Mathematical Model of Morphogen Electrophoresis Through Gap Junctions

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Gap junctional communication is important for embryonic morphogenesis. However, the factors regulating the spatial properties of small molecule signal flows through gap junctions remain poorly understood. Recent data on gap junctions, ion transporters, and serotonin during left–right patterning suggest a specific model: the net unidirectional transfer of small molecules through long-range gap junctional paths driven by an electrophoretic mechanism. However, this concept has only been discussed qualitatively, and it is not known whether such a mechanism can actually establish a gradient within physiological constraints. We review the existing functional data and develop a mathematical model of the flow of serotonin through the early Xenopus embryo under an electrophoretic force generated by ion pumps. Through computer simulation of this process using realistic parameters, we explored quantitatively the dynamics of morphogen movement through gap junctions, confirming the plausibility of the proposed electrophoretic mechanism, which generates a considerable gradient in the available time frame. The model made several testable predictions and revealed properties of robustness, cellular gradients of serotonin, and the dependence of the gradient on several developmental constants. This work quantitatively supports the plausibility of electrophoretic control of morphogen movement through gap junctions during early left–right patterning. This conceptual framework for modeling gap junctional signaling—an epigenetic patterning mechanism of wide relevance in biological regulation—suggests numerous experimental approaches in other patterning systems. Developmental Dynamics 235:2144–2159, 2006.

Key words: gap junctional communication; embryonic patterning; asymmetry; model; electrophoresis; serotonin

INTRODUCTION

The mechanisms that allow an embryo to reliably self-assemble the complex morphology, physiology, and behavior appropriate to its species represent one of the most fundamental and fascinating areas of research in modern science. The events underlying the generation and maintenance of form require a complex web of information flow between cell and tissue subsystems during development. Receptor-mediated signal exchange by means of secreted messenger molecules has been studied extensively in the age of molecular biology. However, another important system of signaling exists: the direct cell–cell exchange of small molecules through gap junctions (Goodenough and Musil, 1993; Lo, 1999).

GAP JUNCTIONAL SIGNALING

Gap junctions can consist of proteins from several gene families, and the cell biology of gap junctions has been described in detail in excellent recent reviews (Goodenough et al., 1996; Falk, 2000). Vertebrate gap junctions are commonly thought to consist of connexins (Sohl and Willecke, 2003, 2004). In contrast, invertebrate gap junctions are made of innexins, a family formerly called OPUS (Barnes, 1994). A variety of small molecules and metabolites are thought to permeate gap junctional paths, including cAMP (Burnside and Collas, 2002;
Several human syndromes have been identified as mutations in connexin genes (Maestrini et al., 1999; Richard et al., 2002), and transgenic mice are beginning to allow the molecular dissection of gap junctional communication (GJJC) function in these contexts (Krutovskikh and Yamasaki, 2000), including the spread of electric waves in cardiac tissue (Kimura et al., 1995; Severs, 1999) and the brain (Budd and Lipton, 1998; Momose-Sato et al., 2003, 2005), and the propagation of signals through gland cells to synchronize hormonal action and secretion (Meda, 1996). Regulation of tumor (Loewenstein and Rose, 1992; Yamasaki et al., 1995; Krutovskikh and Yamasaki, 1997; Li and Herlyn, 2000; Omori et al., 2001) and stem cell (Trosko et al., 2000; Burnside and Collas, 2002; Tazuke et al., 2002; Gibboa et al., 2003; Cai et al., 2004; Wong et al., 2004) behavior is dependent upon gap junction (GJ)-mediated signals.

When open, intercellular channels are generally impermeable to molecules of less than 1 kDa (Loewenstein, 1981), but the exact permeability is a function of (1) precisely which connexin family members form the junction (White and Bruzzone, 1996), (2) the charge and size of the permeant molecule (Landesman et al., 2000; Nicholson et al., 2000), (3) the pH of cytoplasmic and intercellular space (Morley et al., 1997), (4) transjunctional and membrane voltage of the cells (Brink, 2000), (5) the phosphorylation state of the connexin protein subunits (Lampe and Lau, 2000), and (6) the activities of several chemical gating molecules (Granot and Dekel, 1998). Thus, functional GJC is dependant on the existence of compatible hemichannels on the cell's surfaces, the permeability of the hemichannels to the substance, and the open status of the gap junction. Regulated GJC paths serve to establish patterns of iso-potential and/or iso-pH cell fields (Sherman and Rinzel, 1991; Fitzharris and Baltz, 2006; Rocheleau et al., 2006); this epigenetic “prepattern” overlaid upon the embryonic morphology, working in concert with the molecular fields defined by gene expression domains, is an important control element in embryonic patterning (Levin, 2003a; Nuccitelli, 2003; McCaig et al., 2005). This extremely versatile system for communication allows for rapid synchronization among cells in a tissue and the passage of signals, both of which can be regulated at many levels. Thus, it is a perfect conduit for information flow during development, which depends on the ability of cells and tissues to communicate on several time scales (Lo, 1996; Levin, 2001). The converse, however, is also paramount—embryos contain independent compartments that must remain isolated with respect to key signaling molecules for proper morphology to result.

GJC is increasingly recognized as being involved in regulation of patterning signals in vertebrate and invertebrate pattern formation (Bauer et al., 2001, 2002; Starich et al., 2003; Nogi and Levin, 2005; Lehmann et al., 2006). Such morphogenetic control could be exerted by GJC-mediated changes in differentiation (Zhang et al., 2002; Araya et al., 2003, 2005; Gu et al., 2003; Hirschi et al., 2003), proliferation (Paraguassu-Braga et al., 2003; Pearson et al., 2005), or cell migration (Minkoff et al., 1997; Huang et al., 1998; Lecanda et al., 2000; Oviedo-Orta et al., 2002; Kjaer et al., 2004). To integrate GJC-dependent signals into the systems that control complex morphology, it is necessary to understand the factors regulating spatial flows of signals through gap junctions.

