero.

5 Conditions for abstention

We now examine the conditions for abstention. Since such equilibria also arise in a binary voting model, we are interested in comparing the equilibria and abstention rates in this model with the ones in a comparable binary voting model. We will establish the equilibrium conditions in this section and make the comparison in a subsequent section.

Table 1: Expected median types and their associated probabilities when the type θ_i is a high type

Case j	Participants' types	Probability (p_j)	x_j	y_j
$M^- = 0$				
Case 0		$(\beta - \alpha)^2$	$\frac{3}{2}$	θ_i
$M^- = 1$				
Case 11	$\theta_1 \in [1, \alpha]$	$2(\alpha - 1)(\beta - \alpha)$	θ_1	$\frac{\theta_1+\theta_i}{2}$
Case 12	$\theta_1 \in [\beta, \theta_i]$	$2(\theta_i - \beta)(\beta - \alpha)$	θ_1	$\frac{\theta_1+\theta_i}{2}$
Case 13	$\theta_1 \in [\theta_i, 2]$	$2(2 - \theta_i)(\beta - \alpha)$	θ_1	$\frac{\theta_1+\theta_i}{2}$
$M^- = 2$				
Case 21	$\theta_2 \in [1, \alpha]$	$(\alpha - 1)^2$	$\frac{\theta_1+\theta_2}{2}$	$\theta_{(2)}$
Case 22	$\theta_1 \in [\theta_i, 2]$	$(2 - \theta_i)^2$	$\frac{\theta_1+\theta_2}{2}$	$\theta_{(1)}$
Case 23	$\theta_1 \in [\beta, \theta_i]$, $\theta_2 \in [\beta, \theta_i]$	$(\theta_i - \beta)^2$	$\frac{\theta_1+\theta_2}{2}$	$\theta_{(2)}$
Case 24	$\theta_1 \in [1, \alpha]$, $\theta_2 \in [\beta, \theta_i]$	$2(\alpha - 1)(\theta_i - \beta)$	$\frac{\theta_1+\theta_2}{2}$	θ_2
Case 25	$\theta_1 \in [1, \alpha]$, $\theta_2 \in [\theta_i, 2]$	$2(\alpha - 1)(2 - \theta_i)$	$\frac{\theta_1+\theta_2}{2}$	θ_i
Case 26	$\theta_1 \in [\beta, \theta_i]$, $\theta_2 \in [\theta_i, 2]$	$2(\theta_i - \beta)(2 - \theta_i)$	$\frac{\theta_1+\theta_2}{2}$	θ_i

5.1 Moderates vs. extreme types

Proposition 1 *There does not exist an equilibrium where only moderates participate, i.e., there do not exist* α *and* β*, where* $1 < \alpha < \beta < 2$*, such that in equilibrium, only type* $\theta_i \in (\alpha, \beta)$ *participates.*

Proof: Suppose only moderates participate. Let θ^m denotes a moderate, i.e., $\theta_m \in (\alpha, \beta)$. Without loss of generality, assume the number of participants is an odd number. Since only moderates participate, the expected median, denoted by θ_a, should also lie in the interval of (α, β). Consequently, the collective chosen level of the public good is equal to $q^*(\theta_a)$. Let θ_{a-1} and θ_{a+1} denote the expected adjacent types to the median type, where $\theta_{a-1} \leq \theta_a \leq \theta_{a+1}$. Both adjacent types lie in the interval of (α, β). Now consider a high type, $\tilde{\theta} > \beta$. If he participates, θ_a is no longer the median. The new median type is the average of θ_a and θ_{a+1} and thus the level of the public good becomes the average of $q^*(\theta_a)$ and $q^*(\theta_{a+1})$, which is greater than $q^*(\theta_a)$ because $v_\theta \geq 0$ by assumption. Since $v_{q\theta} > 0$, the change in expected utility, v, is greater for a high type than a moderate. In other words, the expected net gain of participating for a high type is greater than any moderate. Therefore, a high type should participate when a moderate does. A similar argument shows that a low type should participate if a moderate does. This completes the proof. ■

Table 2: Conditional probability density functions for a high type agent.

