Dynamic Lending vs. Group Lending in the Ghatak/Stiglitz/Weiss adverse selection model

Christian Ahlin

Brian Waters*

[PRELIMINARY AND INCOMPLETE]

August 2009

Abstract

We compare the efficiency of group lending and dynamic, individualized lending in a simple adverse selection model. We find that the information revelation of both types of unconstrained contracts is similar, but the assumed constraints limit the usefulness of the information revealed in different ways. As a result, group lending can achieve full efficiency under some conditions where dynamic lending fails to. However, when neither type of lending achieves full efficiency, dynamic lending can outperform group lending, by charging high interest rates to borrowers who have defaulted.

*We thank Quan Wen and participants at the Frontiers in the Theory of Economic Development Conference, Mexico 2008. All errors are our own.

1 Introduction

Group lending has been proven theoretically capable of improving the functioning of credit markets among poor borrowers lacking collateral. For example, in a variant of the adverse selection context analyzed by Stiglitz and Weiss (1981), Ghatak (1999, 2000) shows that the propensity of borrowers to group homogeneously by risk can allow a lender to include less risky borrowers in the market.

While group lending is still widely used by micro-lenders, there is speculation that its popularity is waning in favor of individual loan contracts. Indeed, drawbacks of group lending are apparent even in the models justifying it, for example, its increased risk (Stiglitz, 1990) and contagion properties (Besley and Coate, 1995). Further, it seems clear that in some contexts microfinance is possible without group lending, since a number of apparently leading micro-lenders lend exclusively to individuals (e.g. Bank Rakyat Indonesia); see also the evidence in Gine and Karlan (2006). Theoretically, while group lending models typically compare group contracts to individual contracts in a static context, this may undersell individual lending which is often thought to operate via dynamic considerations in a repeated relationship.

The purpose of this paper is to compare the efficiency of group lending with that of dynamic, individualized lending in the Ghatak/Stiglitz/Weiss model. This appears to be a first comparison of group lending with dynamic lending contracts in a well-known framework. In addition, we know of no other dynamic analysis of the adverse selection version of Stiglitz and Weiss (1981); the moral hazard version is analyzed dynamically in Stiglitz and Weiss (1983).

Of course, given the pure adverse selection setting, dynamic lending does not work by providing dynamic incentives for effort, safe project choice, or repayment. Rather, its potential effectiveness comes through enabling better pricing for risk over time as behavior reveals riskiness.

We show a number of results. Thus far we ignore a limited liability constraint (but show it can always be satisfied by adjusting parameters) and impose non-negative gross interest rates and the constraint that borrowers would never prefer to claim failure for themselves or a group member (see Gangopadhyay et al., 2005). In the dynamic lending context, we assume limited commitment on the side of the borrower, that is that the borrower can opt out of future loans.

In a two-type, two-period model of dynamic lending, we find that:

Probabilistically excluding borrowers that default on their first loan from obtaining a second loan can improve efficiency of the loan market. It does this by differentially weeding out risky borrowers, which allows the lender to charge a lower break-even interest rate and may draw in safe borrowers. This increases efficiency when the safe borrowers drawn in outnumber the unlucky safe and risky borrowers excluded from a second loan.

Excluding defaulting borrowers outright is always inefficient compared to dynamic scoring, i.e. charging a finite second-period interest rate that depends on first-period repayment.

Optimal dynamic scoring leads to one of three outcomes, ranked by efficiency, when standard, static individual contracts lead to exclusion of safe borrowers.

1) All safe borrowers are included in both periods. Borrowers pay a high interest rate on their first loan and a zero (low) interest rate on their second loan if they have not (have) defaulted. 2) If 1) is not possible, all safe borrowers are included except in period two if they have defaulted. Borrowers pay a high interest rate on their first loan and a zero (high) interest rate on their second loan if they have not (have) defaulted. 3) If 2) is not possible, safe borrowers are excluded in both periods.

Thus, under dynamic lending interest rates are front-loaded and discounts are back-loaded – terms get better over time, and this can even be true for defaulters.

Group lending (with 2 group members) is more able than dynamic lending (with 2 periods) to achieve perfect efficiency, that is, it includes all safe borrowers all the time over a larger parameter space than dynamic lending. However, dynamic lending can be more efficient than group lending when both fail to achieve full efficiency: in this case group lending will always exclude all safe borrowers, while dynamic lending can include them in all cases except after they default. Thus, the efficiency comparison is ambiguous.

We also provide some results extending the contracts to individual T-period lending and to static group lending to groups of size N. We find that in an optimal T-period dynamic contract, a borrower gets a zero-interest loan in the period after repaying a loan, regardless of past repayment history. Further, the ability of dynamic lending to achieve efficiency increases with T and of group lending to achieve efficiency increases with N. As group size N goes to infinity, group lending approaches perfect efficiency, regardless of the number or distribution of types (ignoring limited liability).

2 Baseline Model and Results

There is a unit measure continuum of risk-neutral agents. Each is endowed with no capital, one unit of labor, a subsistence option, and a project. The subsistence option requires one unit of labor and gives output u. The project requires one unit each of capital and labor. Agents' projects differ in risk, indexed by $p \in \mathcal{P}$. The project of an agent of type p yields gross returns of Y_p ("succeeds") with probability p and yields 0 gross returns ("fails") with probability $1 - p$. The key Stiglitz and Weiss (1981) assumption is that projects have the same expected value but differ in risk, in the sense of second-order stochastic dominance:

$$p \cdot Y_p = \overline{Y}, \quad \forall p \in \mathcal{P} \ . \tag{A1}$$

The higher is p, the lower is the agent's risk.

Given their lack of capital, agents require outside funding to carry out their projects. We assume limited liability, in particular that agents' exposure in any financial contract is limited to project returns. It follows that an agent who fails owes nothing to an outside lender.[1]

[1] This is the only aspect of limited liability we impose throughout much of the paper. In particular, we

Agents' types are observable to other agents, but not to any outside lender. Further, following Ghatak (1999, 2000), we assume the lender can observe output only coarsely: it can distinguish between $Y = 0$ (fail) and $Y > 0$ (succeed), but not between different levels of $Y > 0$. This assumption along with limited liability makes debt contracts the only feasible financial contracts.[2] We also restrict attention to deterministic, pooling contracts.

There is a competitive lending sector, or alternatively a non-profit lender, with access to capital at gross rate $\rho > 0$. Hence, we focus on contracts that maximize total borrower surplus subject to the lender earning expected rate of return ρ on all capital lent. Define

$$R \equiv \frac{\overline{Y} - u}{\rho} \tag{1}$$

as the excess return to capital embodied in these agents' projects. It is assumed that

$$\overline{Y} > \rho + u \qquad \Longleftrightarrow \qquad R > 1. \tag{A2}$$

This implies that all projects are expected to return more than the cost of their inputs, capital and labor. Thus, social surplus is monotonically increasing in the number of projects funded.