Gap Junctions in Left–Right Asymmetry

A considerable amount of data on the function of GJC in vertebrate patterning, including evolutionary conservation in two species, is now available in the field of left–right (LR) patterning (Burdine and Schier, 2000; Levin, 2005). The vertebrate body is based on a bilaterally symmetrical plan; however, the visceral organs and brain display marked and consistent asymmetries in their location or geometry with respect to the embryonic midline. The LR asymmetry is a fascinating example of large-scale embryonic patterning and raises many deep theoretical issues thought to link molecular stereochemistry with multicellular pattern control (Brown and Wolpert, 1990; Wood, 1997). Because no macroscopic force distinguishes right from left, a powerful paradigm has been proposed to leverage large-scale asymmetry from the chirality of subcellular components (Brown and Wolpert, 1990; Brown et al., 1991). In this class of models, some molecule or organelle with a unique chirality is oriented with respect to the anteroposterior and dorsoventral axes, and its chiral nature, thus, is able to nucleate asymmetric processes. Thus, the first developmental event that distinguishes left from right is proposed to take place on a subcellular scale. However, it is now known that a pathway of multicellular fields of asymmetric gene expression directs the laterality of asymmetric organs (Levin, 1998; Yost, 2001). A mechanism must then exist to transduce subcellular signals to cell fields and to transduce information about L or R direction into a cell’s position with respect to the embryonic midline.

In light of evidence for preferentially directional and pH-dependent GJC-mediated transfer in the early Xenopus embryo (Turin and Warner, 1980; Guthrie, 1984; Guthrie et al., 1988; Nagajski et al., 1989), Levin and Mercola tested the hypothesis that GJC could play a role in the mechanism by which asymmetry at the level of a cell can be transduced into embryo-wide asymmetry of gene expression (Levin and Mercola, 1998, 1999). Spatially oriented multicellular GJC paths could allow directional information derived within a cell to be imposed differentially upon its neighbors, resulting in global positional information. Using injections of a system of a small junctionally permeable fluorescent dye together with a large molecule, junctionally impermeable dye (to mark the injected cell and rule out false positives due to cytoplasmic bridges and incomplete cell cleavage), we showed that there is indeed a dorsoventral difference, with a zone of isolation across the ventral midline and good junctional coupling on the dorsal side (Fig. 1A). This asymmetry in ability to transfer dye could be altered in predictable ways by chemical agents that are known to regulate junction permeability. Using dominant-negative and constitutively ac-
tive constructs, it was shown that interruption of the circumferential GJC path, as well as introduction of ectopic open gap junctions across the zone of isolation, both randomized early asymmetric gene expression and position of the asymmetric viscera in frog embryos (in the absence of other defects). Similar data were obtained in the chick—an embryo with very different gastrulation architecture—using antisense oligonucleotides and a variety of surgical manipulations to interrupt the GJC zone (Fig. 1B). Taken together, the results indicated that a zone of circumferential connexin43 expression around the primitive streak (which does not express Cx43) is required for normal laterality.

While much remains to be learned about the individual connexins functioning in the frog embryo and the permeabilities of the native gap junctions for different molecules (Dicaprio et al., 1975; Landesman et al., 2000, 2003), these experiments revealed a very similar picture of the role of GJC in LR patterning in two embryonic systems. In both chicks and frogs, a circumferential large-scale pattern of GJC exists around a zone of isolation. The contiguity of this path is crucial for normal expression of LR-asymmetric genes; interruption of this path by surgical, pharmacological, or molecular-biological methods specifically causes LR randomization. The zone of isolation is likewise crucial for LR patterning. Taken together, these observations suggested a simple model: that some small molecule signal is initially homogenously distributed, but then traverses gap junctions and preferentially accumulates on one side of the zone of isolation when it arrives there (Levin, 2003b; Adams et al., 2006b). This accumulation can then induce asymmetric gene expression using canonical mechanisms. In this model, interfering with the open GJC path results in LR randomization by preventing the movement of the morphogens, whereas introducing GJC through the zone of isolation randomizes asymmetry because there is then no midline barrier that can ensure an asymmetric accumulation of the morphogens.

The evolutionary conservation of LR patterning mechanisms is highly controversial (McGrath and Brueckner, 2003; Tabin and Vogan, 2003; Levin, 2004b); early steps are especially poorly understood, and it is now imperative to integrate available genetic and cell-biological data into comprehensive models that can be tested and applied to different model species. The class of models based on GJC raised two main lines of inquiry.

(1) What controls the unidirectional (chiral) flow of LR information through the gap junctions? Why would morphogens traverse the GJC path counterclockwise rather than equally in both directions (resulting in no net gradient)? Proposing that an electrophoretic mechanism might provide a force for movement of charged determinants through the GJC field, we screened for the involvement of ion transporters in asymmetry. Identifying several ion channels and pumps required for normal laterality, we showed that early frog and chick embryos contain the predicted asymmetries in membrane voltage at the zone of isolation (Levin et al., 2002; Adams et al., 2006b) and that asymmetry is randomized when these physiological LR differences are abolished by equalization of the activity of the implicated H+ and K+ transporters. Thus, one possibility is that the cells on the L vs. R side of the zone of isolation develop different levels of polarization (steady-state membrane voltage level) by differential ion exchange with the outside world. We have demonstrated recently some of the cytoplasmic transport mechanisms that result in such an asymmetric localization of the relevant ion channel and pump proteins (Qi et al., 2005). As long as this difference was actively maintained, an electrophoretic force would be exerted through the other part of the circuit formed by the GJC field (Fig. 1C).

