Can You Find the Center of a Circle?

Abstract
Finding the center of a circle, when you have limited knowledge and capabilities, is an interesting problem that requires the discovery and understanding of a number of geometric concepts. By presenting the problem as a real world problem, such as searching for a skier that got stuck in an avalanche, students should find the activity engaging. A software simulation and a curriculum unit are proposed so that students can explore the problem of finding the center of a circle. The curriculum unit, designed for high school students, includes the simulation of a search for a lost skier using PDAs. The simulation provides a refreshing break from the usual classroom teaching style. It gives students a chance to reason about geometric concepts and improve their problem solving skills in a way that they can find meaningful.

1 Introduction
The motivation for the proposed software stems from a paper in computational geometry. In a research meeting, we discussed a paper entitled “Searching for the Centre of a Circle” (Biedl et al., 2002). The paper discusses the problem of finding the center of a circle in the most efficient way possible when the searcher has a limited number of capabilities. The paper proposed some interesting solutions using simple geometric properties. Though the paper was written by a number of professors and graduate students, I believe that the concepts used are easily understood by high school students.

There are a number of practical applications of finding the center of a circle that will be used in the project to motivate and provide meaning for the students. One application is robotics. In this field, robots are often required to find a particular point with very limited resources. Another application, which the project that I have designed is based on is finding a skier who has been buried by an avalanche. I think that the problem could also be motivated by other searching problems such as finding something at the bottom of the ocean.

1.1 The problem
For the avalanche application, all skiers carry transceivers with them that are set to transmit. If an avalanche should occur, then the skiers that do not get stuck become searchers and turn their transceivers to the receive position. They can then hear the signal from the buried skier. The transceivers are equipped with volume knobs so that searchers can hear the buried
skiers transmission and have a general idea of whether they are getting closer or farther away from the signal. Searchers have no way of measuring absolute distance and they cannot tell in which direction the buried skier lies. What searchers can do is tell whether or not they are in a circular range of the buried skier. It is also assumed that the searchers can mark points that they visit by leaving behind a ski pole or other non-essential item. They can move in straight lines, and they can make 90 and 180 degree turns. Lastly, we assume that searchers are able to determine the halfway point between two marked points assuming that they are on the line between the points.

2 Powerful Ideas

Solving this problem efficiently requires the discovery of many powerful ideas ranging from ideas about geometry to problem solving. Powerful ideas, as described by Papert (Papert, 1991), are powerful in their use, powerful in their connections, and powerful in their roots. Therefore, the powerful ideas relevant to this curriculum are properties of circles and chords, estimation, midpoints, angles, and problem solving with limited resources.

In this curriculum students will need to combine all of these ideas to come up with a successful strategy for finding a buried skier. Therefore, students should find each of these individual ideas powerful in its use. The software is based on the real world problem of finding a lost skier so that students will also find that they are powerful in its connections.

The last property of powerful ideas, that an idea is powerful in its roots, is somewhat more difficult to see than the other properties. Throughout this curriculum unit, students will be applying the ideas of geometry to searching. Searching provides students with a chance to use internal knowledge. There are many different approaches to searching, each affected by students’ prior knowledge of the situation. This gives students a sense of ownership of their ideas. One student’s approach to the problem can be very different from another’s, but both approaches can be successful.

3 The Software

The software proposed here is one that will give students the opportunity to explore the problem of finding a buried skier that has been lost in an avalanche. This software is designed to run on PDAs which gives participants the opportunity to fully simulate the search in an outdoor setting. There are two different interfaces that come with this software. The first is designed for the students and the second is designed for the teachers or advisors of the simulation.

The student version models the transceiver described in section 1.1. The transceiver has very limited capabilities so the interface is simplistic. The only tasks that the students can do with the transceiver are adjust the volume setting and decide to dig for the buried skier. Based on the volume setting of the transceiver, students need to know whether or not they
are in range of the buried skier. This information is displayed clearly in the center of the screen. See figure 1.

Once a student thinks they could be standing over the buried skier, he can decide to dig for the skier. Given that this is a simulation, the student clicks the dig button on their screen to see if they have successfully chosen the correct spot to dig. A message tells them whether or not they have found the skier. See figure 2.  