The analysis will focus on the two-type case: $\mathcal{P} = \{p_r, p_s\}$, with $0 < p_r < p_s < 1$. Let $\theta \in (0, 1)$ be the proportion of risky, and $\overline{p}$ be the mean risk-type: $\overline{p} = \theta p_r + (1 - \theta)p_s$.

Static individual lending. In the static case, standard individual loan contracts involve r paid upon success and 0 paid otherwise. An agent of type $\tau \in \{r, s\}$ will choose to borrow and undertake the project, instead of subsistence, iff

$$\overline{Y} - p_\tau r \geq u \qquad \Longleftrightarrow \qquad r \leq \hat{r}_\tau \equiv \frac{\overline{Y} - u}{p_\tau}\ , \tag{2}$$

ignore the constraint that the amount due when an agent of type p succeeds must be less than Y_p. However, this constraint is satisfied under every condition we derive below for high enough Y_p and u. See Armendariz de Aghion and Gollier (2000) for analysis where this constraint plays a significant role.

[2]There are no enforcement issues by assumption: if a borrower can repay, he does.

where the left-hand side of the first inequality is the expected revenue of the project less the expected loan payment. We can rearrange to derive the type-specific reservation interest rate, $\hat{r}_\tau$, above which a type-τ agent will opt for subsistence; this is the second inequality. It is clear that $\hat{r}_s < \hat{r}_r$: if anyone chooses not to borrow, it will be the safe agents, since they pay the interest rate with higher probability.

Efficiency in this market involves all agents borrowing. If all borrow, the bank charges an interest rate that satisfies

$$\overline{p}r = \rho \qquad \Longleftrightarrow \qquad r = \frac{\rho}{\overline{p}} \ ,$$

since $\overline{p}$ is the expected repayment rate of the average agent and thus $\overline{p}r$ is the expected return on a unit of capital lent. All are willing to borrow at this interest rate iff it is no greater than $\hat{r}_s$, i.e.

$$\frac{\rho}{\overline{p}} \leq \frac{\overline{Y} - u}{p_s} \qquad \Longleftrightarrow \qquad \frac{p_s}{\overline{p}} \leq R \ .$$

This condition is intuitive – efficient lending can be achieved if the degree of asymmetric information, captured by $p_s/\overline{p}$, is not greater than the excess return to capital in this market, R (strictly greater than 1 by assumption A2). In this case, the borrowers get all the surplus from the projects, but risky borrowers are cross-subsidized by, and so earn more than, safe borrowers.

If instead

$$R < \frac{p_s}{\overline{p}} \ , \tag{A3}$$

the lender cannot break even and attract safe agents simultaneously. The next best option is to give up on safe agents and lend only to the risky, charging r to satisfy $p_r r = \rho$. All risky agents borrow at this rate, under assumption A2. Thus, under assumption A3, standard individual lending does not achieve efficiency, as the number of projects funded is θ. Inefficiency arises from the bank's inability to price for risk.[3]

[3]Under perfect information, the bank can break even loan by loan, charging ρ/p_s to safe borrowers and

Static group (paired) lending. As Ghatak (1999, 2000) has shown, joint liability lending can improve efficiency when individual lending breaks down, i.e. under assumption A3. Consider contracts written with *pairs* of borrowers, now with two parameters: r being the amount paid by an agent when he succeeds, and q being the additional amount paid by an agent who succeeds and whose partner fails. The expected payoff of a borrower of type i paired with a borrower of type j, $(i, j) \in \{r, s\}^2$, is then

$$\overline{Y} - p_i r - p_i(1 - p_j)q \;=\; \overline{Y} - p_i[r + (1 - p_j)q] \ .$$

One can think of the bracketed term, $r + (1 - p_j)q$, as the *effective* interest rate in this contract: conditional on success, the payment on one's own loan plus the expected liability payment.

Ghatak shows that the unique stable match involves groups that are homogeneous in risk-type when $q > 0$. As a result, the effective interest rate for a borrower of type p_i is $r + (1 - p_i)q$. Remarkably, this effective interest rate is lower for safer borrowers than for risky borrowers, and can thus more accurately price for risk. Hence, group lending can attain efficient lending in some cases where individual lending cannot.

To see this, first consider the optimal q. Motivated by Gangopadhyay et al. (2005, call it GGL), we impose the constraint that liability for one's partner cannot be more than his debt obligation, i.e.

$$q \leq r \ . \tag{A4}$$

If this were not true, a successful borrower would have the incentive to claim his partner succeeded even when he failed, in order to pay $2r$ rather than $r + q$.[4] If all agents borrow,

$\rho/p_r > \rho/p_s$ to risky borrowers. This achieves efficient and equitable lending, with all surplus going to the borrowers.

[4] There always exists a contract (r, q) that achieves efficient and equitable lending in the two-type case when $p_s + p_r \geq 1$; it solves $p_s[r + (1 - p_s)q] = p_r[r + (1 - p_r)q] = \rho$, i.e. sets $r = q(p_s + p_r - 1) < q$. Efficient (but not equitable) lending can be achieved if $p_s + p_r < 1$ and $R \geq p_s(1 - p_s)/\overline{p}(1 - \overline{p})$; the most equitable such contract sets $r = 0 < q$.

the bank must set r and q to satisfy

$$\overline{p} \cdot r + \overline{p(1-p)} \cdot q = \rho \ , \tag{3}$$

where $\overline{p(1-p)}$ is the population average of $p(1-p)$ and the left-hand side is the expected return on a unit of capital lent. Consider a safe borrower's payoff:

$$\overline{Y} - p_s r - p_s(1-p_s)q \ .$$

It is straightforward to show that the safe borrower's (linear) indifference curve is steeper than the bank's isoprofit line 3, in (r, q) space. The corner solution of $q = r$ thus maximizes the safe borrower's utility for any profit level of the bank. Setting $q = r$ gives borrower payoffs $\overline{Y} - p_\tau(2 - p_\tau)r$ and reservation interest rates

$$\frac{\overline{Y} - u}{p_\tau(2 - p_\tau)} \ , \quad \tau \in \{r, s\} \ .$$

Given that $0 < p_r < p_s < 1$ and that the denominator is a parabola maximized at $p_\tau = 1$, it is clear that including safe borrowers is indeed the binding constraint, so tailoring the contract to maximize their payoff is optimal. With $q = r$ the isoprofit condition becomes

$$\overline{p(2-p)} \cdot r = \rho \qquad \Longleftrightarrow \qquad r = \frac{\rho}{\overline{p(2-p)}},$$

where $\overline{p(2-p)}$ is the population average of $p(2-p)$. All agents are willing to borrow at this interest rate iff it is no greater than the safe borrower's reservation rate, i.e.

$$\frac{\rho}{\overline{p(2-p)}} \leq \frac{\overline{Y} - u}{p_s(2-p_s)} \qquad \Longleftrightarrow \qquad R \geq \frac{p_s(2-p_s)}{\overline{p(2-p)}} = \frac{p_s}{\overline{p} + (p_s - \overline{p})\frac{p_r}{2-p_s}} \ . \tag{4}$$

Otherwise, only risky borrowers can be included. Thus for some values of R, namely $R \in [\frac{p_s}{\overline{p}+(p_s-\overline{p})p_r/(2-p_s)}, \frac{p_s}{\overline{p}})$, group lending is efficient while individual lending is not.