(2) What is the molecular nature of the small-molecule LR signals that might be exchanged between cells on the L and R sides? The ideal candidate would be smaller than the size cut-off of gap junctions (< ~1 kDa), be water-soluble (lipophilic molecules such as retinoic acid do not need gap junctions to move between cells), and be charged (to enable regulation of movement by means of ion pump-dependent voltage gradients as proposed in Levin and Nascone, 1997; Levin et al., 2002; Levin, 2003b). Serotonin fits these criteria, has been demonstrated to go through gap junctions between nervous system cells (Wolszon et al., 1994), and offers the considerable advantage of a well-developed pharmacological tool set (Gaster and King, 1997).

Thus, taking advantage of the large number of well-characterized reagents available to test and characterize its role in LR asymmetry in chick and frog embryos, serotonin’s role in asymmetry, before the formation of

Fig. 3. Dependence of final serotonin concentration on voltage difference. The stationary serotonin concentration cS(x) (given by Eq. (4), final state that can be produced by the mechanism) across the Xenopus embryo is an exponential function and is given here for voltage differences between ~10 mV and ~40 mV. Medial circumferential length indicates position along the path from L to R border in an embryo roughly 1 mm in diameter (see Fig. 2).

Fig. 4. Dependence of overall gradient gain on the electrical charge of morphogen. The serotonin right–left gain, which follows from the stationary concentration given in Eq. (4), exhibits an exponential dependence (shown here on a logarithmic plot) on both the electric charge of the morphogen molecule and the voltage difference. Thus, similar right–left gains (in different biological systems) may be obtained by different combinations of the morphogen charge and potential differences.

Fig. 5. The dependence of overall gradient gain (left–right [L–R] steepness) on voltage difference. The stationary electrophoretic serotonin gain Rs(x) within the embryo also follows an exponential function. It is shown here for the same voltage differences as in Figure 1. The morphogen gain measures how much more serotonin we expect to find within the embryo compared with the left side. Thus, Rs(x = 0) = 1 at the left side, and increases toward the right side. The morphogen’s right–left gain Rs(x = L), that is, the maximum expected ratio of the serotonin concentration at the right side of the embryo with respect to the left side is approximately 2-fold, 5-fold, 10-fold, and so on, for a voltage difference of ~10 mV, ~20 mV, and ~30 mV, respectively. Medial circumferential length indicates position along the path from L to R border in an embryo roughly 1 mm in diameter (see Fig. 2).
Fig. 1. Proposed circumferential movement of morphogens through gap junctional communication (GJC) paths. A: In the animal pole tier of blastomeres of the 16–32 cell frog embryo, a ventral zone of junctional isolation (red line) exists around which the dorsal cells form a long-range cell path coupled by gap junctions. Both the barrier at the ventral midline and the open GJC path are required for normal left–right asymmetry (Levin and Mercola, 1998). Green line, path of junctional communication; blue conduits, gap junctions. B: A similar situation exists with respect to the primitive streak in the early chick blastoderm (Levin and Mercola, 1999). p.s., primitive streak. C: These observations suggested a model whereby asymmetry results from the net unidirectional movement of small molecule morphogens through the circumferential path; the gradient is schematized in blue:red in B and C, and the movement of the morphogen (schematized in yellow) is indicated by the red line in A. One possibility is that this movement is driven by left–right (LR) voltage differences generated across the zone of isolation by differential ion exchange of those cells with the outside medium. D–F: A maternal pool of serotonin is initially symmetrically (homogenously) distributed throughout the early embryo (D; Fukumoto et al., 2005b), but becomes present in a gradient (E), and eventually is restricted to one cell adjacent to the ventral embryonic midline (F). The spatial detection of serotonin was performed using immunohistochemistry (Levin, 2004a).

Fig. 2. Schematic of model of electrophoresis of small molecule morphogens. A,B: The animal tier of blastomeres in the frog can be modeled as a linear array of eight cells coupled by gap junctions (A), if the circular arrangement is visualized as being pulled apart at the ventral zone of isolation (B). We simulate the eight cells, connected by gap junctions, having at each end either the left (L) or right (R) blastomere (cell1, cell8) that terminates at the zone of isolation (red line). These end cells are different with respect to the ion pumps functioning within them; as a result of differential K+/H+ and H+/H+ transport with the outside medium (excess export of positive charges out of the right-most blastomeres), they acquire a consistently biased membrane voltage that is estimated to be 10–20 mV in the frog and chick systems. C: The path can be generalized as an open circuit driven by a battery, along which charged molecules can be electrophoresed.
neurons, was probed recently in chick and frog (Fukumoto et al., 2005a,b). Using analysis of endogenous localization of serotonin and its receptors, as well as gain- and loss-of-function experiments using pharmacological serotonin blockers and dominant-negative and wild-type expression constructs, it was shown that serotonin, and serotonergic signaling through receptor subtypes R3 and R4, were crucial for normal asymmetry. In early Xenopus embryos, serotonin is distributed in a striking radial pattern, beginning homogeneously distributed (Fig. 1D), but eventually forming a gradient (Fig. 1E) and coalescing into a single blastomere on the right side of the midline by the 32/64-cell stage (Fig. 1F), precisely as predicted in the original model (Fig. 1A; Levin and Nascone, 1997). While direct movement of serotonin through gap junctions in this system has not been demonstrated in vivo (serotonin is too small to be modified by fluorescent tags without significantly changing its properties), it has been shown that the ability of serotonin to localize asymmetrically through the GJ-connected blastomeres was dependent upon open gap junctions and upon the function of the H,K-ATPase, and V-ATPase ion pumps. Moreover, evidence was presented for a novel intracellular transducing mechanism for serotonin activity—this is a requirement of the model because serotonin arriving within the target cells’ cytosol has to be able to activate receptor mechanisms there.

Taken together, these data suggested a model that provides a possible answer to the chirality of the morphogen movement: that serotonin moves asymmetrically through the field of GJC-connected cells under an electrophoretic force provided by differential membrane voltages in cells at opposite ends of the circumferential cell field (Levin, 2003b; Levin et al., 2006).

