Incorporating algebra into the elementary grades has become a focus for teachers, principals, and administrators around the country. Algebra is commonly regarded as a gateway to future opportunity (e.g., Moses and Cobb 2001), and elementary mathematics standards at both the state and national levels now reflect this effort to provide students with opportunities to learn critical concepts before middle and high school (California Department of Education 1997; NCTM 2000). However, implementing algebra standards at the elementary level is challenging—how do mathematics educators effectively and meaningfully incorporate algebraic ideas into K–5 curriculum? When elementary teachers are unfamiliar with early algebra, lessons designed and labeled as algebraic may become arithmetic exercises; the algebra then remains hidden from both the teacher and students in the implementation. The result is that the algebra standard is only superficially addressed.

From 2003 to 2006, we were members of a collaborative team of teachers and researchers on a project to develop an algebraic lesson sequence in grades 3–5. The goal of the larger project was to support students’ algebraic reasoning through functions and multiple representations of those functions, including verbal descriptions, tables, graphs, and letter notation (Schliemann, Carraher, and Brizuela 2006; Carraher et al. 2006). In the context of this project, we regularly discussed how our lessons were algebraic. These discussions were informed by the Algebra Standard (NCTM 2000), which emphasizes “relationships among quantities, including functions, ways of representing mathematical relationships, and the analysis of change.”
The vision is that elementary school provides the conceptual foundation for formal algebra in middle and high school. Yet what effective instructional strategies make algebra accessible and meaningful to elementary students? How can we foster algebraic reasoning in the classroom?

This article examines our experience with teaching the Dinner Tables problem in third grade, a lesson commonly used in upper elementary grades for its algebraic potential. The goal was to engage students in recognizing, extending, and predicting the use of patterns. We wanted to ensure that the lesson did not turn into an arithmetic exercise. Using the context of this lesson, we describe how we began to recognize the recurring role of particular instructional strategies throughout the entire lesson sequence. When these strategies were used, the students began to extend and predict using mathematical patterns, instead of completing pattern and function activities by rote without considering the algebra involved. As described, these strategies were the use of (1) unexecuted number expressions, (2) large numbers, and (3) the representational context.

### Three Instructional Strategies for Early Algebra

The Dinner Tables problem was presented as follows: Students' tasks included finding the maximum number of people that can be seated at as many as seven tables in both a function table and in a drawing. Students were then asked to construct a rule or formula that uses the number of tables to determine the number of people. The purpose of this question was to have students consider the input-output relationship of the function. For example, a total of four people can sit at one table; two tables will accommodate six people; three tables can accommodate eight people; and so on (see fig. 1).

This lesson was designed to be algebraic. However, implementation can impact how algebraic a lesson really is. For example, the lesson could have easily turned into a problem of “adding on by twos” when students completed a function table, potentially rendering the lesson an arithmetic task instead of an algebraic one. Three strategies helped keep the algebra in the foreground of the lesson.

#### Using unexecuted number expressions

An unexecuted number expression refers to a non-computed sequence that allows for the consideration of the numbers and operations within it. For example, in the case of three tables, a total of 3 + 3 + 2, or 3 × 2 + 2, people can be seated. Whereas these expressions are equal to one value, 8, the numbers within the expressions became a springboard for discussion in our classroom and a tool to extend and predict. Students said that three tables would accommodate a total of eight people, which was repeated back to students using an unexecuted expression:

**Teacher:** Yeah. We can just count them. There are three on top [the head of the table], three on the bottom [the foot of the table]. And how many are on the sides?

**Richard:** Two.

**Teacher:** So three and three and two is going to give us?

**Students:** Eight.

Students found eight as the total value, but “three and three and two” drew attention to the numbers within the expression and how these numbers map onto the drawing in figure 2. After asking a similar question for four tables, the teacher moved to ten tables. Instead of needing to continue from ten people and add on by twos, students began to generalize the idea by making use of the patterns, moving from $3 + 3 + 2$ and $4 + 4 + 2$ to $10 + 10 + 2$, building off the original problem context that a certain number...
of people sit at the head and foot of the table with two more on the sides.