3 Dynamic, Individual Lending: 2 periods

Assume that agents receive the same endowment (one unit of labor, one project, one subsistence option) in each of two periods. For simplicity, discounting is ignored. The lender offers one unit of capital in each period to borrowers, subject to interest rates that are conditioned on the history of repayment/project success. These contracts boil down to three parameters: (r, r_s, r_f), where r is the interest rate on the first loan and r_s (respectively, r_f) is the interest rate on the second loan for a borrower who has succeeded (respectively, failed) in his first-period project.[5] As before, contracts maximize the total payoffs of the agents subject to the bank earning ρ per unit of capital lent.

We assume one-sided commitment: the lender can fully commit to the two-period contract, but the borrower cannot commit to continuing to borrow. Hence, at date 2 borrowers drop out if the loan terms are such that subsistence is more attractive.[6] Also in the spirit of GGL, we assume non-negativity of (gross) interest rates:

$$r,\, r_s,\, r_f \geq 0\ . \tag{A5}$$

If this were not so, a borrower would in some cases want to claim success rather than failure in order to receive the success bonus.

Efficient lending. Can full efficiency be achieved with dynamic lending when static individual lending breaks down? Define a type-τ agent's two-period payoff from taking the

[5] We ignore a second-period interest rate for a first-time borrower – one can think of such a borrower as starting with a fresh two periods. We also continue to assume that liability is limited only to project returns, and hence rule out use of collateral.

[6] If both lender and borrower can commit to the full 2-period relationship, it is straightforward to show that efficient lending can be achieved exactly when it can be achieved under group lending *without* the GGL constraint A4; see footnote 4xxx. Here, r and r_f play the roles of r and q, respectively. Typically, r_f will be high, r_s will be zero, and r will be relatively low.

loan in period 1 and, only if optimal, in period 2, as $\Pi_\tau(r, r_s, r_f)$; then

$$\begin{aligned}\Pi_\tau(r, r_s, r_f) &= \overline{Y} - p_\tau r + p_\tau \max\{\overline{Y} - p_\tau r_s, u\} + (1 - p_\tau)\max\{\overline{Y} - p_\tau r_f, u\} \\ &= 2\overline{Y} - p_\tau r - p_\tau^2 \min\{r_s, \hat{r}_\tau\} - p_\tau(1 - p_\tau)\min\{r_f, \hat{r}_\tau\}\ .\end{aligned} \tag{5}$$

Here, the max-operators reflect the fact that the agent will not borrow in period 2 if the outside option is better; and the equality uses the fact that $\hat{r}_\tau$ (see equation 2) is the agent's reservation rate, implying $\overline{Y} - p_\tau \hat{r}_\tau = u$.

Full efficiency implies that all agents borrow in both periods, so r_s and r_f cannot exceed $\hat{r}_s$. Thus, a safe borrower's payoff reduces to

$$2\overline{Y} - p_s r - p_s^2 r_s - p_s(1 - p_s) r_f\ . \tag{6}$$

With all agents always borrowing, the lender's zero-profit constraint becomes

$$\overline{p} \cdot r + \overline{p^2} \cdot r_s + \overline{p(1-p)} \cdot r_f = 2\rho\ , \tag{7}$$

where $\overline{p^2}$ is the population average of p^2 and the left-hand side is the expected two-period return from two units of capital lent. It is straightforward to show that the safe borrower's (linear) indifference curve is steeper than the bank's isoprofit line 3 in (r, r_f) space and less steep in (r, r_s) space. Thus, if including safe borrowers is the binding constraint, the contract can do no better than set r_s as low as possible, i.e. $r_s = 0$, and r_f as high as possible, subject to including both types of borrowers, i.e. $r_f = \hat{r}_s$. The full contract would then involve $r_s = 0$, $r_f = \hat{r}_s$, and the r that solves equation 7 given these values:

$$r = \frac{2\rho - \overline{p(1-p)}\hat{r}_s}{\overline{p}} = \frac{2\rho - \overline{p(1-p)}(\overline{Y} - u)/p_s}{\overline{p}}\ .$$

It can be verified that safe borrowers have lower payoffs than risky borrowers under this contract and assumption A3, so including safe borrowers is indeed the binding constraint.

Hence, if this contract cannot include all borrowers in both periods, none can. It remains to be verified that safe borrowers prefer to borrow under this contract, i.e. $\Pi_s(r, 0, \hat{r}_s) \geq 2u$. Some algebra shows that this is true iff

$$R \geq \frac{p_s}{\overline{p} + (p_s - \overline{p})\frac{p_r}{2}}. \tag{8}$$

Thus, for $R \in [\frac{p_s}{\overline{p}+(p_s-\overline{p})p_r/2}, \frac{p_s}{\overline{p}})$, two-period dynamic lending can achieve full efficiency though static individual lending would not. Intuitively, the dynamic contract better prices for risk by varying the second-period interest rate so as to shift the repayment burden toward risky borrowers ($r_s < r_f$).

This contract features back-loaded incentives, with borrowing getting more attractive over time. In particular, when fully efficient lending is achievable though individual lending would break down (condition 8 and assumption A3), even failed borrowers pay a lower rate in period 2 than in period 1: $r > r_f = \hat{r}_s > r_s = 0$.[7] Safe agents borrow in period 1 even though they earn less than their outside option, in anticipation of cheaper future loans. The model thus provides a rationale for the feature of relationship lending that borrowers get better terms over time. Essentially, this back-loading overcomes agents' lack of commitment to keep borrowing over time, which allows the lender to base pricing on progressively better risk information.

One can compare the relative effectiveness of group lending and dynamic lending at achieving efficiency by comparing conditions 4 and 8. Clearly condition 4 is weaker, so that for $R \in [\frac{p_s}{\overline{p}+(p_s-\overline{p})p_r/(2-p_s)}, \frac{p_s}{\overline{p}+(p_s-\overline{p})p_r/2})$, static group (paired) lending can achieve efficient lending while dynamic individual (2-period) lending cannot.