Case j	$f(x_j)$	$f(y_j)$
0	-	-
11	$\frac{1}{\alpha-1}$	$\frac{1}{\alpha-1}$
12	$\frac{1}{\theta_i-\beta}$	$\frac{1}{\theta_i-\beta}$
13	$\frac{1}{2-\theta_i}$	$\frac{1}{2-\theta_i}$
21	$\frac{1}{(\alpha-1)^2}$	$2\,\frac{\theta_2-1}{(\alpha-1)^2}$
22	$\frac{1}{(2-\theta_i)^2}$	$2\,\frac{2-\theta_1}{(\theta_i-2)^2}$
23	$\frac{1}{(\theta_i-\beta)^2}$	$2\,\frac{\theta_2-\beta}{(\theta_i-\beta)^2}$
24	$\frac{1}{(\alpha-1)(\theta_i-\beta)}$	$\frac{1}{\theta_i-\beta}$
25	$\frac{1}{(\alpha-1)(2-\theta_i)}$	-
26	$\frac{1}{(\theta_i-\beta)(2-\theta_i)}$	-

By Proposition 1, we can characterize an abstention equilibrium by finding the boundaries of the two extreme types. Let α and β be such boundaries so a type in either $[1, \alpha]$ or $[\beta, 2]$ participates and a type in (α, β) abstains.

Consider $N = 3$ and the distribution of types are uniform in $[1, 2]$. For each agent i, there are ten cases to be considered. These cases vary depending on agent i's type since the median is determined by its relative standing to the value of θ_i. Furthermore, given extreme types always participate, we have two scenarios to consider: If agent i is a "high type" and if he is a "low type". In the former case, $\theta_i \in [\beta, 2]$ and in the latter case, $\theta_i \in [1, \alpha]$.

Suppose agent i is a high type. For each case j, denote by x_j the median type when agent i abstains and by y_j the median type when agent i participates.

For a high type agent, θ_i, the median types and their associated probabilities are summarized in Table 1. Since agent i does not know the median's type, he forms expectation on the type; the conditional probabilities of the median types are summarized in Table 2. The probabilities and probability density functions for when agent i is a low type are derived in a similar way. With these probabilities and probability density functions, we can find Δ^i, the expected net gain for agent i. If $\Delta^i \geq 0$, then agent i will participate in voting; he will not participate if the net gain is negative:

$$\Delta^i(\theta_i, \alpha, \beta; e, k) \quad = \quad \sum_j p_j \left(J^i_j - NJ^i_j \right),$$

where for each case j in Table 1,

$$J^i_j - NJ^i_j \quad = \quad \int_{\underline{y}_j}^{\bar{y_j}} \left[v(\theta_i, q(y_j)) - kq(y_j) - e \right] f(y_j) dy_j - \int_{\underline{x}_j}^{\bar{x_j}} \left[v(\theta_i, q(x_j)) - kq(x_j) \right] f(x_j) dx_j.$$

In equilibrium, both type α and β are indifferent between participating and not participating. Therefore, Δ^{α} and Δ^{β} are both equal to zero, which jointly determine the equilibrium (α^*, β^*).

The equilibrium boundaries, α^* and β^*, are determined by the cost structure. In other words, α^* and β^* are functions of e and k. However, there is a set of utility functions with a property that k has no effect on agents' participation decision. We would like to exclude such utility functions when examining the effect of k. However, this property of invariance in k simplifies the analysis on the impact of the participation cost, e. We will use such utility functions when they are applicable.

In the rest of the section, we first categorize the set of utility functions with the invariance property. Then we use a utility function with this property to obtain existence results.