Teacher: There are ten tables. How many people will there be? Ashley, do you have a guess? Ashley: Twenty-two. Teacher: You think twenty-two.

Richard: Oh, that’s what I guessed!

Teacher: Can you tell me where that number came from, Ashley?

Ashley: There will be ten on the top, then ten on the bottom, then two on the sides.

As Ashley says that “there will be ten on the top, then ten on the bottom, then two on the sides,” the teacher writes this on the whiteboard both as a final value—22—and also as an unexecuted expression—10 + 10 + 2 (see fig. 3), setting the stage for students to extend the pattern further to predict for 100 tables and then for any number of tables.

Making the unexecuted strand of numbers explicit provides a way to record information from a problem and encourages students to discern emerging patterns. (Kaput and Blanton [2005] provide an example in teacher professional development.) Evaluating the parts of the unexecuted expression is a powerful, strategic tool for instruction in early algebra. In the case of 3 + 3 + 2, what do these numbers represent? In this context, what justifies a particular operation? When considering 3 + 3 + 2 and then 4 + 4 + 2, what patterns begin to emerge? In the classroom, an expression like 3 + 3 + 2 not only retains a connection to the original problem context but also allows students to extend patterns to 4 + 4 + 2 or 5 + 5 + 2 to eventually predict the output for any number of tables—an important goal of the abstraction involved in early algebra. Considering only the final value hides important pieces of mathematics.

Using large numbers

Extending the Dinner Tables problem to include large numbers of tables, such as 100 or 1,000, provides a compelling reason for students to consider the relationship between the input (number of tables) and the output (number of people) and make it abstract—an essential piece of the algebra. One common way to organize this information is in the form of a function table. However, a common complaint about this particular representation is that students often move down columns and rarely move across rows, thereby seldom considering how a given input relates to the corresponding output. Students frequently find the difference from output to output as a way to fill in missing values in the table, a task that can be more arithmetic than algebraic, as the function table in figure 4 shows.

Schliemann, Carraher, and Brizuela (2001) provide a way to promote algebraic reasoning with function tables through the use of large numbers. In a function table, they make use of a break in the

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**Figure 2**

Analyzing the drawings led to generalizing the numeric expression.

<table>
<thead>
<tr>
<th>Drawing</th>
<th>People</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td>4 people</td>
</tr>
<tr>
<td><img src="image2.png" alt="Diagram" /></td>
<td>6 people</td>
</tr>
<tr>
<td><img src="image3.png" alt="Diagram" /></td>
<td>8 people</td>
</tr>
<tr>
<td><img src="image4.png" alt="Diagram" /></td>
<td>10 people</td>
</tr>
</tbody>
</table>

**Figure 3**

A break in the number sequence on the function table allowed students to predict values, which demonstrated algebraic thinking.

<table>
<thead>
<tr>
<th>Table</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>2^2</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>2^2</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>2^(1/2)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>22</td>
<td>(10+10)</td>
</tr>
</tbody>
</table>

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Photograph by Janice Gordon; all rights reserved
sequence of inputs, as illustrated in figure 3 when moving to ten tables. It was possible for students to draw ten dinner tables and count the seats or to keep adding on by twos until reaching the output for ten tables. Drawing 100 dinner tables shows this strategy to be inefficient and time consuming, a fact students quickly grasped. After introducing a large number to have students move from input to output, the teacher had Shayne explain why he and other students said there would be a total of 202 people.

Teacher: Can someone explain? Shayne, do you want to quickly explain how you came up with that number?
Shayne: Because 100 times 2 equals 200, and then 200 plus 2, and [pause] 200 plus 2 equals 202.

Shayne successfully provided a number expression to solve for 100 that did not rely on the response for 99 tables, as some responses for 10 tables relied on the result for 9 tables, and 8 before that, and so on. Rather, he made a generalization to consider how to move from the input to the output.

Teacher: So, he’s saying he multiplied 100 times 2, which is 200, and then he added 2. Why did he do 100 times 2? Who can explain that? Ashley?
Ashley: Because there are 100 on the top and 100 on the bottom.