In terms of achieving full efficiency, group lending thus dominates dynamic lending. It turns out that the borrower's limited commitment in dynamic lending underlies this inferiority. To see this, note that both types of contracts are similar in their incidence on borrowers:

[7] As long as $R < \frac{p_s}{\overline{p}-p^2/2}$, which is true under assumption A3, $r > \hat{r}_s$.

compare expected repayments under group lending, $pr + p(1-p)q$, and under dynamic lending, $pr + p(1-p)r_f$ $(+p^2 r_s = 0)$. In both cases, inclusion of safe borrowers is made easier by lowering r and raising q/r_f, since this shifts repayment obligations toward risky borrowers. In group lending, the constraint is that $q \leq r$ while in dynamic lending it is that $r_f \leq \hat{r}_s$ due to the lack of borrower commitment. The latter constraint turns out to be more binding,[8] so efficient dynamic lending cannot target as much of the repayment burden toward risky borrowers as group lending can.

Optimal lending. When dynamic lending cannot achieve full efficiency, i.e. under the reverse of condition 8, it need not exclude safe borrowers under all circumstances; this is in contrast to static group and individual lending. If it includes safe borrowers in any circumstance, it must include them in the first period.[9] However, it may exclude them in the second period under some or all histories via the level of r_s and/or r_f.

We first show that including safe borrowers can be treated as the binding constraint in every period.

Lemma 1. *In each period and each history, if safe agents choose to borrow, risky agents also choose to borrow.*

Proof. One can derive the reservation interest rate in period 1 given r_s and r_f, call it $\hat{\hat{r}}_\tau$ for a type-τ agent, from the condition $\Pi_\tau(\hat{\hat{r}}_\tau, r_s, r_f) = 2u$. Using payoff 5 and the definition of $\hat{r}_\tau$(see equation 2), $\hat{\hat{r}}_\tau$ can be written

$$\hat{\hat{r}}_\tau = 2\hat{r}_\tau - p_\tau \min\{r_s, \hat{r}_\tau\} - (1-p_\tau)\min\{r_f, \hat{r}_\tau\} \ .$$

We then have

$$\begin{aligned}\hat{\hat{r}}_r - \hat{\hat{r}}_s = \ & 2(\hat{r}_r - \hat{r}_s) + [p_s \min\{r_s, \hat{r}_s\} - p_r \min\{r_s, \hat{r}_r\}] \\ & - [(1-p_r)\min\{r_f, \hat{r}_r\} - (1-p_s)\min\{r_f, \hat{r}_s\}] \ .\end{aligned}$$

[8]As argued above, $\hat{r}_s < r$ in the dynamic contract above when individual lending breaks down.
[9]There is no second-period borrowing option for first-time borrowers by assumption.

It can be verified that the bracketed term on the first line is positive and minimized at $r_s = 0$ and that the bracketed term on the second line is positive and maximized at $r_f = \hat{r}_r$; thus,

$$\hat{\hat{r}}_r - \hat{\hat{r}}_s \geq 2(\hat{r}_r - \hat{r}_s) - (1-p_r)\hat{r}_r + (1-p_s)\hat{r}_s = \hat{r}_r - \hat{r}_s + p_r\hat{r}_r - p_s\hat{r}_s = \hat{r}_r - \hat{r}_s > 0 \ ,$$

where the last equality and the inequality use the definition of $\hat{r}_\tau$. Thus, $\hat{\hat{r}}_s < \hat{\hat{r}}_r$, so if safe borrowers choose to borrow in period 1 ($r \leq \hat{\hat{r}}_s$), so do risky ($r \leq \hat{\hat{r}}_r$).

The same result holds for each history in period 2, since $\hat{\hat{r}}_s < \hat{\hat{r}}_r$ and the reservation interest rate for a one-shot loan is lower for safe than risky agents: $\hat{r}_s < \hat{r}_r$. ■

We next claim that a contract that maximizes safe borrowers' 2-period payoff, $\Pi_s(r, r_s, r_f)$ of equation 5, subject to the lender's zero-profit constraint, can do no better than to set $r_s = 0$. Consider first $r_s \leq \hat{r}_s$. The safe borrower's payoff is

$$\Pi_s(r, r_s, r_f) = 2\overline{Y} - p_s r - p_s^2 r_s - p_s(1-p_s)\min\{r_f, \hat{r}_s\} \ .$$

Define Z as the number of agents who fail in the first period and take a loan in the second period, and Z_p as the number of these agents who repay in the second period.[10] Then the bank's zero-profit constraint is

$$\overline{p} \cdot r + \overline{p^2} \cdot r_s + Z_p \cdot r_f = (1 + \overline{p} + Z)\rho \ .$$

As above, comparison of the safe borrower's (linear) indifference curve and the bank's iso-profit line in (r, r_s) space establishes that a safe borrower prefers r_s to be as low as possible at the expense of the higher r. Thus, $r_s = 0$ is preferred by safe borrowers for $r_s \in [0, \hat{r}_s]$. When $r_s \in (\hat{r}_s, \infty)$, a safe borrower's payoff does not depend directly on r_s– it's so high they opt out of borrowing in the second period when they succeed. However, r_s affects

[10]Then $Z = \theta(1-p_r)1\{r_f \leq \hat{r}_r\} + (1-\theta)(1-p_s)1\{r_f \leq \hat{r}_s\}$ and $Z_p = \theta(1-p_r)p_r1\{r_f \leq \hat{r}_r\} + (1-\theta)(1-p_s)p_s1\{r_f \leq \hat{r}_s\}$.

them indirectly via the bank's profits, so their preference is for the bank to extract as much revenue as possible from risky borrowers via r_s, in order to allow for low r and/or r_f. The bank clearly does this by charging the risky agents' reservation rate, $\hat{r}_r$, extracting all the (strictly positive) surplus from risky agents' borrowing in this state of the world. Thus, in this range safe borrowers uniquely prefer $r_s = \hat{r}_r$. In sum, to maximize safe borrowers' payoff, r_s should be set either to 0 or to $\hat{r}_r$. A direct comparison of these two options reveals that $\Pi_s(r, 0, r_f) > \Pi_s(r', \hat{r}_r, r_f)$ for any value of r_f, where r (respectively, r') sets lender profits to zero given $r_s = 0$ and r_f (respectively, $r_s = \hat{r}_r$ and r_f).

Evidently, the free loan as a reward for success is more valuable than the discount in r that can be offered by extracting successful risky borrowers' period-2 surplus. The intuition is that the discount in r is a convex combination of the net borrowing payoff (extracted from risky borrowers) and the cost of capital (saved by not lending to safe borrowers), while the free loan exactly equals the net borrowing payoff; since the net borrowing payoff is higher than the cost of capital, the free loan is more valuable.