5.2 Expected net benefits that are invariant in k

To find this special set of utility functions, note first that the expected net gain from participating depends on the specification of utility functions. This is because the equilibrium level of the public good is determined by the median type. If θ_i is the median type, his most-preferred level of q is the one that maximizes his utility, i.e., $q_i^* = \arg\max v_i(\theta_i, q) - kq$. In particular, q_i^* is such that $v_q^i = k$, i.e., when the marginal utility of the public good is equal to its marginal cost. Implicitly, k determines the level of q_i^* and thus the agent's utility level and net benefit of participating.

However, there is an exception. For a particular form of utility functions, k does not have any impact on the net benefit and thus agents' participation decision. This takes place when q^*, derived from the utility function, is linear in k. In this case, k has the same negative impact on all types' ideal levels of public good; every type scales down the desired level of the public good the same amount. Hence, a change in k does not alter the net benefit of participating. The following proposition formalizes such a case.

Proposition 2 *The per capita unit cost of the public good has no impact on the net benefit, or equivalently,* $\frac{\partial \Delta^i}{\partial k} = 0$ *for a type* θ_i *agent if*

$$\int \left[\frac{v_q^i(\tilde{q}) - k}{v_{qq}^{\tilde{a}}(\tilde{q})} - \tilde{q} \right] f(\tilde{a}) d\tilde{a} = \int \left[\frac{v_q^i(\bar{q}) - k}{v_{qq}^{\bar{a}}(\bar{q})} - \bar{q} \right] f(\bar{a}) d\bar{a}, \tag{3}$$

where $\tilde{q}$ *denotes the collectively-chosen level of the public good if a type* θ_i *agent participates and* $\bar{q}$ *denotes the level if he does not participate.*

In a special case when the second derivatives are identical across agents, i.e., $v_{qq}^i = v_{qq}^j = \lambda$ for $i \neq j$ and for all $q \in [0, 1]$, Equation (3) reduces to $\int \left[v_q^i(\tilde{q}) - k \right] f(\tilde{a}) d\tilde{a} - \int \left[v_q^i(\bar{q}) - k \right] f(\bar{a}) d\bar{a} = \lambda (\int \tilde{q} f(\tilde{a}) - \int \bar{q} f(\bar{a}))$. That is, the cost of the public good has no impact on the participation decision if the expected net change in the marginal utility of the public good is proportional to the change in the expected level of the public good.

Proof: The most-preferred level of the public good of a type θ_i agent is equal to q_i^* such that $v_q^i = k$. The total derivative of the equation implies that $\frac{dq}{dk} = \frac{1}{v_{qq}^i}$. Denote by $\bar{a}$ the expected median type if a type θ_i agent abstains and by $\tilde{a}$ the expected median if the agent participates. The derivative of B^i with respect to k is thus equal to

$$\begin{aligned}\frac{\partial \Delta^i}{\partial k} &= \int \left[v_q^i(\tilde{q}) \frac{\partial q}{\partial k}(\tilde{a}) - \tilde{q} - k \frac{\partial q}{\partial k}(\tilde{a}) \right] f(\tilde{a}) d\tilde{a} - \int \left[v_q^i(\bar{q}) \frac{\partial q}{\partial k}(\bar{a}) + \bar{q} + k \frac{\partial q}{\partial k}(\bar{a}) \right] f(\bar{a}) d\bar{a} \\ &= \int \left[\frac{v_q^i(\tilde{q}) - k}{v_{qq}^{\tilde{a}}(\tilde{q})} - \tilde{q} \right] f(\tilde{a}) d\tilde{a} - \int \left[\frac{v_q^i(\bar{q}) - k}{v_{qq}^{\bar{a}}(\bar{q})} + \bar{q} \right] f(\bar{a}) d\bar{a}.\end{aligned}$$

By setting the above equation to be zero, we derive the relationship in 3. ■

Example 2 *Consider the utility function,* $v^i(\theta_i, q) = 2\theta_i q - q^2$ *for all* i*. It satisfies the relationship in (3). Furthermore, this utility function is also an example for the special case discussed above; the second derivatives,* v_{qq}^i*, are the same across agents.*

5.3 Abstention equilibrium and comparative statics

In this subsection, we use the utility function in Example 2 to examine equilibrium conditions and properties. This utility function simplifies the analysis significantly while maintaining the main features of equilibria derived from more complicated utility functions. We will use this utility function in later sections for this purpose.