RESULTS

Could It Really Work? A Quantitative Model for Electrophoretic Movement of Serotonin

Roles for electrophoretic movement of morphogens (Cooper, 1984; Cooper et al., 1989; Fear and Stuchly, 1998a–c) have been proposed previously, in the context of follicle–egg systems (Woodruff and Telfer, 1980; Telfer et al., 1981; Bohrmann and Gutzeit, 1987; Woodruff et al., 1988; Woodruff and Cole, 1997; Adler and Woodruff, 2000), self-electrophoresis in Fucus symmetry-breaking (Jaffe et al., 1974; Jaffe and Nuccitelli, 1977), and regeneration in both vertebrate and invertebrate systems (Rose, 1966, 1970; Smith, 1967; Lange and Steele, 1978). The data on LR patterning suggest a mechanism consistent with this class of models, as applied to a GJC-coupled cell field. Specifically, this mechanism is motivated by the dependence of corrected laterality on the GJC path (Levin and Mercola, 1998, 1999), the asymmetric localization of the proposed morphogen (Fukumoto et al., 2005a,b), and the differential biophysical properties of the cells on either side of the zone of isolation (Levin et al., 2002; Adams et al., 2006b). Of interest, theoretical analysis indicates that gap junctional coupling increases the sensitivity of cells to electric fields produced in their milieu (Cooper, 1984; Cooper et al., 1989). However, having proposed this mechanism, we sought to answer the question: Are physiological values for parameters, biological values for parameters, biological values for parameters.

In the Nernst–Planck equation, $D_S$ and $I_S$ are serotonin's diffusion coefficient and electrical mobility respectively, which are related by the Einstein relation $D_S = RTU_v$, where $R$ is the gas constant and $T$ is the temperature of the medium in which the embryo is immersed. Furthermore, $\phi(x)$ is the electric potential as function of position. To gain initial insight, we first consider a hypothetical completely open path between the cells, representing the existence of so many GJ at the cellular interface that the diffusion coefficient $D_S$ has a constant value throughout the embryo along L. The voltage difference across the embryonic cell array $\Delta \phi$ is assumed to be constant in time and to vary linearly along the embryo's circumference such that the cytoplasmic electric field $E$ is given by

$$E = \frac{\partial \phi}{\partial x} = \frac{\Delta \phi}{L}$$

everywhere in the cell array of length $L = 1.5$ mm. This corresponds to an internal electric field of $E = 13.3$ V/m for the voltage difference of $\Delta \phi$. The Nernst–Planck equation

$$\frac{\partial c_S}{\partial t} = \frac{1}{z_S F} \frac{\partial I_S}{\partial x}$$

Here, $z_S$ is serotonin's +2 valence and $F$ is the Faraday constant. Thus, the time-dependent change of the serotonin concentration is brought about by spatial gradients in the serotonin current density $I_S$. The total serotonin current density, in $\text{A/cm}^2$, contains both a diffusive component and an active component (Ohm’s law), that is

$$I_S = -z_S F \left[ D_S \frac{\partial c_S}{\partial x} + u_{\text{gel}} \zeta_{\text{gel}} F c_S \frac{\partial \phi}{\partial x} \right].$$

In the Nernst–Planck equation, $D_S$ and $U_{\text{gel}}$ are serotonin's diffusion coefficient and electrical mobility respectively, which are related by the Einstein relation $D_S = RTU_{\text{gel}}$, where $R$ is the gas constant and $T$ is the temperature of the medium in which the embryo is immersed. Furthermore, $\phi(x)$ is the electric potential as function of position. To gain initial insight, we first consider a hypothetical completely open path between the cells, representing the existence of so many GJ at the cellular interface that the diffusion coefficient $D_S$ has a constant value throughout the embryo along L. The voltage difference across the embryonic cell array $\Delta \phi$ is assumed to be constant in time and to vary linearly along the embryo's circumference such that the cytoplasmic electric field $E$ is given by

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molecular mass has no influence on this stationary profile, but is important for the time constant of its establishment, as will be discussed later. As such, higher-valenced morphogens with presumably higher molecular masses theoretically may be too slow to establish a significant gradient on the relevant time scales of embryonic development, and there may be an overall optimum in the choice of valence, voltage difference, and morphogen mass.

The Nernst equation used throughout the biophysical literature defines an electric field from a stationary concentration ratio of two cellular compartments. The situation is reversed here as the morphogen profile in the embryo exhibits a series of different serotonin concentrations in each individual cell, brought about by the voltage difference between the L and R blastomere. Rather than considering concentrations, it is more instructive to look at the electrophoretic serotonin gain, defined as the concentration ratio $R_S(x) = c_S(x)/c_S(0)$ inside the cell array. At the left side of the embryo (containing the lowest serotonin concentration), we have $R_S(0) = 1$, and, everywhere else, larger values ($R_S(x) > 1$) will be found as shown in Figure 5. It is a metric of how much more serotonin we expect to find at position $x$ with respect to the left side (at $x = 0$). The electrophoretic right–left gain $R_S(L)$ is a particular choice of that function,

$$R_S(L) = \frac{c_S(x = L)}{c_S(x = 0)} = \exp\left(\frac{z_S F \Delta \phi}{R T}\right)$$

and measures how much more serotonin can be found on the right side of the embryo with respect to the left side. As can be observed from Eq. (5), the final steady-state right–left gain depends only on the voltage difference $\Delta \phi$ and not on the system size $L$. The $Xenopus$ and chick embryos both support left–right voltage differences of approximately $\Delta \phi = -20$ mV, suggesting similar serotonin gradients for the stationary states. We consider a value of $R_S(L) = 10$ for the right–left gain as a significant spatial profile for embryonic development (Shvartsman et al., 2002; Pribyl et al., 2003) for which the patterned signal for embryonic development needs to decay by one order of magnitude over the developing field. For $z_S = +2$, a value of $R_S(0) = 10$ indeed occurs for a voltage difference of $\Delta \phi = -29$ mV. This value is remarkably close to the measured $\Delta \phi = -20$ mV.