Finally, let $r_s = 0$ and consider r_f. With $r_f \leq \hat{r}_s$, the safe borrower's payoff is

$$\Pi_s(r, 0, r_f) = 2\overline{Y} - p_s r - p_s(1 - p_s) r_f$$

and the bank's zero-profit constraint is

$$\overline{p} \cdot r + \overline{p(1-p)} \cdot r_f = 2\rho \ .$$

As above, comparison of the safe borrower's (linear) indifference curve and the bank's iso-profit line in (r, r_f) space establishes that a safe borrower prefers r_f to be as high as possible to allow for a lower r. Thus, $r_f = \hat{r}_s$ is preferred by safe borrowers when $r_f \in [0, \hat{r}_s]$. When $r_f \in (\hat{r}_s, \infty)$, the argument is as above: safe borrowers prefer the maximal extraction from risky borrowers in order to fund a lower r. Thus, in this range safe borrowers uniquely prefer the risky borrower's reservation rate, $r_f = \hat{r}_r$. In sum, to maximize safe borrowers' payoff,

r_f should be set either to $\hat{r}_s$ or to $\hat{r}_r$.

The contract with $r_s = 0$ and $r_f = \hat{r}_s$ has been analyzed above and includes safe borrowers in period 1 (and involves fully efficient lending) under condition 8. Consider the contract with $r_s = 0$ and $r_f = \hat{r}_r$. The period-1 rate r solves the bank's zero-profit constraint

$$\overline{p} \cdot r + \theta p_r(1 - p_r) \cdot \hat{r}_r = \rho[1 + \overline{p} + \theta(1 - p_r)] \ ;$$

both left- and right-hand sides reflect the fact that safe borrowers who fail do not borrow in period 2. Solving for r gives

$$r = \frac{2\rho - (1-\theta)(1-p_s)\rho - \theta(1-p_r)(\overline{Y} - u)}{\overline{p}} \ .$$

It remains to be verified that safe borrowers prefer to borrow under this contract, i.e. $\Pi_s(r, 0, \hat{r}_r) \geq 2u$. Some algebra shows that this is true iff

$$R \geq \frac{p_s}{\overline{p} + (p_s - \overline{p})\frac{\theta + (1-\theta)p_s}{1 + \theta + (1-\theta)p_s}}. \tag{9}$$

It is also easy to show that this condition is weaker than condition 8 for fully efficient dynamic (2-period) lending, so for $R \in [\frac{p_s}{\overline{p} + (p_s - \overline{p})[\theta + (1-\theta)p_s]/[1 + \theta + (1-\theta)p_s]}, \frac{p_s}{\overline{p} + (p_s - \overline{p})p_r/2})$, dynamic lending cannot achieve full efficiency but can do better than always excluding safe borrowers.

If condition 9 does not hold, safe borrowers do not borrow at all and the bank lends only to risky (charging e.g. $r = r_s = r_f = \rho/p_r$). This is clear because $r_s = 0$ and $r_f = \hat{r}_s$ or $r_f = \hat{r}_r$ maximize safe agents' 2-period payoff to borrowing. If neither one of these contracts is attractive enough to them, which is true under the reverse of condition 9, since it also implies the reverse of condition 8, they will not borrow. In summary, if condition 8 holds, efficient lending is achievable: all borrow in every period. If it does not but condition 9 holds, partially efficient lending is achievable: all borrow except failed safe borrowers in period 2, for a per-period loan total of $\theta + (1-\theta)(1+p_s)/2$. If condition 9 does not hold, only risky borrow: θ per period.

Interestingly, safe borrowers typically prefer to be priced out of borrowing in period-2 when they fail in period 1. Specifically, for R in the range where individual lending breaks down (assumption A3), extracting all the borrowing surplus from failed risky borrowers (via $r_f = \hat{r}_r$) allows the bank to charge the lowest possible period-1 interest rate. This is all safe borrowers care about when they borrow, since under either contract they get a free loan after succeeding and their reservation payoff after failing (whether or not they borrow). Thus, there is a range of R for which the efficient dynamic contract involves lending to failed safe borrowers even though they would prefer to be priced out of the market. Here, including safe borrowers ($r_f = \hat{r}_s$) raises more surplus than excluding them ($r_f = \hat{r}_r$), but risky borrowers benefit by more than the difference via cheap period-2 loans when they fail.

Comparison of condition 9 for partially efficient dynamic (2-period) lending with condition 4 for efficient group lending gives that the former is strictly weaker iff

$$p_r < \frac{(2 - p_s)[\theta + (1 - \theta)p_s]}{1 + \theta + (1 - \theta)p_s}.$$

When this inequality holds,[11] there exists a range for R (in between the respective bounds of conditions 4 and 9) for which dynamic lending is more efficient than group lending. Here, group lending fails to include safe agents at all, while dynamic lending includes safe agents except in period 2 after a failure. Evidently, the ability of dynamic lending to move away from the always-include/never-include dichotomy of static group lending can give it a decisive efficiency advantage, despite the strong commitment constraint it faces.

Thus, the efficiency comparison between group and dynamic lending is not unambiguous. However, dynamic lending is only more efficient than group lending when it fails to achieve full efficiency, i.e. when it excludes safe borrowers who fail.

[11] One can show that a sufficient condition for this inequality is $p_r \leq 1/2$.

4 Dynamic, Individual Lending: 3 periods

Does the greater information revelation that comes with a longer relationship could allow for efficient lending under weaker conditions? We next extend the previous section's model to 3 periods. Contracts now involve seven interest rates: $(r, r_s, r_f, r_{ss}, r_{sf}, r_{fs}, r_{ff})$, where the subscripts denote the history after which the interest rate is offered. As before, r is the period-1 offer and r_s (r_f) is the period-2 offer after a success (failure). Period-3 rates are, respectively, r_{ss} after two successes, r_{sf} after a success then a failure, r_{fs} after a failure then a success, and r_{ff} after two failures.

We continue to assume one-sided commitment: the lender can fully commit to the three-period contract, but the borrower cannot commit to continuing to borrow. Also, we continue to assume non-negativity of (gross) interest rates:

$$r,\ r_s,\ r_f,\ r_{ss},\ r_{sf},\ r_{fs},\ r_{ff}\ \geq 0\ . \tag{A6}$$

Next, conditions are derived under which fully efficient lending can be achieved, i.e. all agents borrow in every state and every period. Consider safe borrowers' decision to borrow. For them to borrow in period 3 regardless of their history, period-3 interest rates must satisfy

$$r_{ss},\ r_{sf},\ r_{fs},\ r_{ff}\ \leq \hat{r}_s\ , \tag{10}$$

where $\hat{r}_s$ is defined in equation 2. For safe borrowers to choose to borrow in period 2 regardless of their period-1 history, $h \in \{s, f\}$, it must be that

$$\overline{Y} - p_s r_h + p_s[\overline{Y} - p_s r_{hs}] + (1 - p_s)[\overline{Y} - p_s r_{hf}] \geq 2u\ .$$

This can be rewritten as two constraints, one for each history:

$$r_s + p_s r_{ss} + (1 - p_s) r_{sf} \leq 2\hat{r}_s \tag{11}$$

and

$$r_f + p_s r_{fs} + (1 - p_s) r_{ff} \leq 2\hat{r}_s \ . \tag{12}$$

Given that constraints 10-12 are met, a safe agent's 3-period payoff to borrowing is

$$3\overline{Y} - p_s r - p_s^2 r_s - p_s(1 - p_s) r_f - p_s^3 r_{ss} - p_s^2(1 - p_s)(r_{sf} + r_{fs}) - p_s(1 - p_s)^2 r_{ff} \ . \tag{13}$$

Finally, with all agents always borrowing, the lender's zero-profit constraint (ZPC) is

$$\overline{p} \cdot r + \overline{p^2} \cdot r_s + \overline{p(1-p)} \cdot r_f + \overline{p^3} \cdot r_{ss} + \overline{p^2(1-p)}(r_{sf} + r_{fs}) + \overline{p(1-p)^2} \cdot r_{ff} = 3\rho \ , \tag{14}$$

where $\overline{p^3}$, $\overline{p^2(1-p)}$, and $\overline{p(1-p)^2}$ are the population average of p^3, $p^2(1-p)$, and $p(1-p)^2$, respectively.