Recall that (α^*, β^*) is determined by zero expected net benefits of type α and type β:

$$\begin{aligned}\Delta^{\alpha}(\alpha, \beta; e, k) &= 0 && (4) \\ \Delta^{\beta}(\alpha, \beta; e, k) &= 0. && (5)\end{aligned}$$

Since the net benefit derived from the utility function in Example 2 does not depend on k. Δ^{α} and Δ^{β} are specified by e only.

Figure 4 illustrates equations (4) and (5) when $e = 0.1$. Equilibria are where the two curves cross in the upper triangle since $\alpha < \beta$. As seen on the graph, there exist three equilibria, one with abstention and the other two with full participation.[3]

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[3] The numerical solutions are $(\alpha^*, \beta^*) = (1.16701, 1.16701)$, $(1.83299, 1.83299)$, and $(1.33385, 1.66615)$.

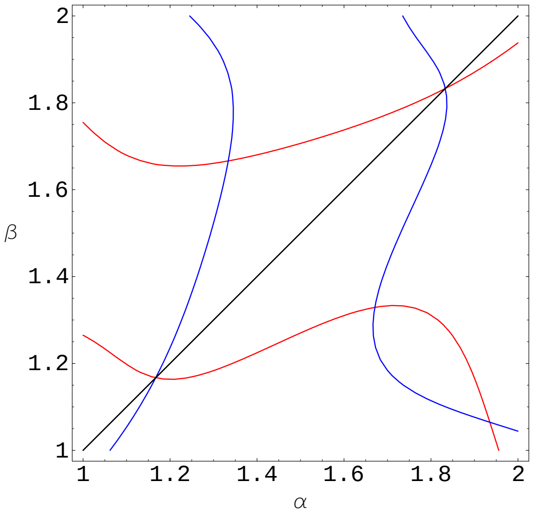

Figure 4: Equilibria for the case when $e = 0.1$. Equilibria are points where the two curves cross.

5.3.1 Participation cost

For abstention equilibrium, e can not be too close to zero for existence. If e were equal to zero, every agent would be indifferent between voting and not voting and thus we would not necessarily have an abstention equilibrium. In this section, we also demonstrate that the higher the participation cost, the higher the rate of abstention.

Proposition 3 *There exists an abstention equilibrium only if* $e \geq \frac{1}{24}$.

Proof: By definition, $\beta > \alpha$; we can rewrite β so that $\beta = \alpha + \epsilon$, $\epsilon > 0$. By substituting this expression in both Equation 4 and Equation 5 and solving for α and e, we get

$$\begin{aligned} \alpha &= \frac{3-\epsilon}{2}; \\ e &= \frac{-1-4\,\epsilon-6\,\epsilon^2-8\,\epsilon^3+7\,\epsilon^4}{24\,(-1-2\,\epsilon+3\,\epsilon^2)}. \end{aligned} \tag{6}$$

Since $\alpha \in [1,2]$, $\epsilon \in [0,1]$. e is an increasing function on this domain of ϵ; Figure 5 illustrates the relationship between e and ϵ in equilibrium. e has a minimum of $\frac{1}{24}$ when $\epsilon = 0$. In other words, if there exists an abstention equilibrium, e has to be greater than or equal to $\frac{1}{24}$. ■

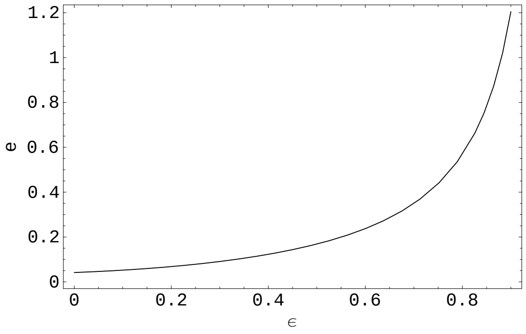

Figure 5: The relationship between e and the equilibrium abstention rate, ϵ.