By incorporating large numbers into the lessons, we found that students sought out new patterns and generalizations in order to answer questions in an algebraic way. Large numbers dissuaded the use of the arithmetic adding on strategy and at the same time promoted algebraic reasoning about the input-output relationship. We frequently used the strategy of large numbers to consider the functional relationship of input to output.

Using representational context
The term representational context refers to the interactions and discourse constructed in a classroom around a particular representation (Ball 1993, p. 157). Representational contexts serve as a way to ground students’ developing mathematical ideas and reasoning. The use of a representational context is prevalent throughout Principles and Standards (NCTM 2000) and in much reform curricula.

In the case of the Dinner Tables problem, third-grade students were able to use an algebraic expression, $n \times 2 + 2$, to represent the problem. Does this necessarily mean they were making connections about this expression related to aspects of the original drawing? What does it mean when students use letter notation, and what is it they are representing? After students brought up this notation in the final class discussion, the teacher asked them what the parts of the expression meant.

Teacher: She has $n$ times two plus two. What does that stand for, Sean?
Sean: Any number!
Teacher: But any number of what?

There was a pause in the classroom as students thought about the question. Most students had picked up on the idea that letters could be used to stand for any number, but now they were being asked to apply this rule to the original context. What is the significance of $n$ in the Dinner Tables problem? After a long pause, one student suggested that $n$ stands for the number of people; others said that $n$ stands for the number of tables. After some consideration, students were able to generate a correct response, but the question that connected the algebraic expression to the original problem context was not trivial.

We found that having a rich problem context was not enough in and of itself to promote algebraic thinking. Rather, strategic and purposeful use of the representational context was necessary to push students to reason algebraically and make connections across representations. By asking what the expression $n \times 2 + 2$ stands for, the teacher helped make the link between the notation and the original problem context. We found this type of question...
essential to providing a motivation for students to use letter notation at all.

In order to recognize when letter notation provides a useful way to represent a situation, connections such as this one across representations—in this case, drawing tables and writing a symbolic expression—are crucial. After establishing that $n$ stands for any number of tables, the teacher then asked about the “times two,” which Alizé—who had struggled with this idea during individual work—now answered with confidence.

Teacher: Why am I multiplying it by two? If I know the number of tables, why do I need to multiply it by two? Alizé?

Alizé: You have to multiply it by two because when you put the tables together, the top and the bottom have the same number of chairs.

Alizé’s explanation provides additional meaning to the expression $n \times 2 + 2$. Applying the rule to the representational context is a way to help students reach a deeper and more flexible understanding of the algebraic expression, one essential reason to develop algebraic reasoning with young students.

Conclusion

Incorporating algebra into an already packed mathematics curriculum can be an elusive task, and educators still have much to learn about how to facilitate this implementation. Most adults learned arithmetic and algebra as separate strands of mathematics, with algebra kept a secret until grade 8 or 9. Therefore, we have little personal experience as learners to relate to our practice. Our collaboration as a researcher and a teacher, respectively, helped us see how algebra can be successfully incorporated into the elementary classroom.

Using the instructional strategies discussed in this article helped make the implementation of early algebra less elusive by allowing students to see patterns, make generalizations, and move across representations—core goals of algebraic reasoning in elementary school.

The Dinner Tables problem in particular is rich with opportunities to generalize and make abstractions. Nevertheless, these strategies can apply to any mathematical domain and in any classroom in which teachers are striving to push kids to think algebraically. Although neither author currently works with the project from which these examples came, we each continue to use these strategies in classrooms.

We encourage all elementary teachers to incorporate algebra into their mathematics classrooms. Keep in mind that learning how to do this is a process, one that is not always transparent. Collaboration with colleagues provides the opportunity to share the effectiveness of instructional strategies, compare stories of student learning, and ultimately learn from one another. The process takes time, but effective instruction in early algebra ultimately creates powerful young mathematical thinkers and provides an essential conceptual foundation for later work in higher mathematics.

References


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