Our approach is to derive a contract that maximizes 3-period borrowing payoff 13 subject to constraints 10-12 and zero-profit constraint 14. If this maximized payoff exceeds $3u$, and risky borrowers also choose to borrow in every period and state, then efficient lending is achieved.

Forming bank isoprofit lines from constraint 14 and (linear) safe borrower indifference curves from payoff 13 (and perhaps constraint 11 or 12), the following facts are readily seen. First, lowering r_s and raising r along the bank's ZPC raises the 3-period payoff 13 and eases constraint 11. Thus, since r is unconstrained in our maximization of the 3-period payoff, setting r_s at the lower boundary is optimal, i.e. $r_s = 0$. Second, lowering r_{ss} and raising r_s along the ZPC raises the 3-period payoff and eases constraints 11 and 10. Thus, setting r_{ss} at the lower boundary is optimal, i.e. $r_{ss} = 0$; otherwise, the 3-period payoff could be raised without violating any constraint by lowering r_{ss} and raising r_s (away from 0). Third, lowering r_{fs} and raising r_f along the ZPC maximizes the 3-period payoff and eases constraints 12 and 10. Thus, setting r_{fs} at the lower boundary is optimal, i.e. $r_{fs} = 0$. Otherwise, if $r_{fs} > 0$ then r_{fs} could be lowered and r_f raised along the ZPC to raise borrowing payoffs; this would even be so if r_f were initially at its upper boundary, defined by constraint 12, since this constraint is relaxed by such an alteration. To summarize, unless $r_s = r_{ss} = r_{fs} = 0$, a safe borrower's 3-period payoff can be increased without violating any constraint.

Consider next r_{ff}. Similar analysis gives that raising r_{ff} and lowering r_f along the bank's ZPC raises the 3-period payoff and eases constraint 12. Thus, either r_f is set at its lower boundary, i.e. $r_f = 0$, or r_{ff} is set at the upper boundary, i.e. $r_{ff} = \hat{r}_s$. Next, raising r_f and lowering r along the ZPC raises the 3-period payoff. Thus, either r is set at its lower boundary, i.e. $r = 0$, or r_f is set at the upper boundary defined by constraint 12. Combining these two sets of facts, if $r > 0$ then $r_f = 2\hat{r}_s - (1-p_s)r_{ff} \geq (1+p_s)\hat{r}_s > 0$, so $r_{ff} = \hat{r}_s$ and $r_f = (1+p_s)\hat{r}_s$.

Finally, raising r_{sf} and lowering r_s along the ZPC raises the 3-period payoff and relaxes constraint 11. But since r_s is at its lower boundary, i.e. $r_s = 0$, this does not restrict r_{sf}. If $p_s + p_r > 1$, then raising r_{sf} and lowering r along the ZPC raises the 3-period payoff. Thus, either r is set at its lower boundary, i.e. $r = 0$, or r_{sf} is set at the upper boundary defined by the more binding of constraints 10 and 12, which given $r_s = r_{ss} = 0$, is $\hat{r}_s$. On the other hand, if $p_s + p_r < 1$, then lowering r_{sf} and raising r along the ZPC raises the 3-period payoff and eases constraints 11 and 10. Thus $r_{sf} = 0$ would be optimal.

The optimal contract is now clear, but depends partly on parameters. We focus attention on the parameter space where individual lending breaks down by making assumption A3. One can show that this implies the lender cannot break even by charging $r = 0$ along with $r_s = r_{ss} = r_{fs} = 0$; thus the optimal contract must involve $r > 0$.[12] This pins down $r_{ff} = \hat{r}_s$ and $r_f = (1+p_s)\hat{r}_s$. We also know that $r_s = r_{ss} = r_{fs} = 0$. Also, if $p_r + p_s > 1$, $r_{sf} = \hat{r}_s$; otherwise if $p_s + p_r < 1$, $r_{sf} = 0$. Finally, r then comes from the ZPC.

[12]If the excess return to borrowers' projects, R, is high enough, the lender can break even with $r = 0$. In this case, one can show that the optimal contract first uses r_{ff}, then if necessary (i.e. if r_{ff} reaches its upper bound of $\hat{r}_s$ without the lender covering costs) r_f, then if necessary (i.e. if r_f reaches its upper bound of $(1+p_s)\hat{r}_s$ without the lender covering costs), and if $p_s + p_r > 1$, r_{sf}. The final resort for recouping costs is r.

Basic features of the contract are not surprising: interest rates are low (zero) after success and tend to be high after failure. There is one exception; when $p_r + p_s < 1$ (which is equivalent to p_s being closer than p_r to 1/2), $r_{sf} = 0$. Here, success is so rare that having one success and one failure is more common for safe than risky borrowers, so the sf history is targeted with a low rate. Essentially, success being rarer means the benefits of success last longer.

When $p_r + p_s > 1$, probably the more likely case, interest rates fully extract the surplus of safe borrowers after failure. After one failure, the interest rate of $(1 + p_s)\hat{r}_s$ extracts the full two-period surplus, charging what safe borrowers would be willing to pay for a single loan *plus* the expected value of the free loan earned after success.

The contract also involves history dependence. In particular, the interest rate does not depend only on the number of successes and failures (a sufficient statistic for the lender's updated estimate of a borrower's type), but also on the order of events. A safe borrower who fails then succeeds gets a zero-interest loan, while a borrower who succeeds then fails gets a loan that extracts his full borrowing surplus. However, the safe borrower who fails then succeeds ends up paying a higher penalty interest rate, one that extracts his full 2-period expected borrowing surplus. In the net, either history may lead to higher payoffs. The agent who succeeds then fails pays r with certainty and $\hat{r}_s$ with probability p_s, while the agent who fails then succeeds pays $(1 + p_s)\hat{r}_s$ with certainty; the comparison boils down to the degree of backloading in the incentives, i.e. how r compares to $\hat{r}_s$.