Note that the denominator of Equation 6 becomes zero when $\epsilon = 1$, or when no one participates. e is infinitely large in this case. Were there 90% abstention in equilibrium, e was around 1.205. In addition, as Figure 5 shows, the equilibrium map between e and ϵ is one-to-one. Therefore, for any given value of e, there exists a unique abstention equilibrium.

Proposition 4 *As the participation costs increases, the abstention rate increases, i.e.,* $\frac{\partial \epsilon}{\partial e} \geq 0$.

Proof: Rewrite Equation 6 to have $G(e, \epsilon) \equiv e - \frac{-1-4\,\epsilon-6\,\epsilon^2-8\,\epsilon^3+7\,\epsilon^4}{24\,(-1-2\,\epsilon+3\,\epsilon^2)} = 0$. Since $0 < \epsilon < 1$, G is a C^1 function on this domain of ϵ. The conditions of the implicit function theorem are met. Hence,

$$\begin{aligned}\frac{\partial \epsilon}{\partial e} &= -\frac{\frac{\partial G}{\partial e}}{\frac{\partial G}{\partial \epsilon}} \\ &= \frac{12\left(1 + 2\,\epsilon - 3\,\epsilon^2\right)^2}{1 + 9\,\epsilon + 24\,\epsilon^2 + 2\,\epsilon^3 - 33\,\epsilon^4 + 21\,\epsilon^5} > 0. \blacksquare\end{aligned}$$

5.3.2 Cost of the public good

The utility function in Example 2 only allows us to study the effect of e. To examine the effect of k, the cost of the public good, we consider a variation of the utility function in Example 2, $v^i = q - \frac{q^2}{2\,\theta_i}$. Equilibria derived from this utility function have a very similar feature to those in the previous case; both abstention and full participation take place in equilibrium. In the neighborhood of an abstention equilibrium, we observe that as k increases, the abstention rate increases as well. Figure 6 illustrates such a case: Original equilibria are where the two solid lines cross and new equilibria are where the two dashing lines cross. On the graph, the new abstention equilibrium is in the northwest of the old abstention

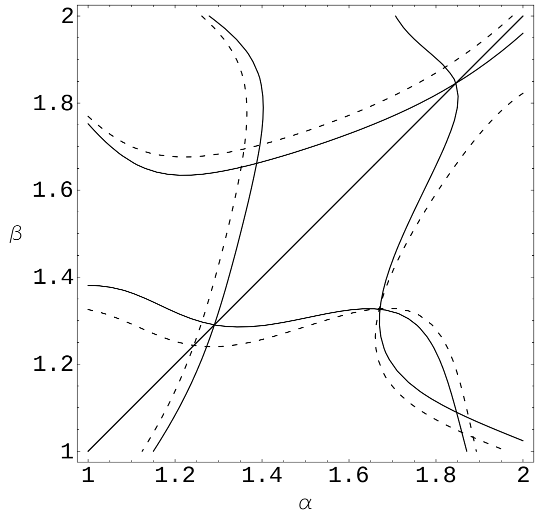

Figure 6: An example of the effect of k on equilibria. $e = 0.4$ and k changes from 0.01 to 0.012.

equilibrium; being further away from the 45-degree line means that the abstention rate is higher.

6 Welfare analysis

Agents' participation decisions are incentive driven and thus the equilibrium outcome may not be socially optimal. We construct a utilitarian social planner's problem to examine the optimality issue. If there were a benevolent social planner that cares about the well-being of everyone in the society, his goal would be to choose a voter composition so that it maximizes the total expected welfare; expectation is due to the fact that he does not know any agent's type. The result we find is that there should be fewer people participating in voting; voluntary participation results in too much participation and thus too much waste of social resources. This result is consistent with the finding in a binary voting game (Chang, 2004).