The interface for the teacher is a little bit more complicated because the teacher has the power to track the progress of all participants in the simulation. In the teacher interface, there is a graphic that shows the location of the buried skier in the center with concentric circles around him. The concentric circles represent the range of the buried skier’s transceiver based on each of the volume settings. Also, displayed on the screen are dots. Each dot represents a student’s location. The teacher can click on a dot and it will bring up a screen displaying information about the student’s search path and when the student has tried to dig. The teacher receives messages in a message box each time a student tries to dig. The message tells the teacher which student dug and whether or not they were successful. The teacher interface is also equipped with two buttons. When the ‘end simulation’ button is clicked it sends out a message to all simulation participants that its time to gather and head back inside. The ‘call participant’ button gives the teacher the ability to call a particular student over to discuss something. See figure 3 for a screen shot from the teacher interface.

Figure 1: A screen shot from the transceiver interface for students.

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1A prototype of this interface is written in java and can be found at http://www.cs.tufts.edu/~kmann03/cd145/finalproject.htm
Figure 2: A screen shot from the transceiver interface where the student chose the wrong place to dig.

Figure 3: A screen shot from the teacher interface.
4 Curriculum

4.1 Goals

By having students use this software for this curriculum unit, students should learn more about topics related to the powerful ideas discussed in section 2. Papert states

When ideas go to school they lose their power, thus creating a challenge for those who would improve learning to find ways to re-empower them. (Papert, 1991)

The goal of this curriculum unit is, therefore, to rise to Papert’s challenge and re-empower these ideas for students. More specifically, by the end of the unit, students’ problem solving ability will increase as well as their ability to reason about certain geometric properties.

Also, this unit covers some of the guidelines from the Massachusetts Curriculum Frameworks. The framework states that in grades 9 and 10, students should be able to apply properties of chords to solve problems. It is also suggested in the framework for grades 11 and 12. In fact, this problem is similar to one of the sample problems given for this subject. The problem states:

Your shot put circle was washed out in a storm. There is only a portion left. You can redraw the circle if you know its center. Explain how you can use geometric construction and the properties of circles to find the center of the original circle. (Education, 2000)

This problem is somewhat similar to the one students will be solving using this software in that it draws on similar knowledge and, therefore, should integrate nicely into the existing curriculum.

4.2 Rationale

The most important skill that students will take away from participating in this simulation is the ability to problem solve. People are required to search with limited resources often in everyday life. The more students are exposed to these types of problems, the better they will do the next time. Students will also realize that there are many different ways to approach solving problems.

4.3 Materials

This simulation requires the use of a large open space. Here we are assuming that the range of the transceivers that we simulate is 30m. Any open space that can contain a circle with radius 15m will work fine. The space need not be flat and, in fact, would model the real world search for a skier better if it were sloped. Any high school sports field or even parking lot will suffice. Since the simulation will be run multiple times with each set of students, a
larger space is ideal. The larger the space, the more variance in the position of the lost skier across multiple instances of the simulation.

Secondly, each student and teacher needs a PDA. The PDA must be equipped with the software described previously, and also the technology to listen for the buried skier’s signal. Infrared, which is standard on many PDAs does not have a large enough range. It is possible that bluetooth, technology could be used and is probably the most cost effective choice. However, many bluetooth devices are designed to be short range in which case a GPS enabled PDA would be required. Unfortunately, adding GPS capabilities to the PDA will increase its price significantly. Using PDAs with GPS capabilities will also make automatic data collection easier.

A laptop, although not absolutely necessary, would be very useful. While students are testing their strategies, the laptop could collect data about the paths they took. After the simulation is over and students are back in the classroom, both the teachers and students could look at the path that they took in relation to the location of the skier and the concentric circles created by the emitted signal of the lost skier’s transceiver.

During the simulation students will be able to mark places that they have visited. Objects to represent these places will be necessary. The best markers would be sticks that can be easily placed in the ground and each student would receive different colored sticks making markers easily distinguishable between students. In the absence of such objects, everyday items will suffice. For example, students can bring in empty milk bottles and draw their initials on them.

For the purposes of learning the program a projector may be necessary. This will allow the teacher to project the screen of a PDA so students can follow along with the teacher when the software is being introduced. Other materials that may be necessary include common classroom items such as paper, a chalkboard, pencils, and folders.