The time-dependent solution of Eq. (1) has a lengthy analytical solution, but with regard to the later more important involvement of GJ also may be found from a numerical integration. Starting from a homogenous serotonin profile (Fig. 1D) throughout the system at $t = 0$, Figure 6A shows the time-dependent change of the serotonin profile for the measured voltage difference of $\Delta \phi = -20$ mV. With the model parameters given in Table 1, it takes approximately 1 hr to reach the stationary state given by Eq. (4). This is consistent with the developmental time available to reach the 32-cell stage, at which time the serotonin localization is complete (Levin and Mercola, 1998; Fukumoto et al., 2005b).

Figure 6B shows the time-dependent evolution of the right–left gain $R_S(L)$ for both the serotonin diffusion coefficient $D_S$ as well as for a comparative didactic study with a smaller diffusion coefficient of $D_S/\sqrt{5}$. The latter uses a $D \propto 1/M$ relationship between the diffusion coefficient and the molecular mass $M$ (Weiss, 1996), which is valid for GJ-permeable molecules ($<1$ kDa) and is meant to represent a hypothetical molecule with five times the molecular mass of serotonin. The stationary serotonin profile in that case would be established only after approximately 3 hr. The time constant $\tau_{\text{stat}}$ to reach the stationary state is diffusion-limited and, hence, scales with $L^2$, which for the chick embryo is a factor of 25 longer time scale compared with $Xenopus$. The stationary morphogen profile, although expected to be equal in both systems as discussed above, is reached in the model only after 25 hr in the chick embryo, which is longer than the time available for the LR patterning step in the chick embryo (Levin, 1998; Raya and Izpisua Belmonte, 2004; Raya et al., 2004). The $\tau_{\text{stat}} \sim L^2/D_S$ scaling law is a lower temporal bound, as GJC can only further delay the formation of the gradient. In particular, $\tau_{\text{stat}}$ depends on the number of GJC sites in the embryo (or equivalently on the number of cells, of which there are approximately 60,000 in the chick embryo). Thus, we expect from this analysis only a weak serotonin gradient in the chick gradient after the available time
TABLE 1. Serotonin, Gap Junction, and Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diffusion constant $D_S$</td>
<td>$3 \times 10^{-10}$ m$^2$/sec (Mastro et al., 1984)</td>
</tr>
<tr>
<td>Serotonin valence $z_S$</td>
<td>+2 (Kema et al., 2000)</td>
</tr>
<tr>
<td>Serotonin radius $r_S$</td>
<td>0.3 nm</td>
</tr>
<tr>
<td>Serotonin molecular mass $M_S$</td>
<td>175 Da</td>
</tr>
<tr>
<td>Voltage difference $\Delta \phi$</td>
<td>20 mV</td>
</tr>
<tr>
<td>Gap junction length $l_{GJ}$</td>
<td>1.6 nm</td>
</tr>
<tr>
<td>Gap junction pore radius $a_{GJ}$</td>
<td>0.6 nm</td>
</tr>
<tr>
<td>Gap junction density $n_{GJ}$</td>
<td>$6 \times 10^{11}$ m$^{-2}$ (Hanna et al., 1980)</td>
</tr>
<tr>
<td>System length $L$</td>
<td>1.5 mm (medial circumference, based on $-1$ mm embryo diameter, calculated as $L=2\pi(d/4)=1.5$ mm)</td>
</tr>
<tr>
<td>Temperature $T$</td>
<td>293 K</td>
</tr>
<tr>
<td>Interfacial area $A_{\text{int}}$</td>
<td>$1.5 \times 10^{-5}$ m$^2$</td>
</tr>
</tbody>
</table>

of 10 hr. In striking agreement, a serotonin gradient has not been observed experimentally so far in the chick embryo (Fukumoto et al., 2005b). Thus, if the system size is too large, then the stationary state given by Eq. (5) may not be reached in the available time. However, the most important metric probably is whether $R_S$ reaches a critical (yet unknown) value $R_{\text{critical}}$, sufficient for triggering the onset of asymmetric LR gene expression.

What is the relevance of GJC to the spatiotemporal generation of the morphogen profile? Without GJs, each cell is isolated and the serotonin profile evolves only over each individual cell but not over the whole embryo. GJs open the path for morphogens to be electrophoretically transported between the cells (Safranyos and Caveney, 1985; Safranyos et al., 1987). With GJC, the time course of establishing the stationary serotonin profile is delayed in comparison with the completely open situation discussed in Figure 6 because of finite GJ density and hindered serotonin diffusion through a GJ pore. But how much longer, and what properties of the GJ are the most relevant for this delay? For simplicity, we use here a single generic connexin-specific GJ, which exhibits $\gamma_{GJ}$ values in the range from 30 pS to 300 pS (Hille, 2001). The serotonin current density through a single GJ in our model is given by the Goldman–Hodgkin–Katz equation (Veenstra, 2000)

$$i_{GJ} = \frac{z_F^2 F D_S A_{\phi_{GJ}}}{l_{GJ} RT} \left[ e^{z_F F \Delta \phi_{GJ}/RT} - 1 \right], \tag{7}$$

which follows from Eq. (2) under the simplifying assumption of a symmetrical membrane without any internal structure along the GJ pore. As such, the ionic mobility and the electrochemical potential can be assumed to be linear over the diffusional distance. Thus, the voltage difference seen by the serotonin concentrations on the left ($c_l$) and right ($c_r$) side of the GJ is $\Delta \phi_{GJ} = E d_{GJ} = 21$ mV. This is an extremely small value with respect to voltage ranges at which nonlinearities and rectifying properties of GJ are typically observed (Veenstra, 2000). The serotonin partition coefficient from cytosol into the pore of the GJ is assumed to be unity. However, we take into account the frictional steric hindrance of the serotonin molecule.
with radius $r_s$ within a finite-size GJ pore. For $\lambda = r_s/r_{GJ}$, we adopt from Levitt (Levitt, 1991)