The utilitarian social planner solves the following problem:

$$\begin{aligned}
&\max_{\alpha,\beta} S \equiv \sum_{i=1}^{3} E[v^i(\theta_i, q(x)) - kq(x)] - eE(\mathbb{M}) \\
= \;&\max_{\alpha,\beta} \sum_{j=1}^{10} \mathbb{P}_j \left\{ \sum_{i=1}^{3} \int\int \left[v_j^i(\theta_i, q_j(x)) - kq_j(x)\right] f_j(x) g_j^i(\theta_i)\; dx\; d\theta_i - e\mathbb{M}_j \right\},
\end{aligned}$$

k	α	β	$\beta - \alpha$
0.1	1.14849	1.68965	0.54116
1	1.02461	1.49363	0.46902
1.2	1.01576	1.45956	0.4438
1.5	1.00487	1.40548	0.40061
1.7	1.00234	1.36907	0.36673

Table 3: Socially optimal levels of participation and abstention for $e = 0.1$.

where $\mathbb{M}$ denotes the number of participants. There are ten cases to be considered when $N = 3$. For example, when $\mathbb{M} = 2$, the participants can both be high types, low types, or one be low type and the other be high type. We index each case by j. $\mathbb{P}_j$ denotes the probability associated with a particular voter composition j. The socially optimal pair of (α, β) is determined by the first-order conditions of the maximization problem.

By substituting with the utility function, $v^i = q - \frac{q^2}{2\theta_i}$ in Example 2, the first order conditions of the maximization problem can be simplified. For $k = 1.2$ and $e = 0.1$, the socially optimal abstention rate should be 44.38% while voluntary participation results in only 33.23% of abstention[4]. Generally speaking, since e only appears in the second term of the maximand, S, a higher participation cost is associated with lower participation. However, k can have either positive or negative impact on the abstention rate. For example, at the socially optimal level, $(1.01576, 1.45956)$, when $e = 0.1$ and $k = 1.2$, a higher level of k should result in more participation of the high type and more overall participation, i.e., $\frac{\partial S}{\partial k} > 0$. However, when $e = 0.1$ and $k = 0.1$, the derivative becomes negative at equilibrium, $(1.14849, 1.68965)$. A higher level of k should associate with less participation in a social optimum. Table 3 provides a few more numerical examples.

7 Comparison with the binary voting model

We will use the utility function given in Example 2 to illustrate how equilibria differ when we relax the assumption of the voting outcomes, from a yes-or-no decision to a choice from a scale. The findings is that the assumption of the voting outcomes matters to the voter turnout. In addition, by setting the scales of the voting outcomes differently, the two models can complement each other in order to amend the over-participation problem we find in the previous section.

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[4] The abstention rate is equal to $\beta - \alpha$. The socially optimal levels of α and β are 1.01576 and 1.45956, respectively. The equilibrium derived from voluntary participation is $(\alpha^*, \beta^*) = (1.33385, 1.66615)$, in which case, both types participate in voting more than in the socially optimal case.

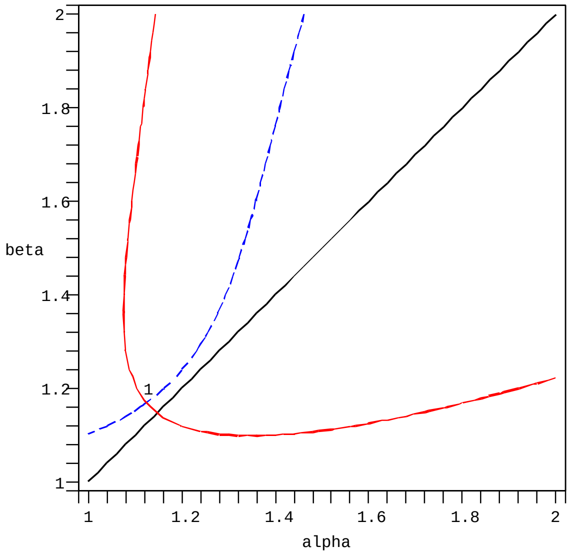

Figure 7: Equilibria for the case when $q \in \{0,1\}$. Equilibria are points where the two curves cross.