4.4 Process

This curriculum unit is flexible and can easily be adapted based on the time available and the ability of students. I will present one possibility here. All classes have a different dynamic and a different level of ability and, therefore, this curriculum proposal should be modified as each teacher sees fit. I would expect this unit to be used in a high school level math class. It is most appropriate for a geometry class. The amount of time spent on the unit is expected to be anywhere from four to eight class periods. Most phases should take 45-60 minutes, or the length of a typical class period. They can be spread out over several weeks or done all in a row.

Also, it is assumed that some basic coverage of circles, chords, angles, and midpoints is has already taken place in class. This unit is designed to get students to take information that they have already been presented, realize the relationship between different concepts, make some observations and apply them to real world situations.
4.4.1 Phase 1: Introduction to the Problem

The first phase is very short and takes place in the classroom. The purpose of this phase is to familiarize students with the PDAs and the simulation program. If students have prior knowledge of PDAs parts of this can be skipped.

The teacher will give each student a PDA with the avalanche simulation program installed. The teacher will also have a PDA and project its screen on the wall so that students can follow along with their own PDAs. The teacher will demonstrate how to start and end the simulation. The teacher will also show students how to dig for a lost skier, how to adjust the volume, and what the interface will look like when the transceiver is in and out of range of the lost skier.

The teacher will then explain the simulation to the students. Students will be told about the lost skier, marking points that they have visited, the fact that the transceiver’s range is a circle and anything else that the teacher might find relevant for their class. For example, a teacher might go over rules for outside behavior and locations of meeting places.

Lastly, the teacher will outline the goals for phase 2, which are to figure out their abilities as a searcher.

4.4.2 Phase 2: Exploration

The next phase will be an exploratory phase and take place outside. During this phase, students will go outside and run the simulation. Students are expected to become familiar with the software and the PDA. Student will not yet be expected to come up with a strategy for finding the skier. In this phase it is expected that students will figure out their capabilities as searcher and become familiar with the area where the simulation takes place. I would expect the students to use a method involving trial and error.

4.4.3 Phase 3: Discussion

Phase 3 will take place back in the classroom. Here the teacher will lead the class in a discussion about the outdoor exploration session. During this discussion its important that the capabilities of searchers be made clear to all students. The teacher should ask questions such as: what kinds of actions can you perform? and is there a way to measure distance? to promote discussion. Through this discussion, as a minimum, students need to understand how to perform the following actions:

- decide whether you are inside or outside of a circle
- mark the point you are currently at
- find the midpoint between two markers
- turn $180^\circ$
During this discussion students may bring other capabilities to the class’ attention. Regardless of whether the capabilities seem relevant, they should be considered. There are numerous ways to solve the problem of finding the lost skier. Creativity should be encouraged as it may lead to some interesting approaches.

Just as the capabilities of the searchers need to be made clear, so do some of the actions that the searchers cannot carry out. For example, it is important that students realize that there is no way to accurately measure distance. Some students may suggest counting their steps as a measure of distance, but they need to be reminded that in the real search for a skier, searchers will be on skis so counting steps is impractical.

4.4.4 Phase 4: Planning

By this phase all students should have a good understanding of the problem they are trying to solve and the resources they have to solve it. Before taking the students back outside, they should be given time to plan. Each student should document at least three different strategies that they plan to test when they go outside. The teacher should emphasize that strategies should yield repeatable results. They also should try to calculate the expected distance that they will travel based on the size of the circle created by the transceiver. This could be done as a homework assignment or during class. The teacher will then collect the documentation and go over each student’s strategies, making comments and suggestions as necessary.

4.4.5 Phase 5: Competition

During this phase students will return to the outdoor site where the exploration phase began. Students will be expected to test the three strategies they presented. Each should be tested a few times to see if their results are repeatable. During this phase, a laptop will record the paths taken by each participant. Once students have tested their three strategies, they will be given time to continue exploring. By following their three proposed strategies it is probable that students will discover things about their strategies which may lead them to new ideas. Students should be given the proper time to adapt their strategies and explore their new ideas.

4.4.6 Phase 6: Results

Once again, students will meet back in the classroom for discussion. Before discussion, students should also be given a chance to analyze their own results. They should compare their documented strategies and estimated distances to the actual distance traveled and comment on discrepancies. This could be assigned as homework.
The teacher will reveal the most successful strategies. The most successful strategies are those that require the searcher to travel the smallest distance. Students responsible for those strategies will be asked to explain their approach at the chalkboard. They will also tell the class how they arrived at that strategy.