$$D_{S,GJ} = \frac{(1 - 2.1054\lambda + 2.0805\lambda^3)}{1 - 0.75857\lambda^5}$$

and find for the serotonin radius of $r_s = 0.3$ nm a reduction of the cytosolic diffusion coefficient in the GJ of $D_{S,GJ} = 0.17 D_S$. The total gap junction current density $I_{S,GJ}$ through the cell-to-cell interfacial area $A_{cc}$ is given by

$$I_{S,GJ} = N_{GJ} A_{GJ} P_0.$$  

Here, the total number of gap junctions $N_{GJ}$ is found from the observed GJ density $n_{GJ}$ (see Table 1) and the interfacial area, and $P_0$ is the probability to find the gap junctions to be open. For simplicity, we assume $P_0 = 1$ in our model. Many specific and important details of gap junction transport such as ionic selectivity, ionic impedance, and the internal structure of the GJ pore are neglected in this simplified description, in which we seek an educated initial understanding of the spatio-temporal properties of the morphogen distribution within the whole embryo, and ask if these results are plausible with respect to the experimental observations.

Figure 7A shows the spatiotemporal dependence of the serotonin concentration as it is influenced by GJC. The profile appears ragged initially, as the gap junctions represent a barrier through a lower effective cross-sectional area $N_{GJ} A_{GJ}$ compared with
As a result, the serotonin gradient in each individual cell is stronger at earlier times compared with a hypothetical completely open cytoplasmic syncytium (Fig. 8), but the gradient over the whole embryo is weaker. For later times, we find a smoothening of the embryonic serotonin profile toward the stationary profile described by Eq. (4). In total, the creation of the stationary profile is delayed compared with the completely open situation. This retardation depends strongly on the total number of GJ at each cellular interface, as detailed in Figure 7B,C, and thus represents a significant bottleneck for the embryonic system. Different values for the gap junctional density n\textsubscript{GJ} can be found or inferred from the literature (see Table 1). Two examples are shown in Figure 7B. For N\textsubscript{GJ} = 105 at the cellular interface, a value estimated by counting GJ plaques on electron microscopy pictures (Hanna et al., 1980), the stationary profile is obtained in 3 hr. This is a plausible time scale in accordance with experimental observations. For N\textsubscript{GJ} = 3 \times 10^3, a number estimated by total conductance measurements between amphibian blastomeres (Spray et al., 1981a), the profile is not reached in 10 hr and appears implausible. Figure 7C shows the right–left gain at 2 hr as a function of the GJ number over 4 orders of magnitude. The right–left gain exhibits nonlinear behavior. It starts from R\textsubscript{GJ}(L) = 1.22 for a negligibly small number of GJs. The marginal gain in that case is the consequence of the morphogen gradient in the individual cell and can be calculated from Eq. (5) for the intracellular voltage difference Δφ\textsubscript{cell} = −2.5 mV. For larger GJ numbers between N\textsubscript{GJ} = 10^3 to N\textsubscript{GJ} = 10^5 at the cellular interface, R\textsubscript{GJ}(L) increases rapidly and then for N\textsubscript{GJ} > 10^4 saturates at the completely open right–left gain.

These results indicate that an electrophoretic mechanism acting upon a GJ-coupled cell field can produce a rich but temporally limited set of distinct local gradients (Figs. 7, 8). The GJs initially hinder the electrophoretic serotonin redistribution such that the gradient across each GJ interface has the opposite sign compared with the intracellular gradients. However, while evolving toward the time-independent smooth steady-state distribution, a sign-reversal occurs and the transcellular gradients across the GJs of two neighboring cells exhibit a value in-between the connected two intracellular gradients. The existence of serotonin gradients across individual cells that differ from the overall transembryonic gradient is a fascinating consequence of this model and supports a previous conjecture that gradients in neurotransmitter concentrations across cells can provide directional signals to cells through differential activation of receptors and transporters on opposite sides of the cell (Levin et al., 2006).

**DISCUSSION**

The present model is a first step toward a quantitative understanding of biophysical processes involved in the embryonic pattern formation. In particular, we quantified the spatiotemporal serotonin profile, which is expected to result from the measured voltage difference between the left and right sides. The influence of GJ in the development of the morphogen gradient is explored. GJs are the key for mediating morphogen signals between adjacent cells and creating a robust and significant morphogen profile within the embryo. In particular, the rate at which the morphogen profile can be established depends strongly on the number of GJs at the cellular interface.

This model reveals that, given reasonable estimates for all of the biophysical constants, a left–right gradient plausibly can be created by the proposed mechanism. Some of the consequences of this model have already been confirmed. For example, it predicts that the L and R sides of the zone of isolation should have different membrane voltage levels. This prediction further suggests that introducing gap junctions in the isolation zone should short-circuit the battery and, thus, randomize the LR axis. Both of these have been observed experimentally (Levin and Mercola, 1998; Levin et al., 2002). This model makes several other predictions that will be tested in the context of left–right patterning in the future, because the amount and distribution of serotonin, gap junctions, and ion transporters can be experimentally varied in this system. However, the analysis presented here can be used to model many other signaling systems in addition to serotonin; for example, other small molecules such as cAMP, ATP, and so on are known to pass through gap junctions (Cotrina et al., 1998, 2000; Braet et al., 2003; Bao et al., 2004) and have been suggested as candidate morphogens participating in the establishment of positional information (Schiffmann, 1989, 1991).