The following example shows that the assumption on the voting outcomes matters for voter composition. Suppose $N = 3$. Let $e = 0.1$ and $k = 1.2$. As shown before in Figure 4, for the case when $q \in [0,1]$, there are two equilibria, one with abstention and the other with full participation. If voting were binary, i.e. $q \in \{0,1\}$, there exists a unique equilibrium, where abstention presents, $(\alpha^*, \beta^*) = (1.1243, 1.1684)$. Figure 7 illustrates such a case. This example shows that the generalization of the assumption that allows voting outcomes to take on more than two possible values results in different voter composition. We can have full participation if voting is no longer binary.

It is interesting to note though, when comparing the abstention equilibrium in the two cases, a binary voting game results in more participation. In the binary case, the probability of abstention is equal to roughly 4% but it becomes 33% when there are more possible voting outcomes. To reach the same amount of abstention, the participation cost has to be reduced to roughly 0.045, more than one forth of the cost in a binary voting model.

The reasoning behind this phenomenon is due to the expected difference between participation and non-participation for a median type. In a binary model, if a median type does not participate, the outcome is either a one or a zero, with a difference of about 0.5 from his ideal level. However, the expected difference is smaller in the model where voting outcomes can take on any value between

0 and 1. On the one hand, this reduces the median type's incentive to participate. On the other hand, participation allows him to get closer to his ideal level. This creates a tradeoff for a median type. Given that the binary model results in more participation, a social planner could adopt a scale model instead to reduce participation so the voter turnout is closer to a social optimum.

However, if we allow levels of the public good to be more extreme in a scale model than those feasible in a binary model, we obtain a converse result. For example, if instead, the levels of the public good are either $\frac{1}{4}$ or $\frac{3}{4}$ in the binary voting model while $q \in [0, 1]$, as before, in the scale model, then abstention rate is higher in the binary voting model.[5] This is because the expected difference from non-participation is no longer reduced in the scale model. Participation becomes more attractive to a median type. In this case, from a social optimal point of view, a social planner could tailor scales and adopt a binary voting model in order to reach a more optimal voter turnout.

8 Conclusion

In a costly voting model with asymmetric information, we study when and what types of agents choose to vote voluntarily. We take on the subject with two different views. First of all, we distinguish two types of voting costs, one explicit and the other intrinsic. For example, in a vote on provision of public goods, agents care not only the time and effort to vote, but also the embedded cost that they must share for carrying out a collectively chosen level of the public good. In a model where the utility derived from the public good can differ across agents, we show that people with stronger preferences always vote. In addition, low turnout is desired from an efficiency point of view.

Secondly, in contrast to most of the voting models in the costly voting literature, we study the case when voting is not binary. This is especially the case for small scale voting. In a committee, voting is used as a way to aggregate preferences in order to select a level of production or provision, instead of simply picking yes or no for a given proposal. We show that full participation can arise when voting is no longer binary. For a concern of political equality, full participation may be desirable. For example, countries like Italy and Belgium have long made voting mandatory. In this case, the condition for full participation may provide an empirical suggestion as to setting the penalty for not voting.

In addition, we show that a binary model and a non-binary model can complement each other from a social optimal point of view. Both models result in over participation but one model performs better than the other under different scale assumptions. By tailoring scales and picking the right model, we could reach a more socially-desired voter turnout.

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[5] Consider a previous example where $e = 0.1$ and $k = 1.2$. If $q \in \{\frac{1}{4}, \frac{3}{4}\}$, the abstention equilibrium in the binary model, (α, β), becomes $(1.0505, 1.0812)$. The abstention rate is significant smaller than the one in the scale model.

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