Then a teacher led discussion should take place to figure out why these strategies were most successful. If not already pointed out by students who presented, the teacher will try to get the class to discover the underlying geometric properties of each strategy.

4.4.7 Phase 7: Testing more strategies

The need for this phase is dependent on the class’ performance in the previous phase. The teacher will need to judge the quality of student’s strategies and whether the class can do better and decide if the geometric concepts remain unclear to students. If so, it would be useful to ask students to propose one or two more strategies, keeping in mind everything they learned in the class discussion. With more background knowledge and more time invested in the projects, new results may arise. Again, students should calculate the expected distance traveled. If time permits, students could be taken outside again to test these new strategies.

4.5 Special Considerations

Because this is an outdoor activity, it will be difficult for one teacher to monitor many students. It might be helpful, depending on class size and behavior, to have extra adults participate in the outdoor portion of the project. Extra adults could help students refine strategies and make sure that students stay on task.

This task requires a lot of mobility. If there are any handicapped students in the classroom, special considerations will have to be made. Depending on the site chosen to complete the task, using wheelchairs may not be possible due to uneven ground.

4.6 Assessment

Assessment of the success of this curriculum unit on students understanding of geometric concepts and problem solving techniques should take several forms. Teachers can give pre and post tests, judge by performance in the competition, involvement in class discussion, and evaluation of the student’s proposed strategies for finding the lost skier.

Pre and post test will provide the teacher with the most concrete data, but the results may not represent everything that students have learned. Questions on the pre and post tests will need to test students ability to reason about circles. Questions should be similar to the one proposed in the Massachusetts curriculum frameworks that was discussed in section 4.1.

Performance in the competition may be a good means of evaluation because it could be a good measure of effort and engagement in the project. Since most teachers are required
to give students grades, this form of evaluation may motivate students and give teachers an initial ranking of students.

Evaluation of the students’ proposed strategies will be rather subjective but should give the teacher the best idea of students’ problem solving abilities. If the students employ sophisticated techniques and arrive at strategies that resemble solutions presented by Biedl (Biedl et al., 2002), then students really have a deep understanding of the geometric concepts as well a strong ability to solve problems. If, however, strategies are ad hoc then students did not really gain enough from the simulation.

The rubric presented by Jonassen (Jonassen, 2000) would provide a good systematic way of evaluating students’ work. The rubric could be applied to both students’ involvement in class discussion and students’ generation of strategies.

### 4.7 Extensions and Variations

The simulation presented here could be extended in numerous ways. As teachers observe the ability of their class, they could make adjustments. Here are a few suggestions:

1. Have students work collaboratively. Students would work in groups to two or three and each group would be assigned one PDA. Students would propose strategies separately, then meet as a group to discuss and combine ideas before the competition.

2. Students work collaboratively with multiple PDAs. Students would work in groups of two or three and each student would be assigned a PDA. Students would propose strategies that take advantage of having multiple transceivers and multiple searchers working together.

3. Students would look for multiple buried skiers where the signals of the buried skiers overlap.

4. A simulation could be applied to a different real world situation such as the robotics problem suggested at the beginning.

5. Add to the capabilities of searchers.

6. Add to the transceiver. For example, have the transceiver output the distance to the skier or the direction to the skier (but not both).

7. Make range of the transceiver an ellipse instead of a circle.

8. Run the simulation in an after school program rather than in a high school classroom.

9. Run multiple versions of the simulation and have students compare results.
5 Conclusion

Finding the center of a circle is an interesting problem that requires the discovery and understanding of a number of geometric concepts. By presenting the problem as a real world problem students should find the activity engaging.

The most important aspect of this software and simulation that it gives students the opportunity to reason about geometric concepts and improve their problem solving skills in a way that they can find meaningful. Having good problem solving skills is invaluable in all aspects of life, from the small things that people encounter in daily life, to problems in work and school to problems in family life.

Developing students’ problem solving skills is a critical part of their education. Often it is very difficult to motivate students to solve problems, especially when they do not see a reason or an application for that problem. This software takes an interesting real life problem that has been studied by many and that expert skiers are trained to solve. Properties of chords and circles can seem useless when presented in a textbook. This software and curriculum unit give that meaning, giving students a chance to identify with the problem and make discoveries on their own.

References