Assume that $p_s + p_r > 1$ for the remainder of the section. The ZPC gives r as

$$r = \frac{3\rho - \overline{p(1-p)}(1 + p_s)\hat{r}_s - [\overline{p^2(1-p)} + \overline{p(1-p)^2}]\hat{r}_s}{\overline{p}} = \frac{3\rho - \overline{p(1-p)}(2 + p_s)\hat{r}_s}{\overline{p}} .$$

It can be verified that safe borrowers have no greater payoffs than risky borrowers under this contract and assumption A3, at every history except after one success, at which history risky agents strictly prefer to borrow. Thus, if this contract attracts safe borrowers in period 1, fully efficient lending is achieved; if not, fully efficient lending cannot be achieved. Some algebra shows that safe borrowers do indeed borrow in period 1, i.e. their 3-period payoff 13 is not less than $3u$, iff

$$R \geq \frac{p_s}{\overline{p} + (p_s - \overline{p})\frac{p_r(2+p_s)}{3}}. \tag{15}$$

Thus, for $R \in [\frac{p_s}{\overline{p}+(p_s-\overline{p})p_r(2+p_s)/3}, \frac{p_s}{\overline{p}+(p_s-\overline{p})p_r/2})$, three-period dynamic lending can achieve full efficiency though two-period dynamic lending (and static individual lending) would not.

One can again compare the relative effectiveness of group lending and dynamic lending at achieving efficiency. It turns out that condition 15 is weaker than condition 4, so that for $R \in [\frac{p_s}{\overline{p}+(p_s-\overline{p})p_r(2+p_s)/3}, \frac{p_s}{\overline{p}+(p_s-\overline{p})p_r/(2-p_s)})$, dynamic individual (3-period) lending can can achieve efficient lending while static group (paired) lending cannot. Evidently, the additional information revelation from one extra period is enough to overcome the advantages of group (paired) lending.

The contract again features back-loading of incentives, especially when efficient lending is barely achievable. In particular, when condition 15 holds with equality, we have $r = (1 + p_s + p_s^2)\hat{r}_s$. In this case, the period-1 interest rate exceeds all others even though all borrowers are equally likely to pay it. Interest rates decline over time for all borrowers, even those who repeatedly fail, with one exception: borrowers who succeed then fail. Part of what drives this back-loading is the limited borrower commitment – borrowers are started at high rates and promised improving borrowing terms over time to keep them in the relationship and allow for better risk pricing.

The main exception to back-loading occurs as R increases away from the bound in condition 15. A higher R allows for a lower r (linearly) to cover the bank's costs. As R increases, the corresponding r can drop below $(1 + p_s)\hat{r}_s$ and even $\hat{r}_s$ (even under assumption A3 that individual lending breaks down). Recall that R is the excess return to capital in this market; it increases when expected project payoffs ($\overline{Y}$) increase, or outside options (u) or lenders' cost of capital (ρ) decrease. Thus, among a high return population or after a downward spike in lenders' cost of capital, optimal contracts could involve a relatively low initial rate, and higher rates over time if borrowers fail. Even then, however, the highest rates come after one failure, not two failures.

4.1 Group Lending: 3-person groups

One wonders if extending group-size up from 2 might affect the comparison between dynamic and group lending. Group size has been addressed in the context of this model by Ghatak (1999). Ghatak uses the following generalization of joint liability: a successful borrower must pay a fixed amount q for *each* fellow group member who fails. Homogeneous matching continues to obtain, and it turns out that in this setup group size is irrelevant for efficiency of lending. This is because with one partner, a safe borrower's expected liability payment upon success is $q(1-p_s)$, while with $n-1$ partners, it is $q(n-1)(1-p_s)$. Hence, what is attainable with groups of size n and joint liability q is also attainable with pairs and joint liability $q' = q(n-1)$.

Here we examine groups of size 3 in a static setting and derive the optimal form of joint liability. Let r_k, $k \in \{0,1,2\}$, be the payment due from a borrower who succeeds and of whose fellow group members k fail. Symmetry is implicitly assumed, i.e. that the payment due from a borrower does not depend on which borrower it is or which partner(s) failed. We also impose non-negativity and some amount of joint liability:

$$0 \leq r_0 \leq r_1 \leq r_2 \qquad \textit{and} \qquad r_0 < r_1 \;\; \textit{or} \;\; r_1 < r_2 \; . \tag{A7}$$

Thus the payment due from a borrower is monotonically increasing in the number of partners that fail, strictly so at least somewhere.[13] Finally, we impose the following GGL-type constraint:

$$3r_0 \geq 2r_1 \geq r_2 \; . \tag{A8}$$

This ensures that the revenue of the bank is not increasing in the number of failures in the

[13] Here r_0 corresponds to the r of the group-size 2 analysis (section 2) and r_1 to $r+q$. The joint liability assumption is thus equivalent to $q > 0$, as was assumed. This assumption is also satisfied by Ghatak's n-person extension, where $r_k = r + k \cdot q$ with $q > 0$.

group. If this did not hold, borrowers in at least one state of the world would want to falsely claim success in order to reduce the total liability of the group.

The expected payoff of a borrower of type i who matches with borrowers of types j and k is then

$$\overline{Y} - p_i p_j p_k r_0 - [p_i(1-p_j)p_k + p_i p_j(1-p_k)]r_1 - p_i(1-p_j)(1-p_k)r_2 \ .$$

The cross-partial of this payoff with respect to p_i and p_x, $x \in \{j,k\}$, is

$$p_x(r_1 - r_0) + (1-p_x)(r_2 - r_1) \ ,$$

strictly positive under assumption A7. Thus, groups form homogeneously in risk-type and a borrower of type $\tau \in \{r,s\}$ has payoff

$$\overline{Y} - p_\tau^3 r_0 - 2p_\tau^2(1-p_\tau)r_1 - p_\tau(1-p_\tau)^2 r_2 \ . \tag{16}$$

We proceed by maximizing the safe borrower's payoff subject to constraint A8 and the bank's zero-profit constraint assuming all agents borrow:

$$\overline{p^3} \cdot r_0 + 2\overline{p^2(1-p)} \cdot r_1 + \overline{p(1-p)^2} \cdot r_2 = \rho \ ; \tag{17}$$

here the left-hand side is the expected return on a unit of capital lent. We will then show that the contract also satisfies constraint A7 and that it gives safe borrowers lower payoffs than risky, implying that it is the contract best positioned to achieve lending efficiency.

Analysis of the bank isoprofit line and (linear) safe borrower indifference curves demonstrates the following facts. First, raising r_1 and lowering r_0 along the ZPC raises the safe borrower's payoff. Thus, r_0 must be set at its lower boundary defined by constraint A8, i.e. $r_0 = 2r_1/3$. If instead $r_0 > 2r_1/3$, safe borrower payoff could be improved by raising r_1

and lowering r_0 along the ZPC while still satisfying constraint A8. Second, raising r_2 and lowering r_1 along the ZPC raises the safe borrower's payoff. Thus, by similar logic r_2 must be set at its upper boundary defined by constraint A8, i.e. $r_2 = 2r_1$.