This analysis also provides an explanation for the extraordinary robustness (Eldar et al., 2002; England and Cardy, 2005; Houchmandzadeh et al., 2005; Mizutani et al., 2005) of the left–right system. Under normal conditions, less than 1% of frog embryos has any laterality defect, and this proportion is stable against a large number of pharmacological, physiological, and surgical manipulations (Nascone and Mercola, 1997; Levin et al., 2002). Indeed, short pulses of reagents that functionally interfere with the known early steps do not result in randomization, revealing the ability of the system to regulate and restore LR signals when perturbed (Levin and Mercola, 1998). Our results indicate that the magnitude of the final right–left gradient gain for a specific morphogen depends only on the voltage difference, not on the number of GJs, two-dimensional (2D) geometry, or on an assumption of a constant electric field everywhere in the path. This independence of a large number of parameters and molecular details reveals a novel method of generating a gradient that does not rely on feedback loops or transcriptional cascades for robustness and stability (Freeman and Gordon, 2002; Eldar et al., 2003, 2004).

**Comparison to Other Morphogen Gradients**

Our model takes a somewhat wider view of the concept of “morphogen,” because in this system, serotonin is likely to signal to downstream events in early blastomeres before the generation of specific “cell types” by differential (Ashe, 2006 #8011). Nevertheless, it is useful to compare the gradient produced by the electrophoretic system to morphogen behavior in other systems to ask what magnitude of gradient is likely to be
developmentally relevant. While quantitative studies of morphogen distributions are not many, one molecule presents itself as a good candidate for comparison. Auxin, a plant hormone that bears a striking similarity to serotonin, is a positional signal in several plant patterning events (Sabatini et al., 1999; Vroemen et al., 1999; Baluska et al., 2003; Friml, 2003; Barlow, 2005) and is involved in establishment of bilateral symmetry in plants (Lee and Evans, 1985; Liu et al., 1993; Zgurski et al., 2005). Indeed, many of the same players that have been implicated in left–right asymmetry (K⁺ channels, plasma membrane H⁺ flux, cell membrane transporters, and gradients of serotonin/auxin, regulatory roles of pH, fuscoccin, etc.) are now known to be crucial components of auxin signaling (Arend et al., 2002; Coenen et al., 2002; Pasternak et al., 2002; Hager, 2003; Rober-Kleber et al., 2003; Wind et al., 2004). As with serotonin, there is thought to be a soluble auxin receptor (Dharmasiri et al., 2003; Woodward and Bartel, 2005a,b). Most interestingly, auxin has been suggested to move under electrophoretic control (Goldsworthy and Rathore, 1985; Rathore and Goldsworthy, 1985; Rathore et al., 1988; Rathore and Robinson, 1989; Wang et al., 1989; Rathore et al., 1991; Fischer et al., 1997; Fischer-Iglesias et al., 2001; Rober-Kleber et al., 2003).

Paponov et al. (2005) proposed an auxin-flow canalization hypothesis: auxin flow, starting by diffusion, induces formation of polar transport system. This system in turn promotes auxin transport, leading to canalization of auxin flow along a narrow column of cells, ultimately controlling differentiation. This kind of positive feedback amplification loop (Jaffe et al., 1997; Fischer-Iglesias et al., 1998, 1999; Rathore et al., 1991; Fischer et al., 1997; Fischer-Iglesias et al., 2001; Rober-Kleber et al., 2003).

Testing the Model: Future Prospects

The model developed above is a starting point to understanding early developmental physiology. For the first time, it enables specific testable predictions to be made regarding the distribution of serotonin and ultimate laterality following perturbations such as manipulation of GJC levels, voltage gradient, serotonin levels, etc. It can also provide useful analysis of other morphogens that may be proposed by future discoveries. The model will be refined by subsequent work; direct measurements still needed are the density of gap junctions at blastomere interfaces, and the quantification of serotonin levels in each blastomeres as a function of time. Moreover, several layers of complexity and feedback loops need to be incorporated. The ability of serotonin to localize to a specific blastomere depends on GJC and H,K-ATPase function, which may provide an electrotropic force for moving charged small molecules (such as serotonin) between cells (Levin, 2004b). Conversely, several ion transporters are controlled by 5HT, most notably 5HT-R3 and SERT (Mariq et al., 1991; Quick, 2003).

Additionally, the increased understanding of pH gradients in embryos (Turin and Warner, 1980; Gillespie and McHanwell, 1987; Gillespie and Greenwell, 1988; Guthrie et al., 1988; Dickens et al., 1989; Grandin and Charbonneau, 1991) and their contribution to GJC in LR patterning (Adams et al., 2005b) must be included, as must possible unidirectional gap junctions (Robinson et al., 1993; Xin and Bloomfield, 1997). These increased layers of complexity can be accommodated in the model through the $P_0$ parameter in Eq. (9).

The next important step will be to develop a detailed model that includes the activity of the ion pumps responsible for the generation of the voltage gradient and the complex inter-relation between $P_0$ and membrane voltage, and then to self-consistently solve for the generation of the morphogen profile. This can be approached using recently developing modeling methods (Gowrishankar and Weaver, 2003; Gowrishankar et al., 2006),
which are straightforwardly extendable to the creation of more realistic 2D and 3D models of embryonic morphology. pH is a crucial component, because gap junctions are sensitive to intracellular pH (Spray et al., 1981b; Francis et al., 1999); however, asymmetry in Xenopus is stable against changes in pH\textsubscript{int} ranging from 11 to 5 (Adams et al., 2006a), providing additional opportunities for exploring robustness aspects of the electrophoretic mechanism.