Thus, the optimal contract involves $3r_0 = 2r_1 = r_2$. That is, full liability is optimal:[14] the bank collects the same revenue from the group of borrowers in every state of the world, except the one where all three fail. The intuition is straightforward. Efficiency is enhanced by targeting repayment toward risky borrowers as much as possible, and the way to do so is to charge more when there are more failures. Given that the GGL constraint rules out anything beyond full liability, full liability remains the way to target maximal repayment toward states of the world with greater numbers of failure.

Letting $r_0 = r$, we have $r_1 = (3/2)r$ and $r_2 = 3r$. Putting these values into the ZPC gives

$$r = \frac{\rho}{1 - \overline{(1-p)^3}} ,$$

where $\overline{(1-p)^3}$ is the population average of $(1-p)^3$. Clearly this contract satisfies constraint A7. The payoff of a borrower of type τ becomes

$$\overline{Y} - p_\tau^3 r - 3p_\tau^2(1-p_\tau)r - 3p_\tau(1-p_\tau)^2 r = \overline{Y} - [1-(1-p_\tau)^3]r .$$

Clearly this payoff is lower for safe borrowers than for risky. Thus the necessary and sufficient condition for fully efficient lending is that the safe borrower payoff exceeds u, which is equivalent to

$$R \geq \frac{1-(1-p_s)^3}{1-\overline{(1-p)^3}} = \frac{p_s}{\overline{p} + (p_s - \overline{p})\frac{p_r(3-p_r-p_s)}{p_s^2+3(1-p_s)}} . \tag{18}$$

One can verify that this condition is weaker than the one for 3-period dynamic lending to achieve efficiency, condition 15, and hence weaker than both the paired group lending condition and the 2-period dynamic lending condition. Thus, the additional information from increased group size raises efficiency.

[14]Full liability was also the optimal contract in the 2-person group case: $q = r$.

What leads to the differences between joint liability lending (group size of 3) and dynamic lending ($3-period$)? Note that both types of contracts reveal to the lender the same amount of information, via three draws from an agent's probability of success. These draws are essentially panel data in the dynamic lending case, and in the group lending case cross-sectional data, which are informative of each agent's type due to homogeneous matching. In this regard, the two types of lending are quite similar.[15]

The main difference, then, is in the constraints facing lenders under both systems. This is the GGL constraint for group lending, which sets boundaries on rates in bad states of the world, and for dynamic lending, which prohibits subsidies for success. However, dynamic lending faces an additional limited commitment constraint, which significantly restricts the lender's ability to extract revenue after information has been revealed. This proves decisive in dynamic lending's lesser ability to achieve fully efficient lending.

[Compare the two with GGL without limited commitment.]

Of course, dynamic lending has another advantage, which is a larger set of states of the world in which to vary the interest rate. Thus, when both types of lending fail to achieve fully efficient lending, dynamic lending can dominate group lending by including safe borrowers in some histories and excluding them in others. This is true in the $n = T = 2$ case, and we conjecture it to be true in the $n = T = 3$ case as well.

5 Conclusion

[TO BE COMPLETED]

[15] Indeed, ignoring constraints, the 3-period dynamic contract could replicate the group lending outcome by setting $r_{ss} = 3r_0$, $r_{sf} = r_{fs} = 3r_1$, and $r_{ff} = 3r_2$, with every other rate set to zero.

References

[1] Christian Ahlin and Neville Jiang. Can micro-credit bring development? *Journal of Development Economics*, forthcoming.

[2] Christian Ahlin and Robert M. Townsend. Using repayment data to test across theories of joint liability lending. *Economic Journal*, 117(517):F11–F51, February 2007a.

[3] Christian Ahlin and Robert M. Townsend. Selection into and across contracts: Theory and field research. *Journal of Econometrics*, 136(2):665–698, February 2007b.

[4] Christian Ahlin and Brian Waters. Dynamic versus group lending with adverse selection. April 2007.

[5] Beatriz Armendariz de Aghion and Jonathan J. Morduch. *The Economics of Microfinance*. MIT Press, Cambridge MA, 2005.

[6] Abhijit V. Banerjee, Timothy Besley, and Timothy W. Guinnane. Thy neighbor's keeper: The design of a credit cooperative with theory and a test. *Quarterly Journal of Economics*, 109(2):491–515, May 1994.

[7] Eric Bellman. Invisible hand: Entrepreneur gets big banks to back very small loans. *Wall Street Journal*, page A1, May 15, 2006.

[8] Timothy Besley and Stephen Coate. Group lending, repayment incentives and social collateral. *Journal of Development Economics*, 46(1):1–18, February 1995.

[9] Morris H. DeGroot. *Probability and Statistics*. Addison Wesley, 2nd edition, 1986.

[10] Jeremy T. Fox. Estimating matching games with transfers. Working Paper, January 9, 2006.

[11] Shubhashis Gangopadhyay, Maitreesh Ghatak, and Robert Lensink. On joint liability lending and the peer selection effect. *Economic Journal*, 115(506):1005–1015, October 2005.

[12] Maitreesh Ghatak. Group lending, local information and peer selection. *Journal of Development Economics*, 60(1):27–50, October 1999.

[13] Maitreesh Ghatak. Screening by the company you keep: Joint liability lending and the peer selection effect. *Economic Journal*, 110(465):601–631, July 2000.

[14] Maitreesh Ghatak and Timothy W. Guinnane. The economics of lending with joint liability: Theory and practice. *Journal of Development Economics*, 60(1):195–228, October 1999.

[15] Jean Dickinson Gibbons and Subhabrata Chakraborti. *Nonparametric Statistical Inference*. Marcel Dekker, New York, 4th edition, 2003.

[16] Joseph Kaboski and Robert M. Townsend. Borrowing and lending in semi-urban and rural Thailand. Mimeo, 1998.

[17] Patrick Legros and Andrew F. Newman. Monotone matching in perfect and imperfect worlds. *Review of Economic Studies*, 69(4):925–942, October 2002.

[18] Paolo Mauro. Corruption and growth. *Quarterly Journal of Economics*, 110(3):681–712, August 1995.

[19] Jonathan Morduch. The microfinance promise. *Journal of Economic Literature*, 37(4):1569–1614, December 1999.

[20] Loic Sadoulet. Equilibrium risk-matching in group lending. Working paper, June 1999.

[21] Joseph E. Stiglitz. Peer monitoring and credit markets. *World Bank Economic Review*, 4(3):351–366, September 1990.

[22] Joseph E. Stiglitz and Andrew Weiss. Credit rationing in markets with imperfect information. *American Economic Review*, 71(3):393–410, June 1981.