The model makes several specific predictions. First, the influx of the LR morphogens into cells (as distinct from the well-understood serotonin receptor activation on the cell surface) must be able to control cell fate. This appears to be borne out in other systems, as the function of the SERT importer is involved in tumor growth (Nordenberg et al., 1999; Serafeim et al., 2002), as is serotonin (Dizeyi et al., 2004). An important aspect is the identification of gene transcription modulated by the intracellular arrival of GJC-dependent morphogens into cells. Although these studies are just beginning in our laboratory in the context of the asymmetric gene cascade, others have begun to characterize, using transcriptome analysis, the genetic targets of gap junction-mediated signaling (Iacobas et al., 2003, 2004, 2005; Kim et al., 2005). It is now abundantly clear that a large number of important gene networks are potentially controlled by GJC signaling and these networks represent excellent targets for future molecular dissection. For example, recent analysis links the expression of Syndecan-2 with connexin43 in mouse embryos (Iacobas et al., 2005), which is the first glimpse of conservation of regulatory circuits among the known roles of GJC and Syndecan-2 in lower vertebrates and mammals in establishment of left–right asymmetry (Kramer et al., 2002; Kramer and Yost, 2002; Fukumoto and Levin, 2005). The model predicts that microarray analysis of Xenopus embryos receiving intracellular injections of serotonin on the ventral side will reveal genes up- and down-regulated by this signal. Such transcripts will be an essential link between the serotoninergic morphogen and downstream asymmetric gene cascades (Levin, 1998).

In light of the current controversy over early LR mechanisms, this class of models needs to be extended to asymmetry in the chick, which is more similar to the embryonic architecture of most mammals, and where the cells are far smaller but more numerous than the early frog blastomeres. The architecture of chick embryos, and the details of serotonin localization, differ significantly between frog and chick (Fukumoto et al., 2005b), providing a fertile ground for assessing the generality of this model within the evolution of developmental mechanisms. Indeed, even among amphibian embryos, early embryo sizes can vary sharply (Berger and Roguski, 1978; Ninomiya et al., 2001; Rasanel et al., 2005). Our model (Eq. 5) suggests that the magnitude of the final left–right gain at a given voltage is independent of system size; however, starting initially from a homogenous morphogen distribution, the time scale on which the final right–left gain is reached does depend on the system size. Furthermore, the generation of a constant voltage gradient in much larger systems will be progressively more difficult. Thus, the investigation of the timing, voltage gradient, and final serotonin distribution in other amphibians provides an important opportunity for testing this class of models.

As this field matures, the model must incorporate more details about the ion flux at the ends of the zone of isolation, to be able to simulate the bidirectional relationship between 5HT movement in an electric field and the regulation of ion transporters (and the resulting electromotive force across the field) by serotonin. The resulting ion flux and field gradient distribution’s effect on GJC permeability should also be incorporated. Such models, when augmented by empirical testing, will allow a deep understanding of whether the activation of asymmetric ion flux by serotonin and the unidirectional movement of serotonin and other morphogens due to an electrophoretic force, thus, may be a positive-feedback loop that could magnify small asymmetries on the cellular level into asymmetry on the scale of cell fields. Other possible developmental systems where such models may be relevant include Drosophila imaginal disks and the chick limb (Weir and Lo, 1984; Coelho and Kosher, 1991). The proposed electrophoretic mechanism may also be related to dipolar alignment of long, thin molecules larger than the usual 1-kDa limit to facilitate their passage through gap junction channels (Woodruff, 2005) and will have to incorporate knowledge of the specific types of GJ present in a given tissue as well as the nature of putative morphogens, as radically different permeabilities of gap junctions to different types of molecules have been reported (Bevans et al., 1998; Goldberg et al., 1999, 2002; Nicholson et al., 2000). Ultimately, it will be necessary to incorporate not only upstream mechanisms responsible for generating the voltage gradients but also downstream mechanisms by which gradients are sensed by cell fields and transduced into stable cascades of differential gene expression. Detection of such biological signals at thresholds in light of developmental noise represent future efforts of considerable general importance (Weaver et al., 2000; Eldar et al., 2003).

Significant molecular genetic work remains to be done to address the fascinating and likely widely-relevant question of how intracellular movement of small signaling molecules couples to transcription cascades, such as the Nodal–Lefty–Pitx cassette known to be a conserved element of asymmetry in vertebrates (Yost, 2001; Palmer, 2004). Consistent with our model, which relies on the cytoplasmic activity of the LR morphogen, there is some evidence for a novel intracellular receptor of serotonin (Tamir and Gershon, 1981; Fukumoto et al., 2005b). The molecular machinery responsible for linking cytoplasmic serotonin to downstream activations of gene expression is an important area of future investigation and may reveal additional candidates for electrophoretic movement of morphogenetic signals.

This electrophoretic signaling system is a powerful and versatile patterning mechanism. Its analysis is the first step in providing quantitative, testable, extensible models that can synthesize the available molecular and physiological data in a wide range of systems. These data provide specific hypotheses that will drive exciting experimental work as patterning roles of gap junctions continue to be uncovered.
EXPERIMENTAL PROCEDURES

The time-dependent continuity Eq. (1) together with the Nernst–Planck Eq. (2) represent a partial differential equation (PDE), which can be solved by standard numerical methods (Press et al., 1994). A discretization of time and space allows representation of the PDE in a Finite Difference Scheme, in which the first (time and space) and second (space) derivatives of the serotonin concentration can be expressed. We use a nonhomogeneous discretization in space with a small spacing $\Delta_x = l_{GA}$ across the gap junctional region and a larger spacing $\Delta_x >> l_{GA}$ in the intracellular space.

The time step $dt$ is basically determined by a stability criterion and that criterion is related to the discretization size in physical space. We typically use $dt < 0.1$ sec. When the simulation is run for 3 hr (that is the real physical time for the events described in the model), 3 hr/$dt$ iteration steps are needed. The real time it takes the computer to simulate those 3 hr is only a couple of minutes on a standard workstation.

The simulation starts off at $t = 0$ with a homogenous serotonin distribution, and iteratively evolves in time until the final stationary serotonin state is reached. The serotonin concentration is saved at many time points in-between and thus allows display of the time-dependent change of the distribution in Figures 6–8.

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REFERENCES


